From Optimal Control to Maximum Principle

Agropoli, September 12 - 14, 2018

Keynote Speakers

Yves Achdou (Université Paris-Diderot)

Optimal control of conditioned processes

Abstract: We consider a class of optimal control problems of conditioned processes. This topic has been first introduced by P-L Lions in his lectures at Collge de France. The necessary optimality conditions, reminiscent of mean field games, consist of a system involving a non local Fokker-Planck equation (forward in time) with Dirichlet conditions and a non local Hamilton-Jacobi-Bellman equation (backward in time) with Dirichlet conditions. A large part of the material in the present talk comes from the lectures of P-L Lions. We will study in details particular aspects of the theory and discuss numerical simulations. In particular, we will address a class of control problems driven by the principal eigenvalue problem associated with a Fokker-Planck equation with Dirichlet conditions, which arises naturally in the long time asymptotics.

Henri Berestycki (EHESS)

Predators-prey model with competition: emergence of territoriality in animal behavior

Abstract: In this talk I report on joint work with Alessandro Zilio. It concerns a new predator-prey model of Lotka - Volterra type that aims at showing how territories and packs are formed among certain predators. This model rests on basic principles of predators? prey interaction and competition. In mathematical terms, we focus on asymptotic regimes, especially the one when the competition parameter becomes large.

Xavier Cabré (ICREA and UPC - Barcelona)

Nonlocal minimal cones and a gradient estimate for nonlocal minimal graphs

Abstract: The talk will be concerned with s-minimal surfaces, that is, hypersurfaces of $\mathbb{R}^n$ with zero nonlocal mean curvature. These are the equations associated to critical points of the fractional s-perimeter. We prove that half spaces are the only stable s-minimal cones in $\mathbb{R}^3$ for s sufficiently close to 1. We will then turn to nonlocal minimal graphs in any dimension, for which we establish a gradient estimate. It leads to their smoothness, a result that was only known for n=1,2 (but without a quantitative bound); in higher dimensions only their continuity had been established.

Pierre Cardaliaguet (Université Paris Dauphine)

First order Mean Field Games with state constraints

Abstract: We study first order Mean Field Game (MFG) problems (optimal control problems with infinitely many small agents) in which each agent has a constraint on its state. This class of problems appears naturally in the applications (pedestrian models, macroeconomics, etc...). In contrast with the unconstrained model, the density of the agents develops singularities and the generic uniqueness of the optimal trajectories no longer holds. It is however possible to prove the some regularity for relaxed equilibrium configurations and to describe them in terms of an (almost) standard MFG system. Joint work with P. Cannarsa and R. Capuani.

Ivar Ekeland (Université Paris-Dauphine)

A new approach to regularity theory for systems of PDEs

Abstract: Traditionally regularity is proved by finding weak solutions and then proving that they are smooth. Instead, I will work directly in the (Fréchet) space of indefinitely differentiable function, and prove existence in this space

Françoise Demengel (Université de Cergy-Pontoise)

Extremal functions for an embedding from some anisotropic space, involving the "one Laplacian"

Abstract: In this lecture i propose first an overview of problems linked to anisotropic Sobolev spaces and anisotropic pde. In a second time i will

develope the existence of Extremal functions for the Sobolev embedding from $W^{1,\vec{p}}(\mathbb{R}^N)$ into $L^{p^\star}(\mathbb{R}^N)$, where $p_i = 1$ for some $i \in [1, N]$ and $p^\star$ is the Sobolev critical exponent defined as

$$\frac{N}{p^\star} = \sum \frac{1}{p_i} - 1$$

Cyril Imbert (École Normale Supérieure, Paris)

Decay estimates for large velocities in the Boltzmann equation without cut-off

Abstract: I will present a joint work with C. Mouhot and L. Silvestre concerned with establishing pointwise decay estimates in the velocity variables for solutions of the Boltzmann equation without cut-off under the assumption that mass, energy and entropy are under control.

Hitoshi Ishii (Tsuda University)

The vanishing discount problem for Hamilton-Jacobi equations in Euclidean n space

Abstract: I will present recent joint work with A. Siconolfi which concerns the vanishing discount problem for Hamilton-Jacobi equations in Euclidean n space. Under appropriate assumptions, which, in particular, imply the compactness of the (projected) Aubry set of the associated ergodic problem, the convergence of the whole family of solutions of the discount problems, as the discount factor tends to zero, is established.

