

1 Calcolo di limiti

Calcolare i seguenti limiti:

$$1 \quad \lim_{n \rightarrow +\infty} \left(\frac{3n+5}{n+3} \right)^5$$

$$2 \quad \lim_{x \rightarrow +\infty} \frac{\sin x + x}{x^3 + 3}$$

$$3 \quad \lim_{x \rightarrow +\infty} \left(\frac{x^3}{x^2 + 1} - x \right)$$

$$4 \quad \lim_{x \rightarrow +\infty} x(\sin x - 2)$$

$$5 \quad \lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1}$$

$$6 \quad \lim_{n \rightarrow +\infty} \frac{3n^2 + 5n + 1}{5n^2 + 2n + 7}$$

$$7 \quad \lim_{n \rightarrow +\infty} \frac{1 + 2 + \dots + n}{n^2}$$

$$8 \quad \lim_{n \rightarrow +\infty} \frac{1 + 2^2 + \dots + n^2}{n^3}$$

$$9 \quad \lim_{n \rightarrow +\infty} \frac{2^n + (-1)^n}{2^n}$$

$$10 \quad \lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x^2 - 6x + 9}$$

$$11 \quad \lim_{x \rightarrow +\infty} \left(\sqrt{2+x^3} - \sqrt{1+2x^2+x^3} \right)$$

$$12 \quad \lim_{x \rightarrow +\infty} \left(\sqrt[3]{2+x^3} - \sqrt[3]{1+2x^2+x^3} \right)$$

$$13 \quad \lim_{x \rightarrow +\infty} \frac{\log_2(e^x + 1)}{x + \sin x}$$

$$14 \quad \lim_{x \rightarrow 0} \frac{\cos x^3 - 1}{\sin x^5}$$

$$15 \quad \lim_{t \rightarrow +\infty} \frac{\sqrt{t^6 + 1} (\cos \frac{2}{t} - 1)}{t}$$

$$16 \quad \lim_{x \rightarrow +\infty} \frac{\ln(1 + 2e^{x^3})}{x^3}$$

$$17 \quad \lim_{x \rightarrow 0} \frac{2 - 2 \cos x}{\sin x^2 - 3 \sin x - x^2}$$

$$18 \quad \lim_{x \rightarrow 0} \frac{\log_a(x+2) - \log_a 2}{x}$$

$$19 \quad \lim_{x \rightarrow 0} \frac{e^{\operatorname{tg}^3 x} - 1}{x (\cos x - e^{x^2})}$$

$$20 \quad \lim_{t \rightarrow +\infty} \frac{t^3 \log \left(1 + \frac{3}{t^2} \right)}{\sqrt{t^2 + 1}}$$

$$21 \quad \lim_{t \rightarrow +\infty} \frac{\sqrt{t^6 + 1} (\cos \frac{2}{t} - 1)}{t}$$

$$22 \quad \lim_{x \rightarrow 0} \frac{\log(1 + \operatorname{arctg} x)^x}{e - e^{\cos^4 x}}$$

$$23 \quad \lim_{x \rightarrow 0^+} (1 + \sin(x^\alpha))^{1/x} \text{ al variare di } \alpha > 0$$

$$24 \quad \lim_{x \rightarrow 0} \frac{(\cos x)^{\tan x} - 1}{x^3},$$

$$25 \quad \lim_{x \rightarrow +\infty} x^2 (e^{\frac{1}{x}} - 1)^{\frac{3}{2}}$$

$$26 \quad \lim_{x \rightarrow +\infty} x^{\frac{1}{3}} (e^{\frac{1}{x}} - 1)^{\frac{1}{2}}$$

$$27 \quad \lim_{x \rightarrow 0} \sin(x^{\frac{2}{5}}) \frac{\log(1 + x^{\frac{1}{5}})}{4x^{\frac{3}{5}}}$$

$$28 \quad \lim_{x \rightarrow +\infty} \sin\left(\frac{1}{x^{\frac{4}{7}}}\right) (\log(1 + \frac{e}{x^{\frac{2}{7}}}))^{-2}$$

$$29 \quad \lim_{x \rightarrow +\infty} \sqrt{x} \sqrt{\left(1 + \frac{1}{x}\right)^{\frac{1}{4}} - 1}$$

$$30 \quad \lim_{x \rightarrow 0} \frac{(1 + x^{\frac{3}{2}})^{\frac{2}{3}} - 1}{e^{x^{\frac{3}{2}}} - 1}$$

$$31 \quad \lim_{x \rightarrow 0} \frac{|4^x - 1 - (\log 2) \sin x|}{x^3 - |x|^2}$$

$$32 \quad \text{Determinare } c \text{ in } \mathbb{R} \text{ tale che } \lim_{x \rightarrow +\infty} \left(\frac{x+c}{x-c} \right)^x = 4.$$

$$33 \quad \lim_{n \rightarrow +\infty} \operatorname{tg} \frac{n\sqrt{n} + 1}{2n\sqrt{n} - \sqrt{n}}$$

