

Esercizio 1

Calcolare i seguenti limiti

$$\lim_{x \rightarrow +\infty} e^x - x^2$$

$$\lim_{x \rightarrow +\infty} \log(x) - \sqrt{x}$$

$$\lim_{x \rightarrow +\infty} \frac{\log(\sqrt{x+1})}{x}$$

$$\lim_{x \rightarrow +\infty} \frac{\log(x^2+1)}{e^x}$$

$$\lim_{x \rightarrow +\infty} \frac{x}{e^x - 1}$$

Esercizio 2

Usando i limiti notevoli

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

e $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$, verificare che

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{arctg}(x)}{x} = 1$$

$$\lim_{x \rightarrow \pm \infty} \left(1 + \frac{\alpha}{x}\right)^x = e^\alpha$$

$$\forall \alpha \in \mathbb{R}$$

$$\lim_{x \rightarrow 0} \frac{\log_a(1+x)}{x} = \frac{1}{\log(a)}$$

$$\forall a > 0 \text{ e } a \neq 1$$

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log(a)$$

Esercizio 3

Calcolare i limiti seguenti

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{x^4 + 9x} - \sqrt{x^4 + 1}}{x^2 + 2x}$$

$$\lim_{x \rightarrow +\infty} (2x^5 + 1)^{\frac{1}{x}}$$

$$\lim_{x \rightarrow +\infty} \frac{x^2}{2\sqrt{x}}$$

$$\lim_{x \rightarrow +\infty} (2^x + 3^x)^{\frac{1}{x}}$$

$$\lim_{x \rightarrow +\infty} \log(1+x+x^3) - 3 \log(x)$$

$$\lim_{x \rightarrow +\infty} \left(\frac{x^3}{x^2+1} - \frac{x^3-1}{x^2} \right)$$

$$\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x}$$

$$\lim_{x \rightarrow 0} \frac{\sin(2x)}{\operatorname{tg}(3x)}$$

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$$\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{(\sin(3x))^2}$$

$$\lim_{x \rightarrow 0} \frac{\cos(e^x - e^{-x}) - 1}{x^2}$$

$$\lim_{x \rightarrow 5} \frac{x - 5}{\sqrt{x} - \sqrt{5}}$$

$$\lim_{x \rightarrow 0^+} x \log(x)$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(x) + \operatorname{tg}(x)}{e^x - 1}$$

$$\lim_{x \rightarrow +\infty} x (\log(x+1) - \log(x))$$

$$\lim_{x \rightarrow +\infty} (x+1) (e^{1/x} - 1)$$

$$\lim_{x \rightarrow +\infty} \left(\frac{x+2}{x+1} \right)^x$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{\log(1-x) + \log(1+x)}$$

$$\lim_{x \rightarrow 0^+} \left(\frac{1}{x} + \log(\sin x) \right)$$

$$\lim_{x \rightarrow 0^+} \left(\frac{1}{\sin x} + \log x \right)$$

$$\lim_{x \rightarrow +\infty} \frac{x + 2 \cos(x)}{3x}$$

Esercizio 4

Calcolare, se esistono, i seguenti limiti, o altrimenti i limiti destro e sinistro

$$\lim_{x \rightarrow 0} \frac{x \sin(x)}{|x|}$$

$$\lim_{x \rightarrow 0} \frac{x \cos(x)}{|x|}$$

$$\lim_{x \rightarrow 0} \frac{\log |x|}{x}$$

$$\lim_{x \rightarrow 0} |x|^{\frac{1}{x}}$$

$$\lim_{x \rightarrow 0} \frac{2^x + \frac{1}{x}}{1 + 2^{1/x}}$$

$$\lim_{x \rightarrow 0} \frac{\log(1+x)}{\sqrt[3]{x^5}}$$

$$\lim_{x \rightarrow 0} \frac{x^2 + x + |x|}{x}$$