ES1

$$\lim_{x \to -\infty} \frac{x^4 - 5x^7 + x^3}{x + 2 + 4x^5} =$$

$$= \lim_{x \to -\infty} \frac{x^7 \left(\frac{x^4}{x^7} - 5 + \frac{x^3}{x^7} \right)}{x^5 \left(\frac{x}{x^5} + \frac{2}{x^5} + 4 \right)} =$$

$$= \lim_{x \to -\infty} \frac{-5}{4} x^2 = -\infty .$$

ES2

$$\int_{-1}^{2} \left(e^{3x} - x^2\right) dx = \left[\frac{e^{3x}}{3} - \frac{x^3}{3} \right]_{-1}^{2} = \frac{e^6}{3} - \frac{8}{3} - \frac{e^{-3}}{3} - \frac{1}{3} =$$

$$= \frac{e^6 - e^{-3}}{3} - \frac{9}{3} = \frac{e^6 - e^{-3}}{3} - 3$$

ES 3

Si calcoli il polinomio di Taylor di ordine 3 di $\operatorname{sen}(2x)$ nel punto $\frac{\pi}{4}$.

$$f(x) = \operatorname{sen}(2x)$$

$$f^{(1)}(x) = 2\cos(2x)$$

$$f^{(2)}(x) = -4\operatorname{sen}(2x)$$

$$f^{(3)}(x) = -8\cos(2x)$$

$$f\left(\frac{\pi}{4}\right) = \operatorname{sen}\left(\frac{\pi}{2}\right) = 1$$

$$f^{(1)}\left(\frac{\pi}{4}\right) = 2\cos\left(\frac{\pi}{2}\right) = 0$$

$$f^{(2)}\left(\frac{\pi}{2}\right) = -4\operatorname{sen}\left(\frac{\pi}{2}\right) = -4$$

$$f^{(3)}\left(\frac{\pi}{2}\right) = -8\cos\left(\frac{\pi}{2}\right) = 0$$

$$P_3(x) = 1 + 0\cdot\left(x - \frac{\pi}{4}\right) - \frac{4}{2!}\left(x - \frac{\pi}{4}\right)^2 + \frac{0}{3!}\cdot\left(x - \frac{\pi}{4}\right)^3 =$$

$$= 1 - 2\left(x - \frac{\pi}{4}\right)^2$$