$$34 \quad \lim_{n \rightarrow +\infty} \frac{n \sin n + 5}{n^2 + \sqrt{n}}$$

$$35 \quad \lim_{n \rightarrow +\infty} (2^n - n)$$

$$36 \quad \lim_{n \rightarrow +\infty} \frac{2^n + 4^n}{3^{n+1} + 5^n}$$

$$37 \quad \lim_{n \rightarrow +\infty} \frac{n2^n}{3^n}$$

$$38 \quad \lim_{n \rightarrow +\infty} \frac{n^2}{n^2 + (\log n)^{1000} - n \sin(n^{500})}$$

$$39 \quad \lim_{n \rightarrow +\infty} \frac{n \log n}{2^n}$$

$$40 \quad \lim_{n \rightarrow +\infty} e^n \operatorname{sen} \frac{1}{n^{100}}$$

$$41 \quad \lim_{n \rightarrow +\infty} \frac{n(1 - \cos \frac{1}{n})}{\operatorname{sen} \frac{1}{n}}$$

$$42 \quad \lim_{n \rightarrow +\infty} \frac{n+1}{\sqrt{n}+2} \operatorname{tg} \frac{1}{\sqrt{n}}$$

$$43 \lim_{n \rightarrow +\infty} \operatorname{tg} \frac{\pi n^2}{4n^2 - 5n}$$

$$44 \lim_{n \rightarrow +\infty} \operatorname{tg} \frac{\pi n^2}{2n^2 - 5n}$$

$$45 \lim_{n \rightarrow +\infty} (3n + 2) \operatorname{sen} \frac{\pi n - 5}{n + 7}$$

$$46 \lim_{n \rightarrow +\infty} (2n + 5) \cos \frac{\pi n^2 + 7}{2n^2 + n}$$

$$47 \lim_{n \rightarrow +\infty} \frac{n^4}{n + 1} \operatorname{sen} \frac{\log n}{n^4}$$

$$48 \lim_{n \rightarrow +\infty} \frac{(n + 1)^{n+1}}{(n + 2)^n} \operatorname{sen} \frac{1}{n}$$

$$49 \lim_{n \rightarrow +\infty} \left(1 - \frac{1}{n^2}\right)^n$$

$$50 \lim_{n \rightarrow +\infty} \left(1 - \frac{1}{n}\right)^{n^2}$$

$$51 \lim_{n \rightarrow +\infty} \left(1 + \sin^2 \frac{1}{n}\right)^n$$

$$52 \lim_{n \rightarrow +\infty} \cos^n \frac{1}{n}$$

$$53 \lim_{n \rightarrow +\infty} \cos^{n^2} \frac{1}{n}$$

$$54 \lim_{n \rightarrow +\infty} \left(1 + \sin \frac{1}{n}\right)^n$$

$$55 \lim_{n \rightarrow +\infty} (n^{\sqrt{n}} - 2^n)$$

$$56 \lim_{n \rightarrow +\infty} \sqrt{n} \left(\operatorname{sen} \frac{2}{\sqrt{n}} - \frac{1}{\sqrt{4n + 3}} \right)$$

$$57 \lim_{n \rightarrow +\infty} n \left(1 - \cos \frac{1}{\sqrt{n}} + e^{-n} \right)$$

$$58 \lim_{n \rightarrow +\infty} \left(\frac{\log(2n) + 1}{\log(2n) - 5} \right)^{e^n}$$

$$59 \lim_{n \rightarrow +\infty} \left(\frac{4 - 3 \log n}{2 - 3 \log n} \right)^{e^{n+2}}$$

$$60 \lim_{n \rightarrow +\infty} \left(\frac{1}{3} \log(n^2 + 1) - \log \sqrt[3]{n^2 + 2} \right) \operatorname{sen} n$$

$$61 \lim_{n \rightarrow +\infty} \frac{n + 1}{\sqrt{n}(n + 2)} (\operatorname{sen} n - \cos n)$$

$$62 \lim_{n \rightarrow +\infty} \left(\cos \frac{1}{n^2} \right)^{\sqrt{4n^8 + 7n + 3}}$$

$$63 \lim_{n \rightarrow +\infty} \frac{n}{\log n} \left(n^{1/n} - 1 \right)$$

$$64 \lim_{n \rightarrow +\infty} \frac{1}{n} (\sqrt{n^2 + 2} - \sqrt{2n^2 + 5})$$

$$65 \lim_{n \rightarrow +\infty} (\sqrt[4]{n + 1} - \sqrt[4]{n + 5})$$

$$66 \lim_{n \rightarrow +\infty} \frac{n^{\frac{5}{2}} + n^2 (\log n)^3 - n (\log n)^5}{\log \log 2n + n^{\frac{1}{2}} (n^4 + \log n)^{\frac{1}{2}}}$$

$$67 \lim_{n \rightarrow +\infty} \frac{n^{(\log n)^3} + e^{(\log n)^2}}{1 + (\log n)^n}$$

