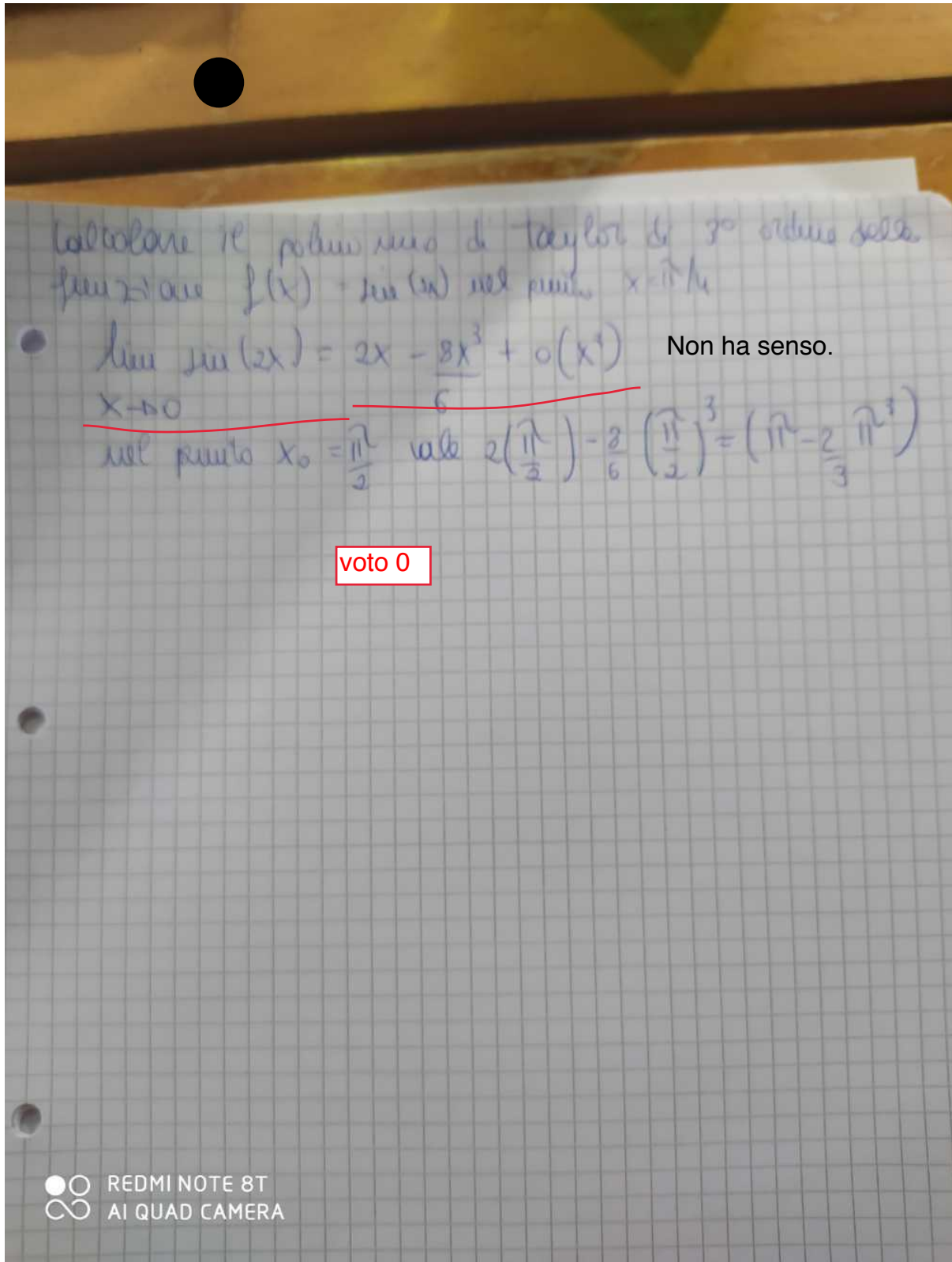


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Word count: 0



17720_3

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es 3)

