Secondo scritto 2021

i)
$$\lim_{x\to 0} \frac{e^{3x}-e^{-3x}}{(x+1)(e^x-e^{-x})}$$

$$\frac{e^{3\cdot 0}-e^{-3\cdot 0}}{(0+1)\cdot(e^0-e^{-0})} = \frac{1-1}{1\cdot(1-1)} = \frac{0}{1\cdot 0} = \frac{0}{0}$$

non contribuisce alla forma indeterminata

$$\lim_{x\to 0} \frac{e^{3x}-e^{-3x}}{(x+1)(e^x-e^{-x})} = \lim_{x\to 0} \frac{1}{x+1}\cdot\frac{e^{3x}-e^{-3x}}{e^x-e^{-x}}$$

$$= \underbrace{\lim_{x\to 0}\frac{1}{x+1}}\cdot\lim_{x\to 0}\frac{e^{3x}-e^{-3x}}{e^x-e^x}$$

$$= 1\cdot\lim_{x\to 0}\frac{e^{3x}-e^{-3x}}{e^x-e^x}$$

$$= \lim_{x\to 0}\frac{e^{3x}-e^{-3x}}{e^x-e^x}$$

de l'H.
$$= \lim_{x\to 0}\frac{3\cdot e^{3x}-(-3)e^{-3x}}{e^x-(-1)e^{-x}} = \lim_{x\to 0}\frac{3e^{3x}+3e^{-3x}}{e^x+e^x} =$$

$$= \frac{3\cdot e^{3\cdot 0}+3\cdot e^{3\cdot 0}}{e^0+e^0} = \frac{3+3}{1+1} = 3$$

ii) $$\int (\overbrace{x+2}^{y}) \log (\overbrace{x+2}^{y}) \underbrace{dx}_{dy}$$

$\log = \log_e$

$$x+2 = y$$

$$\underset{\substack{\| \\ dx}}{d(x+2)} = dy$$

$$\int \underbrace{y \overbrace{\log(y)}^{f} dy}_{dg}$$

$$f \,,\, dg \longrightarrow df \,,\, g = \int dg$$

$$f\,dg \rightsquigarrow g\,df$$

$$f = \log(y) \longrightarrow df = d \log y = \frac{1}{y} dy$$

$$dg = y\,dy \longrightarrow g = \int y\,dy = \frac{y^2}{2}$$

$$\int f\,dg = fg - \int g\,df$$

$$\int \log(y) \cdot y\,dy = \log(y) \cdot \frac{y^2}{2} - \int \frac{y^{\cancel{2}}}{2} \cdot \frac{1}{\cancel{y}} dy$$

$$= \log(y) \cdot \frac{y^2}{2} - \int \frac{y}{2} dy$$

$$= \frac{y^2}{2}\log(y) - \frac{1}{2}\int y\,dy$$

$$= \frac{y^2}{2}\log(y) - \frac{1}{2}\cdot\frac{1}{2}y^2 + c$$

$$= \frac{1}{2}y^2\left(\log(y) - \frac{1}{2}\right) + c$$

$$= \frac{1}{2}(x+2)^2\left(\log(x+2) - \frac{1}{2}\right) + c$$

Verifica: $\frac{d}{dx}\left(\frac{1}{2}(x+2)^2\left(\log(x+2) - \frac{1}{2}\right)\right)$

$$= \frac{1}{2}\frac{d}{dx}\left((x+2)^2\left(\log(x+2) - \frac{1}{2}\right)\right.$$

$$= \frac{1}{2}\left(2\cdot(x+2)\cdot\left(\log(x+2) - \frac{1}{2}\right) + (x+2)^{\cancel{2}}\cdot\frac{1}{\cancel{x+2}}\right)$$

$$= \frac{1}{2}\left(2(x+2)\log(x+2) - \cancel{(x+2)} + \cancel{(x+2)}\right)$$

$$= (x+2)\log(x+2)$$ ✓

$d(x+2)^2 = 2(x+2)\cdot d(x+$

$= 2(x+2)dx$

$\to \frac{d}{dx}(x+2)^2 = 2(x+2)$

$d\log(x+2) = \frac{1}{x+2}\,d(x+2)$

$= \frac{1}{x+2}\,dx$

$$\frac{d}{dx}\log(x+2) = \frac{1}{x+2}$$

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iii)

