

Advanced Topics in Analysis

Laurea Magistrale in Matematica & Matematica per le Applicazioni

Lecture diary 2022/2023

January 24, 2023

Lecture 1-2 (September 29, 2022) Introduction to the course. Preliminaries to the notion of viscosity sub and supersolution. Vanishing viscous approximation of first order HJ equations.

Lecture 3-4 (October 3, 2022) Definition of viscosity sub and supersolution for fully nonlinear second order elliptic PDEs. Some equivalent choices of test functions. The key stability result of the notion of viscosity sub and super solution. An example.

Lecture 5-6 (October 6, 2022) Viscosity solution theory for first order HJ equations: sub and superdifferentials and relation with C^1 sub and supertangents; definition of viscosity sub and supersolution via super and subdifferentials; properties of sub and superdifferentials. 1dimensional examples and discontinuity points of the derivative of a viscosity solution to $H(x, u') = 0$ in Ω .

Lecture 7-8 (October 10, 2022) Comparison principles for a first order HJ equation set on a bounded open set. First comparison principle: statement and some remarks on the assumptions.

Lecture 9-10 (October 13, 2022) Proof of the first comparison principle.

Lecture 11-12 (October 17, 2022) Second comparison principle: statement and proof. Third comparison principle: statement and proof.

Lecture 13-14 (October 20, 2022) Third comparison principle: a second proof. Remarks and extension. Comparison principle for a first order HJ equation set on the Euclidean space: assumptions and statement.

Lecture 15-16 (October 24, 2022) A comparison principles for a first order HJ equation set on the euclidean space \mathbb{R}^N .

Lecture 17-18 (October 27, 2022) Existence results. The case of coercive Hamiltonians on \mathbb{R}^N .

Lecture 19-20 (October 31, 2022) Clarke generalized gradient in finite dimension. Convex Hamiltonians: equivalence of the notion of viscosity subsolution and almost everywhere subsolution.

Lecture 21-22 (November 3, 2022) Convex Hamiltonians: equivalence among four different notion of subsolution for locally Lipschitz functions; remarks on the stability of the notion of viscosity subsolution for the inf operator. Eikonal equations on bounded domains: the model case $|Du| = 1$ in Ω . Solvability of the associated Dirichlet problem.

Lecture 23-24 (November 7, 2022) Eikonal equations of the form $H(x, Du) = 0$ in a bounded domain Ω . The associated intrinsic length and intrinsic semidistance.

Lecture 25-26 (November 10, 2022) Properties of the intrinsic length functional and of the intrinsic semi-distance.

Lecture 27-28 (November 17, 2022) The intrinsic semi-distance vs. viscosity subsolutions of the Eikonal equation $H(x, Du) = 0$ in Ω . Lax-type representation formulae for the associated Dirichlet problem.

Lecture 29-30 (November 24, 2022) Evolutive H-J equations (with autonomous Hamiltonians): notion of discontinuous viscosity sub and supersolution. Properties of the sup-convolution in time of an upper semicontinuous viscosity subsolution.

Lecture 31-32 (November 28, 2022) Evolutive H-J equations (with autonomous Hamiltonians): comparison principle; existence and uniqueness of the solution of the evolutive H-J equation with a uniformly continuous initial datum.

Lecture 33-34 (December 1, 2021) Convex Analysis: definition of convex function; convexity vs. continuity/Lipschitz continuity. Subdifferentials of a convex function. Local minima of a convex function are global minima. The finite dimensional case: semicontinuous properties of the subdifferential. Subdifferential of a convex function vs. Clarke's generalized gradient.

Lecture 35-36 (December 5, 2021) Fenchel transform, bidual function, Fenchel's inequality. Lax-Oleinik formula and value function: motivation of the formula. The value function is a viscosity subsolution of the associated evolutive H-J equation.

Lecture 37-38 (December 12, 2022) Lipschitz regularity of the value function. Existence of absolutely continuous Lagrangian minimizers. Regularity of Lagrangian minimizers: the Tonelli case.

Lecture 39-40 (December 15, 2022) Regularity of Lagrangian minimizers: the Tonelli case. Lipschitz regularity of Lagrangian minimizers for non-smooth Lagrangians.

Lecture 41-42 (December 19, 2022) Homogenization of periodic Hamilton-Jacobi equations.

Lecture 41-42 (January 10, 2023) Weak KAM Theory: part 1.

Lecture 43-44 (January 12, 2023) Weak KAM Theory: part 2.

Lecture 45-46 (January 17, 2023) Weak KAM Theory: part 3.