$$68 \lim_{n \rightarrow +\infty} \frac{(1 + e^{\frac{2}{3}n})^{\frac{3}{2}} + 3^n}{(1 + 2^n)^3 + e^n}$$

$$69 \lim_{n \rightarrow +\infty} \left(\frac{n - 5}{n + 1} \right)^n$$

$$70 \lim_{n \rightarrow +\infty} \frac{n}{\log(n^6 + \sqrt{n})}$$

$$71 \lim_{n \rightarrow +\infty} \frac{n}{\log(1 + e^n)}$$

$$72 \lim_{n \rightarrow +\infty} (\sqrt{n^2 + 5n} - n)$$

$$73 \lim_{n \rightarrow +\infty} \frac{\sqrt{2n + (-1)^n}}{\sqrt{2n - (-1)^n}}$$

$$74 \lim_{n \rightarrow +\infty} n^2 \operatorname{sen} \frac{2\pi}{(n + 5)^2}$$

$$75 \lim_{n \rightarrow +\infty} \sqrt[n]{e^n + \log n}$$

$$76 \lim_{n \rightarrow +\infty} (n^{\sqrt{n}} - (\sqrt{n})^n)$$

$$77 \lim_{n \rightarrow +\infty} \frac{3n^2 + (\cos n)n \log n}{(3 + n)\sqrt{n^2 + 1}}$$

$$78 \lim_{n \rightarrow +\infty} \left(2 - \frac{3}{n} \right)^{\log n}$$

$$79 \lim_{n \rightarrow +\infty} \frac{\sqrt{n^4 + 3n^3} - \sqrt{n^4 - 2n^3}}{n}$$

$$80 \lim_{x \rightarrow +\infty} x^{10} \ln(1 + 2e^{-x^3})$$

$$81 \lim_{n \rightarrow +\infty} \frac{\sqrt[p]{p} - 1}{n - 1}, \quad p > 0,$$

$$82 \lim_{x \rightarrow +\infty} [x\sqrt{1 + x^2} - \sqrt{x^2 - 1}],$$

$$83 \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^{\frac{1+x}{x}},$$

$$84 \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^2},$$

$$85 \lim_{x \rightarrow 0} \left(\frac{x^2 - 2x + 3}{x^2 - 3x + 2} \right)^{\frac{\sin x}{x}},$$

$$86 \lim_{x \rightarrow \infty} (\sqrt{x^2 + 8x + 3} - \sqrt{x^2 + 4x + 3}),$$

$$87 \lim_{x \rightarrow 1} \frac{x^3 - 6x^2 + 11x - 6}{x^2 - 3x + 2},$$

$$88 \lim_{x \rightarrow -\infty} x^{-10} 3^{-(x+1)} 2^x,$$

$$89 \lim_{x \rightarrow 0} \frac{\sin 2x}{\log(x+1)}.$$

2 Risposte ad alcuni esercizi

1: 243;

2: 0;

3: 0;

4: $-\infty$;

5: 2;

6: $\frac{3}{5}$;

7: $\frac{1}{2}$;

8: $\frac{1}{3}$;

9: 1;

10: non esiste, il limite per $x \rightarrow 3^+$ vale $+\infty$, mentre il limite per $x \rightarrow 3^-$ vale $-\infty$;

11: $-\infty$;

12: $-\frac{2}{3}$;

13: $\frac{1}{\ln 2}$;

14: 0;

15: -2;

16: 1;

17: 0;

18: $\frac{1}{2 \ln a}$;

19: $-\frac{2}{3}$;

20: 3;

21: -2;

22: $\frac{1}{2e}$;

23: 1 se $\alpha > 1$, e se $\alpha = 1$, $+\infty$ se $0 < \alpha < 1$;

24: $-\frac{1}{2}$;

25: $+\infty$;

26: 0;

27: $\frac{1}{4}$;

28: $\frac{1}{e^2}$;

29: $\frac{1}{2}$;

30: $\frac{2}{3}$;

31: $-\infty$;

32: $c = \ln 2$;

33: $\operatorname{tg} \frac{1}{2}$;

34: 0;

35: $+\infty$;

36: 0;

37: 0;

38: 1;

39: 0;

40: $+\infty$;

41: $\frac{1}{2}$;

42: 1;

43: 1;

44: $-\infty$;

45: $3(7\pi + 5)$;

46: $\frac{\pi}{2}$;

47: 0;

48: $\frac{1}{e}$;

49: 1;

50: 0;

51: 1;

52: 1;

53: $e^{-1/2}$;

54: e ;

55: $-\infty$;

56: $\frac{3}{2}$;

57: $\frac{1}{2}$;

58: $+\infty$;

59: 0;

60: 0;

61: 0;

62: e^{-1} ;

64: $1 - \sqrt{2}$;

65: 0;

66: 1;

67: 0;

68: 0;

69: e^{-6} ;

70: $+\infty$;

71: 1;

72: $\frac{5}{2}$;

73: 1;

74: 2π ;

75: e ;

76: $-\infty$;

77: 3;

78: $+\infty$;

79: $\frac{5}{2}$;

80: 0;

88: $+\infty$.