Calcolare il polinomio di Taylor di ordine 3 della funzione $f(x) = \sin(2x)$ nel punto $x_0 = \frac{\pi}{4}$

$$f(x) = \sin(2x) \quad f'(x) = 2 \cos(2x) \quad f''(x) = -4 \sin(2x)$$

$$f(x_0) = \sin \frac{\pi}{2} = 1 \quad f'(x_0) = 2 \cos \frac{\pi}{2} = 0 \quad f''(x_0) = -4 \sin \frac{\pi}{2} = -4 \cdot 1 = -4$$

$$f'''(x) = -2 \cdot 2 \cdot \cos(2x) = -4 \cos(2x)$$

$$f'''(x_0) = -4 \cdot \cos \frac{\pi}{2} = 0$$

$$P_3(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)(x-x_0)^2}{2!} + \frac{f'''(x_0)(x-x_0)^3}{3!}$$

$$= 1 + 0 + \frac{(-4) \cdot \left(x - \frac{\pi}{4}\right)^2}{2} + 0 = 1 - \left(x - \frac{\pi}{4}\right)^2$$

Un errore di calcolo nella derivata seconda.

voto 3.7

17720_3
Word count: 0

CALCOLARE IL POLINOMIO DI TAYLOR DI ORDINE 3
DELLA FUNZIONE

$$f(x) = \sin(2x) \quad \text{NEL PUNTO } x_0 = \frac{\pi}{4}$$

$$f'(x) = 2 \cos(2x)$$

$$f''(x) = -4 \sin(2x)$$

$$f'''(x) = -8 \cos(2x)$$

$$f\left(\frac{\pi}{4}\right) = \sin\left(2 \cdot \frac{\pi}{4}\right) = 1$$

$$f'\left(\frac{\pi}{4}\right) = 2 \cos\left(2 \cdot \frac{\pi}{4}\right) = 0$$

$$f''\left(\frac{\pi}{4}\right) = -4 \sin\left(2 \cdot \frac{\pi}{4}\right) = -4$$

$$f'''\left(\frac{\pi}{4}\right) = -8 \cos\left(2 \cdot \frac{\pi}{4}\right) = 0$$

$$P_3(x) = 1 + 0 \cdot \left(x - \frac{\pi}{4}\right) - \frac{4}{2!} \left(x - \frac{\pi}{4}\right)^2 + \frac{0}{3!} \left(x - \frac{\pi}{4}\right)^3$$

$$= 1 - \frac{4}{2} \left(x^2 - 2 \cdot \frac{\pi}{4} x + \frac{\pi^2}{4^2}\right)$$

$$= 1 - 2x^2 + \pi x - \frac{\pi^2}{8}$$

$$P_3(x) = 1 - 2x^2 + \pi x - \frac{\pi^2}{8}$$

voto5

17720_3
 Word count: 0

$f(x) = \sin 2x$
 $f'(x) = 2 \cos x$
 $f''(x) = 0$
 $f'''(x) = 0$

$P_n(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!} (x-x_0)^2 + \frac{f'''(x_0)}{3!} (x-x_0)^3$

$P_3(x) = \sin \frac{\pi}{2} + 2 \cos \frac{\pi}{4} (x - \frac{\pi}{4}) + \frac{0}{2!} (x - \frac{\pi}{4})^2 + \frac{0}{3!} (x - \frac{\pi}{4})^3$

$P_3(x) = 1 + 0$

$P_3(x) = 1$

voto 0.5

Errori di calcolo molto gravi.
 Molti errori non possono essere verificati perché i calcoli non sono riportati.

17720_3

Word count: 0

$f(0) = \sin(0)$ A cosa serve?

$f(0) = 0$

$f'(x) = \cos(x) + \sin(x)$ NO

$f'(x_0) = \cos\left(\frac{\pi}{4}\right) + \sin\frac{\pi}{4}$

$f'(x_0) = 0 + \frac{1}{\sqrt{2}}$ Troppi errori.