$$f(x) = \log\left(\sqrt{\frac{(x-2)^2-1}{3}}\right)$$

Dominio :

$$x \to x-2 \to (x-2)^2 \to (x-2)^2-1 \to \frac{(x-2)^2-1}{3} \to \sqrt{\frac{(x-2)^2-1}{3}} \to \log\left(\sqrt{\frac{(x-2)^2-1}{3}}\right)$$

$$\frac{(x-2)^2-1}{3} \geq 0$$

$$\sqrt{\frac{(x-2)^2-1}{3}} > 0$$

$$\Updownarrow$$

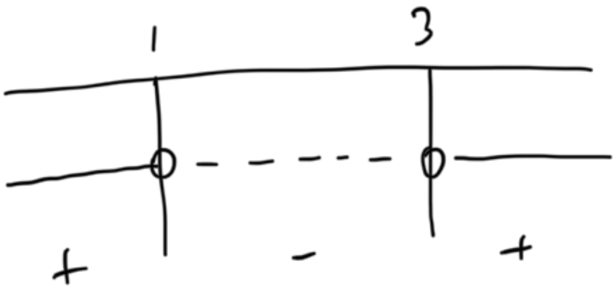
$$\frac{(x-2)^2-1}{3} > 0$$

$$\frac{(x-2)^2-1}{3} > 0$$

$$\frac{(x-2)^2-1}{3} > 0 \iff (x-2)^2-1 > 0 \implies$$

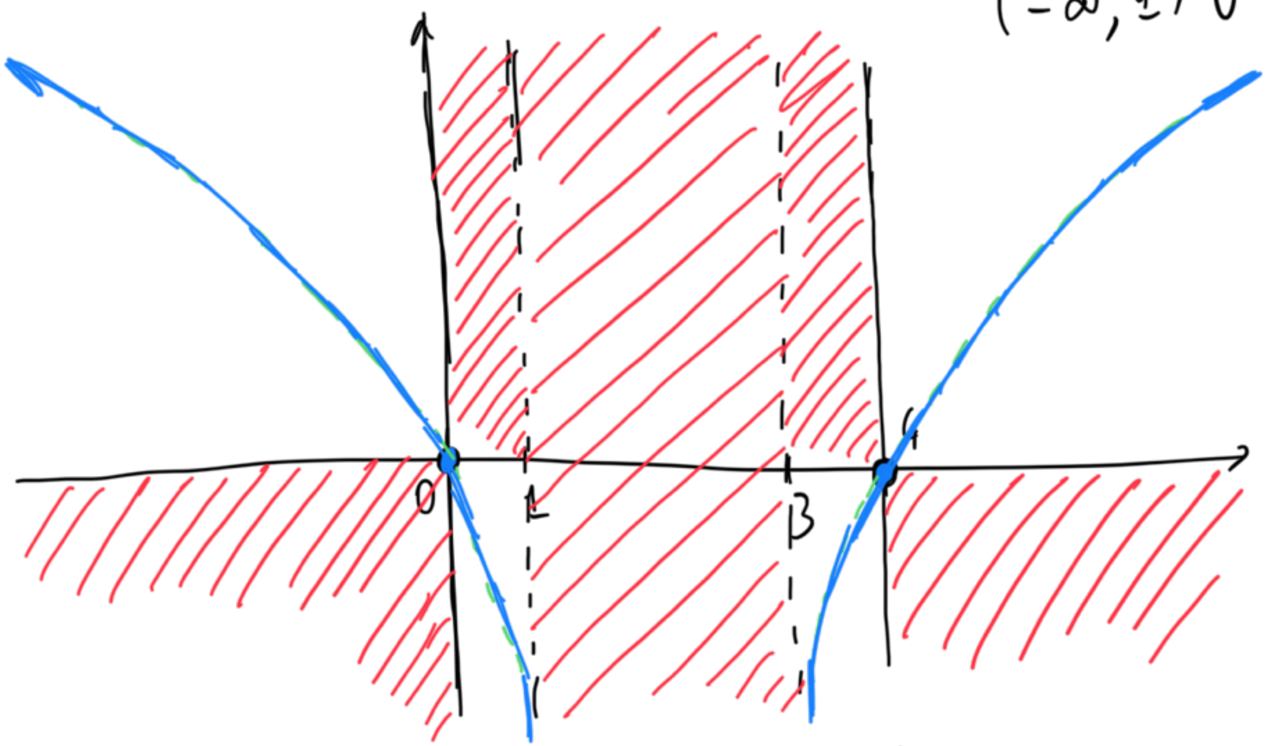
$$x^2-4x+3>0$$

$$x_{1/2} = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4\pm 2}{2} \begin{cases} 1 \\ 3 \end{cases}$$