Pierre-Louis Lions (Collège de France and Université Paris Dauphine)

On Mean Field Games (again!)

Abstract: We discuss some very recent developments in MFG theory and Models.

Umberto Mosco (Worcester Polytechnic Institute)

On a fully discrete sand-pile-type model for self-organized-criticality

Abstract: Certain large dissipative system have an intrinsic property: they drive themselves to a critical state which acts as a finite time attractor of the

dynamics. These phenomena are generically referred to as self-organized-criticality. Computational evidence of this behavior was first brought to evidence in cellular automata models for avalanches and sand piles on two-dimensional finite regular lattices by Bak, Tang, Wiesenfeld and by Dahr and Creutz in the late eighties and early nineties. Soon afterwards the automata models were described as discrete nonlinear difference equations by Bantay-Janosi and Carlson-Swindle. The theory was then carried out in continuous variables by V. Barbu in 2009-2010, as a multi-valued nonlinear diffusion equation in $\mathbf{R}^2$. He proved that the supercritical region is absorbed in finite-time into the critical one. The theory was further developed in 2009-2016 by Barbu, Da Prato, Röckner, Gess in the framework of stochastic nonlinear pdes. In our talk we shall present a new model for self-organized-criticality. Our model is purely discrete and it is set as a toppling process on synchronized infinite grids in space and time. The process is described by a system of nonlinear difference equations of impulsive diffusion type. The purpose of synchronization is to implement the principle that spatial interactions occurring at shorter and shorter range, as in sand piles and avalanches, are best observed at faster and faster ticking times. We prove that the process comes to an end in a finite time and we obtain an a-priori estimate of the finite extinction time in terms of the data. Our analytic theory bridges the gap between the finite-lattice cellular automata of physics and the continuum mathematical models based on PDEs.

Tristan Rivière (ETH Zurich)

The Cost of the Sphere Eversion and the 16π *Conjecture*

Abstract: How much does it cost...to knot a closed simple curve? To cover the sphere twice? to realize such or such homotopy class? ...etc. All these questions consisting of assigning a "canonical" number and possibly an optimal "shape" to a given topological operation are known to be mathematically very rich and to bring together notions and techniques from topology, geometry and analysis. In this talk we will concentrate on the operation consisting of turning inside out the 2 sphere in the 3 dimensional space. Since Smale's proof in 1959 of the existence of such an operation the search for effective realizations of such eversions has triggered a lot of fascination and works in the math community. The absence in nature of matter that can interpenetrate and the quasi impossibility, up to the advent of virtual imaging, to experience this deformation is maybe the reason for the difficulty to develop an intuitive approach on the problem. We will present the optimization of Sophie Germain conformally invariant elastic energy for the

eversion. Our efforts will finally bring us to consider more closely an integer number together with a mysterious minimal surface.

Jean-Michel Roquejoffre (Université Paul Sabatier, Toulouse)

The logarithmic delay in Fisher-KPP type equations

Abstract: The Fisher-KPP (Kolmogorov, Petrovskii, Piskunov) equation is one of the simplest looking reaction-diffusion equations. In one space dimension, the solution starting from a Heaviside initial datum will converge, up to a nontrivial logarithmic time delay, to a travelling wave. This result was proved by Bramson in the early 80's, using elaborate probabilistic arguments. We will discuss a PDE proof of this result, and present some extensions, such as sharp asymptotics and multi-dimensional models.

Luca Rossi (EHESS)

The Landis conjecture for elliptic operators

Abstract: The Landis conjecture, proposed in the 80s, states that if a solution of the equation $\Delta u + V(x)u = 0$ decays faster than a suitable exponential then it must be identically equal to zero. It can be viewed as a unique continuation property at infinity (UCI). The conjecture has been disproved by Meshkov in the case of complex-valued functions, but it remains open in the real case. In this talk, I will recall some partial results obtained by Kenig and collaborators. Next, I will present the proof of the UCI and of the Landis conjecture for radial operators. Finally, I will discuss the validity of the UCI under the restriction on the sign either of the solution or of the generalized principal eigenvalue.

Panagiotis E. Souganidis (University of Chicago)

Qualitative properties of stochastic viscosity solutions

Abstract: I will present results about the qualitative properties of stochastic viscosity solutions, including domain of dependence and finite speed of propagation, intermittent regularisation, long time behavior and lack of regularity. All the results are joint work with P.-L. Lions; the work about the domain of dependence and speed of propagation is also joint with P. Gassiat and B. Gess.