Shult Sets and Translation Ovoids of $H(3, q^2)$

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A *Hermitian surface*, denoted by $\mathcal{H} = H(3, q^2)$, is defined to be the set of all isotropic points of a non–degenerate unitary polarity defined on $PG(3, q^2)$. The *generators* of $\mathcal{H}$ are the lines of $PG(3, q^2)$ lying totally on the surface, and the points of $\mathcal{H}$ together with its generators form a finite polar space (as well as one of the classical generalized quadrangles). The dual polar space is the five–dimensional elliptic quadric $\mathcal{Q}^-(5, q)$ of order q. An *ovoid* on $\mathcal{H}$ is a set of points having exactly one point in common with every generator of the polar space. In recent years many inequivalent ovoids of the Hermitian surface have been found, but very few "translation ovoids" are known. Here a *translation ovoid* is an ovoid admitting a collineation group which fixes one of its points, fixes all the generators incident with that point, and acts regularly on the remaining points of the ovoid.

In this talk we describe newly discovered infinite families of translation ovoids on the Hermitian surface, and various connections with locally Hermitian 1–spreads of $\mathcal{Q}^-(5, q)$ and semifield spreads of $PG(3, q)$. Our technique is to start with carefully chosen sets of points in the Desarguesian affine plane $AG(2, q^2)$, an idea first formulated by Ernie Shult.

Two previously known examples of translation ovoids on the Hermitian surface are the so–called classical and semiclassical examples, the first being simply a non–tangent planar section of the Hermitian surface, and the second being obtained by taking all the chords passing through some point of an embedded Baer elliptic quadric $Q^-(3, q)$ which is permutable with the Hermitian surface. We discuss how these examples arise from Shult sets in $AG(2, q^2)$, and mention some characterization results in the classical and semiclassical cases. The Klein correspondence will be used heavily throughout the talk.