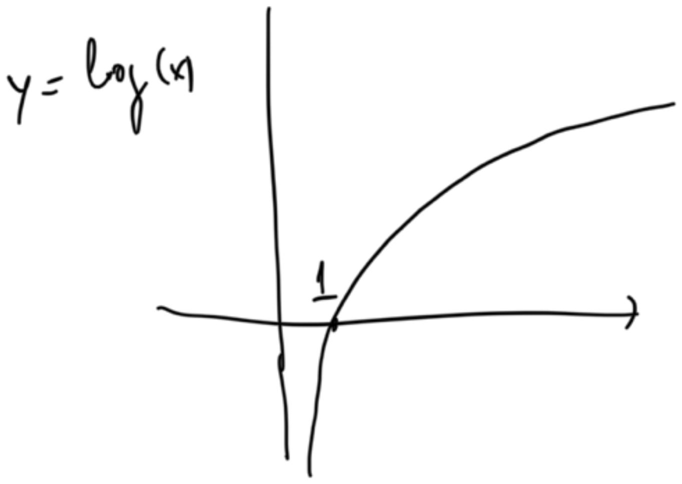


Dominio : $x < 1$ e $x > 3$

$(-\infty, 1) \cup (3, +\infty)$

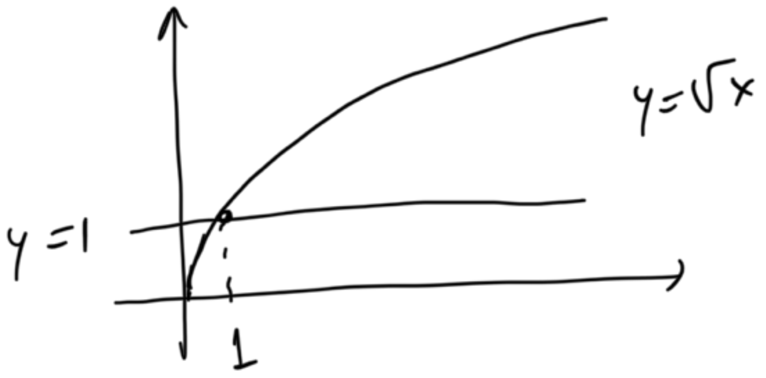


Segno: $\log\left(\sqrt{\frac{(x-2)^2-1}{3}}\right) \geq 0$



$\log(x) \geq 0 \iff x \geq 1$

$$\sqrt{\frac{(x-2)^2-1}{3}} \geq 1$$



$\sqrt{x} = 1 \iff x = 1$

$\sqrt{x} \geq 1 \iff x \geq 1$

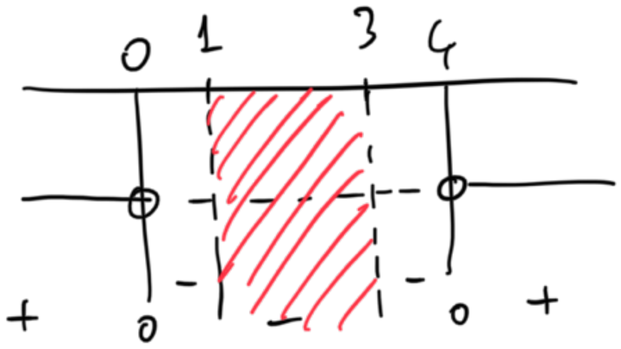
$$\frac{(x-2)^2-1}{3} \geq 1$$

$(x-2)^2 - 1 \geq 3$

$$(x-2)^2-4 \geq 0 \quad \to \quad x^2-4x \geq 0$$

$$x(x-4) \geq 0$$

$$x_{1,2} = 0, 4$$



Limiti agli estremi del dominio

$$\lim_{x \to -\infty} f(x) = \log\sqrt{\frac{(-\infty-2)^2-1}{3}} = \log\sqrt{\frac{+\infty}{3}} = \log\sqrt{+\infty}$$

$$= \log(+\infty)$$

$$= +\infty$$

$$\lim_{x \to 1^-} f(x) = \log\sqrt{\frac{(1-2)^2-1}{3}} = \log\sqrt{\frac{0}{3}} = \log(0) = -\infty$$

