Minimal surfaces, Plateau's problem, and related questions

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Outlines of the lectures [1]

Lecture I

Minimal surfaces as conformally parametrized mappings. Branch points. Plateau boundary conditions and the class $\mathcal{C}(\Gamma)$ *of admissible mappings. The Courant–Lebesgue lemma. Plateau's problem. The problem of area minimization. Dirichlet's integral D and area A. Solution of "*$D \to \min$ *in* $\mathcal{C}(\Gamma)$*". The relation* $\inf_{\mathcal{C}(\Gamma)} A = \inf_{\mathcal{C}(\Gamma)}(D)$ *and the simultaneous problem "*$A \to \min$ & $D \to \min$ *in* $\mathcal{C}(\Gamma)$*".* Basic regularity results: boundary regularity; absence of branch points for minimizers. Uniqueness and non–uniqueness. Open problems.

Lecture II

Free boundary problems. Plateau problem for surfaces of prescribed mean curvature $H(x)$ *(H–surfaces). Proof of the boundary regularity. Asymptotic expansion about boundary branch points. Gauss–Bonnet theorem for branched surfaces. Enclosure theorems.* The "thread problem". Miscellaneous results for minimal surfaces with free boundaries. *Isoperimetric inequalities.* Open problems.

Lecture III

Construction of unstable minimal surfaces via the mountain pass lemma: the Courant–Shiffman approach, and Heinz's theory of quasi–minimal surfaces. *Stable minimal surfaces. Field embedding.* Tomi's finiteness theorem, and Nitsche's 6π–theorem. The global Korn–Lichtenstein theorem as a generalization of Riemann's mappings theorem. Open problems.

Lecture IV

The "Douglas problem" for configurations of several boundary curves. Cases of nonsolvability. Condition of cohesiveness. Solution of the simultaneous Douglas problem "$A \to \min$ & $D \to \min$*" if Douglas sufficient condition is satisfied.* Examples. Open problems.

[1] The topics written in Italics will be presented with proofs