Probability ACSAI 2023-24
L. Bertini and V. Silvestri

Week 1

Exercise 1. Prove the following identities between sets:

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$
$$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$
$$(A \cap B)^{\mathrm{c}} = A^{\mathrm{c}} \cup B^{\mathrm{c}}$$

Exercise 2. Let A, B be two sets. Prove that the following statements are equivalent:

i) $A \subset B$
ii) $A \cap B = A$
iii) $A \cup B = B$

Exercise 3. Let A, B and C be sets, not necessarily disjoint. Write $|A \cup B \cup C|$ in terms of the cardinalities of A, B, C and of their intersections.

Exercise 4. After interviewing 50 students, the following data are collected: 25 students studied French, 20 studied German and 5 studied both languages. Compute:

1) the number of students who only studied French,

2) the number of students who only studied German,

3) the number of students who did not study any of the two languages.

Exercise 5. After interviewing 60 people, the following data are collected: 25 people read Mickey Mouse, 26 people read Tex, 23 people read Diabolik. Moreover, 9 people read both Mickey Mouse and Tex, 11 people read both Mickey Mouse and Diabolik, 8 people read both Tex and Diabolik. Finally, 3 people read all the three journals. Compute:

1) how many people read only Mickey Mouse,

2) how many people read only Tex,

3) how many people read only Diabolik,

4) how many people read at least one journal,

5) how many people read exactly one journal,

6) how many people read none of the journals.

Exercise 6. Let $(\Omega, \mathbb{P})$ be a probability space, and let A, B and C be events. It is known that $A \cap B \cap C = \emptyset$, $\mathbb{P}(A \cap C) = 1/5$ and $\mathbb{P}(B \cap C) = 2/5$.

1) Compute $\mathbb{P}((A \cup B) \cap C)$

2) What are the possible values of $\mathbb{P}(A \cap B)$? (For example, can it be $\mathbb{P}(A \cap B) = 1$?)

Exercise 7.

1) If $\mathbb{P}(A) = 1/3$ and $\mathbb{P}(B^c) = 1/4$, can A and B be disjoint events?

2) If $\mathbb{P}(A) = 1/4$ and $\mathbb{P}(A \cup B) = 3/4$, what is the value of $\mathbb{P}(B)$ when A and B are disjoint?

3) If $\mathbb{P}(A) = \mathbb{P}(B) = 3/8$, can it be the case that $\mathbb{P}(A \cup B) = 1/4$? How about $\mathbb{P}(A \cup B) = 7/8$?

4) Let $\mathbb{P}(A) = 3/4$ and $\mathbb{P}(B) = 3/8$. Check that $1/8 \leq \mathbb{P}(A \cap B) \leq 3/8$.

5) Prove the following inequality:

$$\mathbb{P}(A \cap B) \geq \mathbb{P}(A) + \mathbb{P}(B) - 1$$

Exercise 8. Two fair dice, one red and one blue, are tossed.

1) Describe the sample space Ω.

2) Describe the following events as subsets of Ω:
- the red die gives 5
- one of the dice gives 5
- both dice give 5
- no die gives 5
- the sum of the two dice equals 5.

3) Compute the probability of the above events.