$$\lim_{x \to 3^+} f(x) = \log\sqrt{\frac{(3-2)^2-1}{3}} = \log\sqrt{\frac{0}{3}} = -\infty$$

$$\lim_{x \to +\infty} f(x) = \log\sqrt{\frac{(+\infty-2)^2-1}{3}} = \log(+\infty) = +\infty$$

Derivata

$$f'(x) = \frac{d}{dx}\log\sqrt{\frac{(x-2)^2-1}{3}} =$$

$$d \log \sqrt{\frac{(x-2)^2-1}{3}} = \frac{1}{\sqrt{\frac{(x-2)^2-1}{3}}} d\sqrt{\underbrace{\boxed{\frac{(x-2)^2-1}{3}}}_{t}}$$

$$= \frac{1}{\sqrt{\frac{(x-2)^2-1}{3}}} \cdot \overset{\frac{1}{2\sqrt{t}}}{\frac{1}{2\sqrt{\frac{(x-2)^2-1}{3}}}} \cdot \overset{dt}{d\left(\frac{(x-2)^2-1}{3}\right)}$$

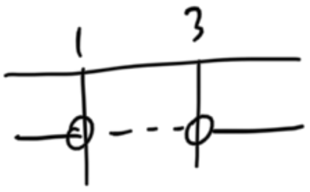
$$= \frac{1}{\cancel{2}\left(\frac{(x-2)^2-1}{\cancel{3}}\right)} \cdot \frac{\cancel{2}(x-2)\cdot 1}{\cancel{3}}$$

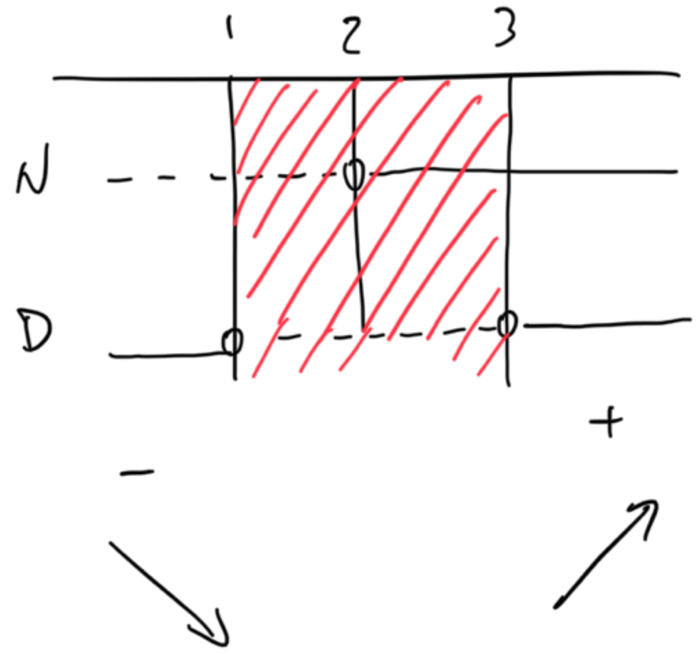
$$d\sqrt{t} = d\,t^{\frac{1}{2}} = \frac{1}{2} t^{-\frac{1}{2}} dt = \frac{1}{2\sqrt{t}} dt$$