$f'(x_0) = \frac{1}{\sqrt{2}}$ voto 0

$f''(x) = -\sin(x) + \cos(x)$

$f''(x_0) = -\sin\frac{\pi}{4} + \cos\frac{\pi}{4}$

$f''(x_0) = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$f''(x_0) = 0$

$f'''(x) = -\cos(x) - \sin(x)$

$f'''(x_0) = -\cos\frac{\pi}{4} - \sin\frac{\pi}{4}$

$f'''(x_0) = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{-1-1}{\sqrt{2}} = \frac{2}{\sqrt{2}}$

$P_m(x) = 0 + \frac{1}{\sqrt{2}}\left(x - \frac{\pi}{4}\right) + \frac{0}{2!}\left(x - \frac{\pi}{4}\right)^2 + \frac{2}{\sqrt{2}}\frac{\left(x - \frac{\pi}{4}\right)^3}{3!}$

$= +\frac{1}{\sqrt{2}}x - \frac{\pi}{4\sqrt{2}} + \frac{2}{\sqrt{2}}\frac{\left(x - \frac{\pi}{4}\right)^3}{6}$

17720_3
Word count: 0

3) CALCOLARE IL POLINOMIO DI TAYLOR DI ORDINE 3 DELLA FUNZIONE

$$f(x) = \sin(2x) \quad x_0 = \frac{\pi}{4}$$

$$f' = \cos(2x) \cdot 2 = 2\cos(2x)$$

$$f'' = 2 \cdot (-\sin(2x)) \cdot 2 = -4\sin(2x)$$

$$f''' = -4 \cos(2x) \cdot 2 = -8\cos(2x)$$

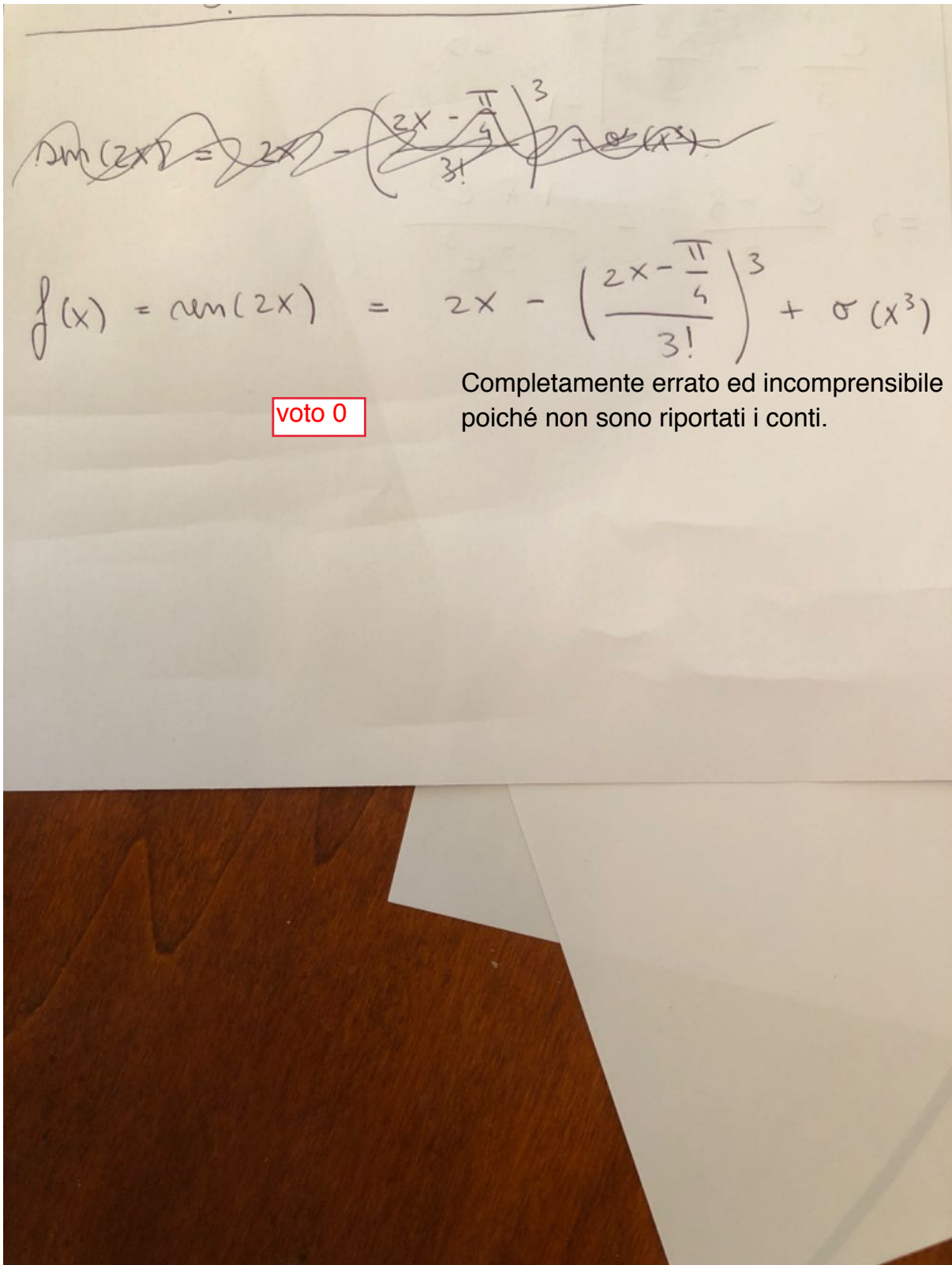
$$P_m(x) = \sin\left(\frac{\pi}{2}\right) + 2\cos\left(\frac{\pi}{2}\right) \cdot \left(x - \frac{\pi}{4}\right) + \frac{-4\sin\left(\frac{\pi}{2}\right) \cdot \left(x - \frac{\pi}{4}\right)^2}{2!} + \frac{-8\cos\left(\frac{\pi}{2}\right) \cdot \left(x - \frac{\pi}{4}\right)^3}{3!}$$