$$= \frac{x-2}{x^2-4x+3}$$

N: $x-2 \geq 0$ $\quad x \geq 2$

D: $x^2-4x+3 \geq 0$





iv)

$$\begin{cases} -k x - 2y + (-2k-4) z = -k+1 \\ -(k+1) x + (k-2) y + (-k-5) z = -k+1 \\ (k+1) x + 2y + (2k+5) z = k \end{cases}$$

$$\left( \underbrace{\begin{matrix} -k & -2 & -2k-4 \\ -(k+1) & k-2 & -k-5 \\ k+1 & 2 & 2k+5 \end{matrix}}_{A} \middle| \underbrace{\begin{matrix} -k+1 \\ -k+1 \\ k \end{matrix}}_{b} \right)$$

$$rg(A) \overset{?}{=} rg(A|b)$$

$$\det A = \det \begin{pmatrix} -k & -2 & -2k-4 \\ -(k+1) & k-2 & -k-5 \\ k+1 & 2 & 2k+5 \end{pmatrix} =$$

$II_r \to II_r + III_r$

$$= \det \begin{pmatrix} -k & -2 & -2k-4 \\ 0 & k & k \\ k+1 & 2 & 2k+5 \end{pmatrix}$$

$$= k \det \begin{pmatrix} -k & -2 & -2k-4 \\ 0 & 1 & 1 \\ k+1 & 2 & 2k+5 \end{pmatrix}$$

$III_r \to III_r + I_r$

$$= k \cdot \det \begin{pmatrix} -k_{+} & -2 & -2k-4 \\ 0_{-} & 1_{+} & 1_{-} \\ 1 & 0 & 1 \end{pmatrix} \leftarrow$$

$$= k \cdot \left( 1 \cdot \det \begin{pmatrix} -k & -2k-4 \\ 1 & 1 \end{pmatrix} - 1 \det \begin{pmatrix} -k & -2 \\ 1 & 0 \end{pmatrix} \right)$$

$$= k\left((-k+2k+4)-2\right) = k(k+2)$$

$$\det A = 0 \iff k=0 \quad \text{oppure} \quad k=-2$$

$$\det A \neq 0 \;\Updownarrow\; \operatorname{rg} A = 3$$

| $k$ | $\operatorname{rg} A$ | $\operatorname{rg}(A\|b)$ | soluz. |
|---|---|---|---|
| $k \neq 0,-2$ | 3 | 3 | $\exists!$ (0 parametri) |
| $k=0$ | 2 | 3 | $\nexists$ soluz. |
| $k=-2$ | 2 | 2 | $\infty$ soluz. dipendenti da 1 parametro. |

$k=0$

$$\left(\begin{array}{ccc|c} 0 & \boxed{-2} & -4 & 1 \\ -1 & -2 & -5 & 1 \\ 1 & 2 & 5 & 0 \end{array}\right)$$

$$\det\begin{pmatrix} 0 & -2 \\ -1 & -2 \end{pmatrix} = 0-2 = -2 \neq 0 \rightarrow \operatorname{rg} A = 2$$

$$\det\begin{pmatrix} 0 & -2 & 1 \\ -1 & -2 & 1 \\ 1 & 2 & 0 \end{pmatrix} \overset{II_r \to II_r + III_r}{=} \det\begin{pmatrix} 0 & -2 & 1 \\ \underset{-}{0} & \underset{+}{0} & \underset{-}{1} \\ 1 & 2 & 0 \end{pmatrix} \leftarrow$$