$$P_m = 1 + 2 \cdot 0 \cdot \left(x - \frac{\pi}{4}\right) + \frac{-4 \cdot 1 \cdot \left(x - \frac{\pi}{4}\right)^2}{2} + \frac{0 \cdot \left(x - \frac{\pi}{4}\right)^3}{3} = 1 - 2 \left(x - \frac{\pi}{4}\right)^2$$

voto 5

17720_3

Word count: 0



[REDACTED]

2020-07-17

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Word count: 0

[REDACTED]

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Word count: 0

Poteva semplificare $4/2=2$

voto 4.8

3) ricerca di polinomi di Taylor di ordine 2 della funzione $f(x) = \sin(x^2)$ nel punto $x_0 = \frac{\pi}{4}$

$$f(x) = \sin(x^2)$$

$$f'(x) = 2x \cos(x^2) = 1 \quad (x = \frac{\pi}{4})$$

$$f''(x) = 2 \cos(x^2) - 4x^2 \sin(x^2) = 2 \cos(\frac{\pi}{4}) - 4(\frac{\pi}{4})^2 \sin(\frac{\pi}{4}) = 2 \cdot \frac{\sqrt{2}}{2} - 4 \cdot \frac{\pi^2}{16} \cdot \frac{\sqrt{2}}{2} = \sqrt{2} - \frac{\pi^2 \sqrt{2}}{4}$$

$$f''(\frac{\pi}{4}) = \sqrt{2} - \frac{\pi^2 \sqrt{2}}{4}$$

$$P_2(x) = \sin(\frac{\pi}{4}) + 1(x - \frac{\pi}{4}) + \frac{1}{2}(\sqrt{2} - \frac{\pi^2 \sqrt{2}}{4})(x - \frac{\pi}{4})^2$$

$$= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}(x - \frac{\pi}{4}) + \frac{\sqrt{2}}{4}(x - \frac{\pi}{4})^2 - \frac{\pi^2 \sqrt{2}}{8}(x - \frac{\pi}{4})^2$$



17720_3
Word count: 0

$f(x) = \sin(2x)$ nel punto $x_0 = \frac{\pi}{4}$
 ~~$P_n(x) = \cos(2x)$~~

$$P_n(x) = \frac{\pi}{4} + \frac{(-\cos(2x)) \left(\frac{\pi}{4} + \cos(2x) \right)^2}{2}$$

$$P_n(x) = \frac{\pi}{4} + \frac{(-\cos(2x)) \left(\frac{\pi}{4} - \cos(2x) \right) \left(\frac{\pi}{4} + \cos(2x) \right)}{2}$$

$$P_n(x) = \frac{\pi}{4} - \frac{\cos(2x)}{2}$$

Non è neanche un polinomio ...