$$= -1 \det\begin{pmatrix} 0 & -2 \\ 1 & 2 \end{pmatrix} = -2 \neq 0$$

$$\Rightarrow \operatorname{rg}(A|b) = 3$$

$k = -2$

$$\left(\begin{array}{ccc|c} \boxed{2} & -2 & 0 & 3 \\ 1 & -4 & -3 & 3 \\ -1 & 2 & 1 & -2 \end{array}\right)$$

$$\det\begin{pmatrix} 2 & -2 \\ 1 & -4 \end{pmatrix} = -8 + 2 = -6 \neq 0 \qquad rg(A) = 2$$

$$\downarrow$$

$$\det\begin{pmatrix} 2 & -2 & 3 \\ 1 & -4 & 3 \\ -1 & 2 & -2 \end{pmatrix} \overset{III_r \to III_r + II_r}{=} \det\begin{pmatrix} 2 & -2 & 3 \\ 1 & -4 & 3 \\ 0 & -2 & 1 \end{pmatrix}$$

$$= -2 \det \begin{pmatrix} 2_{+} & 1 & 3 \\ 1_{-} & 2 & 3 \\ 0_{+} & 1_{-} & 1_{+} \end{pmatrix} \leftarrow$$

$$= -2 \left( -1 \det\begin{pmatrix} 2 & 3 \\ 1 & 3 \end{pmatrix} + \underline{1} \det\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \right)$$

$$= -2 \left( -1 \cdot (6 - 3) + (4 - 1) \right)$$

$$= -2 \left( -1 \cdot 3 + 3 \right) = 0$$

$$\Rightarrow rg(A|b) = 2$$

Verificare che per $k = 1$ il sistema ha un'unica soluzione e risolverlo.

Con Gauss

$$\left(\begin{array}{ccc|c} -1 & -2 & -6 & 0 \\ -2 & -1 & -6 & 0 \\ 2 & 2 & 7 & 1 \end{array}\right)$$

$$\xrightarrow{\mathrm{I} \to -\mathrm{I}} \left(\begin{array}{ccc|c} 1 & 2 & 6 & 0 \\ -2 & -1 & -6 & 0 \\ 2 & 2 & 7 & 1 \end{array}\right)$$

$$\xrightarrow[\mathrm{III}_r \to \mathrm{III}_r - 2\mathrm{I}_r]{\mathrm{II}_r \to \mathrm{II}_r + 2\mathrm{I}_r} \left(\begin{array}{ccc|c} 1 & 2 & 6 & 0 \\ 0 & 3 & 6 & 0 \\ 0 & -2 & -5 & 1 \end{array}\right)$$

$$\mathrm{II}_r \to \frac{1}{3}\mathrm{II}_r \swarrow$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 6 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -2 & -5 & 1 \end{array}\right)$$

$$\begin{array}{l} \mathrm{I}_r \to \mathrm{I}_r - 2\mathrm{II}_r \\ \mathrm{III}_r \to \mathrm{III}_r + 2\mathrm{II}_r \end{array} \Big\downarrow$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 \end{array}\right)$$

$$\mathrm{III}_r \to -\mathrm{III}_r \Big\downarrow$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\begin{array}{l} \mathrm{I}_r \to \mathrm{I}_r - 2\mathrm{II}_r \\ \mathrm{II}_r \to \mathrm{II}_2 - 2\mathrm{III}_r \end{array} \Big\downarrow$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \end{array}\right)$$

$$\searrow \mathrm{II}_r \to \mathrm{II}_r + \mathrm{III}_r$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 6 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & -2 & -5 & 1 \end{array}\right)$$

$$\Big\downarrow \begin{array}{l} \mathrm{I}_r \to \mathrm{I}_r - 2\mathrm{II}_r \\ \mathrm{III}_r \to \mathrm{III}_r + 2\mathrm{II}_r \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 4 & -2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -3 & 3 \end{array}\right)$$

$$\Big\downarrow \mathrm{III}_r \to \frac{-1}{3}\mathrm{III}_r$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 4 & -2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\Big\downarrow \begin{array}{l} \mathrm{I}_r \to \mathrm{I}_r - 4\mathrm{III}_r \\ \mathrm{II}_r \to \mathrm{II}_r - \mathrm{III}_r \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\left.\begin{matrix} 0 & 0 & 1 & \mid & -1 \end{matrix}\right|$$

$$\searrow \qquad \swarrow$$

$$\begin{cases} x = 2 \\ y = 2 \\ z = -1 \end{cases}$$

con Cramer

$$x = \frac{\det A_x}{\det A} = \frac{6}{3} = 2 \qquad A = \begin{pmatrix} -1 & -2 & -6 \\ -2 & -1 & -6 \\ 2 & 2 & 7 \end{pmatrix}$$

$$y = \frac{\det A_y}{\det A} = \frac{6}{3} = 2 \,;\, \checkmark \quad A_x = \begin{pmatrix} 0 & -2 & -6 \\ 0 & -1 & -6 \\ 1 & 2 & 7 \end{pmatrix}$$

$$z = \frac{\det A_z}{\det A} = \frac{-3}{3} = -1 \,;\, \checkmark \quad A_y = \begin{pmatrix} -1 & 0 & -6 \\ -2 & 0 & -6 \\ 2 & 1 & 7 \end{pmatrix}$$

$$A_z = \begin{pmatrix} -1 & -2 & 0 \\ -2 & -1 & 0 \\ 2 & 2 & 1 \end{pmatrix}$$

$$\det A = \det \begin{pmatrix} -1 & -2 & -6 \\ -2 & -1 & -6 \\ 2 & 2 & 7 \end{pmatrix} =$$

$$= -\det \begin{pmatrix} 1 & 2 & 6 \\ -2 & -1 & -6 \\ 2 & 2 & 7 \end{pmatrix} \overset{\substack{II_r \to II_r + I_r \\ III_r \to III_r - I_r}}{=}$$

$$\overset{I_c \to I_c + II_c}{}$$

$$= -\det \begin{pmatrix} 1 & 2 & 6 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} =$$

$$= -\det\begin{pmatrix}3 & 2 & 6\\ 0 & 1 & 0\\ 1 & 0 & 1\end{pmatrix} \leftarrow \; = -\det\begin{pmatrix}3 & 6\\ 1 & 1\end{pmatrix} = +3$$

$$\det A_x = \det\begin{pmatrix}\overset{\downarrow}{0} & -2 & -6\\ 0 & -1 & -6\\ 1 & 2 & 7\end{pmatrix} = \det\begin{pmatrix}-2 & -6\\ -1 & -6\end{pmatrix}$$

$$= 6$$

$$\det A_y = \det\begin{pmatrix}-1 & \overset{\downarrow}{0} & -6\\ -2 & 0 & -6\\ 2 & 1 & 7\end{pmatrix} = -\det\begin{pmatrix}-1 & -6\\ -2 & -6\end{pmatrix}$$

$$= -(6-12) = 6$$

$$\det A_z = \det\begin{pmatrix}-1 & -2 & \overset{\downarrow}{0}\\ -2 & -1 & 0\\ 2 & 2 & 1\end{pmatrix} = \det\begin{pmatrix}-1 & -2\\ -2 & -1\end{pmatrix}$$

$$= 1-4 = -3$$

$u = -2$

$$\left(\begin{array}{ccc|c}2 & -2 & 0 & 3\\ 1 & -4 & -3 & 3\\ -1 & 2 & 1 & -2\end{array}\right)$$

Con Gauss

$$\mathrm{II}_r \leftrightarrow \mathrm{I}_r \quad \left(\begin{array}{ccc|c}1 & -4 & -3 & 3\\ 2 & -2 & 0 & 3\\ -1 & 2 & 1 & -2\end{array}\right) \xrightarrow[\mathrm{III}_r \to \mathrm{III}_r + \mathrm{I}_r]{\mathrm{II}_r \to \mathrm{II}_r - 2\mathrm{I}_r}$$

$$\left(\begin{array}{ccc|c} 1 & -4 & -3 & 3 \\ 0 & 6 & 6 & -3 \\ 0 & -2 & -2 & 1 \end{array}\right) \xrightarrow{\mathrm{II}_r \to \frac{1}{3}\mathrm{II}_r}$$