voto 0

17720_3
Word count: 0

esercizio 3

calcolare il polinomio di Taylor di ordine 3 della funzione $f(x) = \sin(2x)$ nel punto $x_0 = \frac{\pi}{4}$

$$f''(x) = -2\sin(2x) \cdot 2 = -4\sin(2x)$$

$$f'(x) = \cos(2x) \cdot 2 = 2\cos(2x)$$

$$f'''(x) = -4\cos(2x) \cdot 2 = -8\cos(2x)$$

$$P_m(x) = f\left(\frac{\pi}{4}\right) + f'\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right) + \frac{f''\left(\frac{\pi}{4}\right)}{2!}\left(x - \frac{\pi}{4}\right)^2 + \frac{f'''\left(\frac{\pi}{4}\right)}{3!}\left(x - \frac{\pi}{4}\right)^3$$

~~Polinomio di Taylor~~

Ha trascritto male la derivata seconda.

$$P_m(x) = \left(\sin\left(2 \cdot \frac{\pi}{4}\right)\right) + \left(\cos\left(2 \cdot \frac{\pi}{4}\right) \cdot 2\right)\left(x - \frac{\pi}{4}\right) + \frac{(-2\sin\left(2 \cdot \frac{\pi}{4}\right))}{2!}\left(x - \frac{\pi}{4}\right)^2 + \frac{(-4\cos\left(2 \cdot \frac{\pi}{4}\right))}{3!}\left(x - \frac{\pi}{4}\right)^3 =$$

$$P_m(x) = 1 + 0\left(x - \frac{\pi}{4}\right) + \frac{(-2)^1}{2!}\left(x - \frac{\pi}{4}\right)^2 + \frac{(0)}{3!}\left(x - \frac{\pi}{4}\right)^3 =$$

$$P_m(x) = 1 - 1\left(x - \frac{\pi}{4}\right)^2 = 1 - \left(x - \frac{\pi}{4}\right)^2$$

voto 4.3

17720_3

Word count: 0

CALCOLARE IL POLINOMIO DI TAYLOR IN ORDINE 3
 DELLA FUNZIONE $f(x) = \sin(2x)$ $x_0 = \frac{\pi}{4}$

$f(x) = \sin(2x)$

$f'(x) = 2\cos(2x)$ $(f \cdot g)' = f'g + f \cdot g'$

$f''(x) = \cos(2x) + 2 \cdot (-2\sin(2x)) =$
 $= \cos(2x) - 4\sin(2x)$

$f'''(x) = -2\sin(2x) - \sin(2x) - 4 \cdot 2\cos(2x)$
 $= -2\sin(2x) - \sin(2x) - 8\cos(2x)$

$P_n(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 =$
 $= \sin(2 \cdot 0) + 2\cos(2 \cdot 0)(x-0) + \frac{\cos(2 \cdot 0) - 4\sin(2 \cdot 0)}{2!}(x-0)^2 +$
 $\frac{-2\sin(2 \cdot 0) - \sin(2 \cdot 0) - 8\cos(2 \cdot 0)}{3!}(x-0)^3 =$
 $= 0 + 2x + \frac{1}{2} + \frac{8}{6} =$
 $= 0 + 2x - \frac{3+8}{6} = 0 + 2x - \frac{11}{6}$

Errori nelle derivate.

voto 1

17720_3
Word count: 0

$$n=3 \quad x_0 = \frac{\pi}{4}$$

$$f(x) = \sin(2x) \quad \Rightarrow \quad f(x_0) = \sin\left(\frac{\pi}{2}\right) = 1$$

$$f'(x) = 2\cos(2x) \quad \Rightarrow \quad f'(x_0) = 2\cos\left(\frac{\pi}{2}\right) = 0$$

$$f''(x) = -4\sin(2x) \quad \Rightarrow \quad f''(x_0) = -4\sin\left(\frac{\pi}{2}\right) = -4$$

$$f'''(x) = -8\cos(2x) \quad \Rightarrow \quad f'''(x_0) = -8\cos\left(\frac{\pi}{2}\right) = 0$$

$$P_n(x) = 1 + 0\left(x - \frac{\pi}{4}\right) + \frac{-4}{2}\left(x - \frac{\pi}{4}\right)^2 + \frac{0}{6}\left(x - \frac{\pi}{4}\right)^3$$

$$P_n(x) = \cancel{\left(x - \frac{\pi}{4}\right)^2} \quad 1 - 2\left(x - \frac{\pi}{4}\right)^2$$

voto 5