$$\left(\begin{array}{ccc|c} 1 & -4 & -3 & 3 \\ 0 & 2 & 2 & -1 \\ 0 & -2 & -2 & 1 \end{array}\right) \xrightarrow{\mathrm{III}_r \to \mathrm{III}_r + \mathrm{II}_r}$$

$$\left(\begin{array}{ccc|c} 1 & -4 & -3 & 3 \\ 0 & 2 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -4 & -3 & 3 \\ 0 & 2 & 2 & -1 \\ 0 & 0 & 1 & t \end{array}\right)$$

$$\mathrm{II}_r \to \frac{1}{2}\mathrm{II}_r \quad \left(\begin{array}{ccc|c} 1 & -4 & -3 & 3 \\ 0 & 1 & 1 & -1/2 \\ 0 & 0 & 1 & t \end{array}\right)$$

$$\to \mathrm{I}_r \to \mathrm{I}_r + 4\mathrm{II}_r \quad \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1/2 \\ 0 & 0 & 1 & t \end{array}\right)$$

$$\to \begin{array}{l} \mathrm{I}_r \to \mathrm{I}_r - \mathrm{III}_r \\ \mathrm{II}_r \to \mathrm{II}_r - \mathrm{III}_r \end{array} \quad \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1-t \\ 0 & 1 & 0 & -1/2 - t \\ 0 & 0 & 1 & t \end{array}\right)$$

$$\begin{cases} x = 1 - t \\ y = -\frac{1}{2} - t \\ z = t \end{cases}$$

Con Cramer

$$\left(\begin{array}{ccc|c} 2 & -2 & 0 & 3 \\ 1 & -4 & -3 & 3 \\ \hline -1 & 2 & 1 & -2 \end{array}\right)$$

$$\det\begin{pmatrix} 2 & -2 \\ 1 & -4 \end{pmatrix} = -8+2 = -6 \neq 0$$

$$\left(\begin{array}{ccc|c} 2 & -2 & 0 & 3 \\ 1 & -4 & -3 & 3 \\ 0 & 0 & 1 & t \end{array}\right) \leftarrow z = t$$

$$x = \frac{\det A_x}{\det A} = \frac{\cancel{6}(t-1)}{-\cancel{6}} = 1-t \qquad y = \frac{\det A_y}{\det A} = \frac{6t+3}{-6} = -t-\frac{1}{2}, \qquad z = \frac{\det A_z}{\det A} = \frac{-\cancel{6}t}{-\cancel{6}} = t$$

$$\det\begin{pmatrix} 2 & -2 & 0 \\ 1 & -4 & -3 \\ 0 & 0 & 1 \end{pmatrix} = \det\begin{pmatrix} 2 & -2 \\ 1 & -4 \end{pmatrix} = -6 \quad \Bigg\}$$

$$\det\begin{pmatrix} 3 & -2 & 0 \\ 3 & -4 & -3 \\ t & 0 & 1 \end{pmatrix} \leftarrow = t \det\begin{pmatrix} -2 & 0 \\ -4 & -3 \end{pmatrix} + \det\begin{pmatrix} 3 & -2 \\ 3 & -4 \end{pmatrix}$$

$$= t \cdot 6 - 6 = 6(t-1)$$

$$\det\begin{pmatrix} 2 & 3 & 0 \\ 1 & 3 & -3 \\ 0 & t & 1 \end{pmatrix} \leftarrow \quad = -t \det\begin{pmatrix} 2 & 0 \\ 1 & -3 \end{pmatrix} + \det\begin{pmatrix} 2 & 3 \\ 1 & 3 \end{pmatrix}$$

$$= 6t + 3$$

$$\det\begin{pmatrix} 2 & -2 & 3 \\ 1 & -4 & 3 \\ 0 & 0 & t \end{pmatrix} \leftarrow \quad = t \det\begin{pmatrix} 2 & -2 \\ 1 & -4 \end{pmatrix} = -6t$$