1. Groupoids

~~Def.: A groupoid is a category where all morphisms are isomorphisms.~~
~~Ex.: Group + equiv. rel.~~

Def.: Let C be a category. A groupoid object in C is a 7-tuple $X_\bullet = (X_0, X_1, s, t, i, \varepsilon, m)$, where X_0 (objects) and X_1 (morphisms) are objects in C, with morphisms $s, t : X_1 \to X_0$ (source and target), $\varepsilon : X_0 \to X_1$ (identity map), $i : X_1 \to X_1$ (~~st~~ inverse) and $m : X_1 {}_s\!\times_t X_1 \to X_1$ (composition), subject to the following relations:

$$
\begin{tikzcd}
X_0 \arrow[r, "\varepsilon"] \arrow[d, "\varepsilon"'] \arrow[dr, "id"] & X_1 \arrow[d, "t"] \\
X_1 \arrow[r, "s"'] & X_0
\end{tikzcd}
\qquad
\begin{tikzcd}
X_1 \arrow[r, "i"] \arrow[d, "id"'] & X_1 \arrow[d, "s"] \arrow[dl, "i"'] \\
X_1 \arrow[r, "t"'] & X_0
\end{tikzcd}
\qquad
\begin{tikzcd}
X_1 \arrow[d, "s"'] & X_1 {}_s\!\times_t X_1 \arrow[l, "p_1"'] \arrow[r, "p_2"] \arrow[d, "m"] & X_1 \arrow[d, "t"] \\
X_0 & X_1 \arrow[l, "s"] \arrow[r, "t"'] & X_0
\end{tikzcd}
$$

(inverse) (identity)

$$
\begin{tikzcd}
X_1 \arrow[r, "i \times id"] \arrow[d, "s"'] & X_1 {}_s\!\times_t X_1 \arrow[d, "m"] & X_1 \arrow[l, "id \times i"'] \arrow[d, "t"] \\
X_0 \arrow[r, "\varepsilon"'] & X_1 & X_0 \arrow[l, "\varepsilon"]
\end{tikzcd}
\qquad
\begin{tikzcd}
X_0 {}_{id}\!\times_t X_1 \arrow[d, "\varepsilon \times id"'] & = X_1 = & X_1 {}_s\!\times_{id} X_0 \arrow[d, "id \times \varepsilon"] \\
X_1 {}_s\!\times_t X_1 \arrow[r, "m"] & X_1 \arrow[u, phantom, "\shortparallel"] & X_1 {}_s\!\times_t X_1 \arrow[l, "m"']
\end{tikzcd}
$$

(associativity)

$$
\begin{tikzcd}
X_1 {}_s\!\times_t X_1 {}_s\!\times_t X_1 \arrow[r, "m \times id"] \arrow[d, "id \times m"'] & X_1 {}_s\!\times_t X_1 \arrow[d, "m"] \\
X_1 {}_s\!\times_t X_1 \arrow[r, "m"] & X_1
\end{tikzcd}
$$

Write $X_1 \rightrightarrows X_0$ for $(X_0, X_1, s, t, i, \varepsilon, m)$.

Rmk.: Note that we thus implicitly require that $X_0 {}_s\!\times_t X_1$ exists. E.g. for C = smooth mflds with smooth morphisms this can be achieved by requiring s and t to be ~~smooth immersion~~ submersion.

~~Def~~ Def./Ex.: A small groupoid is a small category, where every morphism is an isomorphism. A groupoid thus is a groupoid object in the category Set. Call Gpoid the category of groupoids.

Ex.: A group G, thought of as a category is a groupoid. An equivalence relation $R \subseteq X \times X$ on a set X can be thought of as a groupoid:

	G	$R \subseteq X \times X$
X_0, X_1	$\{*_G\}, \{g \in G\}$	X, R
s, t	$G \ni g \to *_G$	$p_1, p_2 : R \to X$
ε	$*_G \to e \in G$	$X \ni x \to (x, x)$
i	$g \to g^{-1}$	$(x, y) \to (y, x)$
m	$(g, r) \to g \cdot r$	$((x, y), (y, z)) \to (x, z)$

~~Rmk.: Suppose we can view objects of C as sets (?).~~

Ex.: A Lie groupoid is a groupoid in $C = \mathrm{Mfd}$. To get the existence of fibered products, we furthermore require that s, t are ~~surj~~ submersions. (sufficient, not ~~nec.~~ necessary. For failure consider $\mathbb{R} \xrightarrow{x^2} \mathbb{R} \xleftarrow{x^2} \mathbb{R}$).

Ex.: The transformation groupoid of a group action:
Suppose ~~we have~~ a top. (alg., Lie) - group G acts on a top. space (scheme, mfd.). U
We take ~~$X_0 = U \times G$ and $X_1 =$~~ $X_0 = U$ and $X_1 = U \times G$
with ~~$s = pr_1 : U \times G \to U$ and~~ $s(u, g) = u$ and $t(u, g) = u \cdot g$,
$e(u) = (u, e_G)$, $i(u, g) = (u \cdot g, g^{-1})$ and $m((u, g), (u \cdot g, h)) = (u, g \cdot h)$.
Write $U \rtimes G$ for this groupoid.

2. Categories Fibered in Groupoids

Def.: • $\mathcal{C}$ category. A category over $\mathcal{C}$ is a category $\mathcal{F}$ together with a functor $p: \mathcal{F} \to \mathcal{C}$

• ~~The fibre~~ For $U \in Ob(\mathcal{C})$, the fiber $\mathcal{F}(U)$ is ~~defined~~ the sub-category of $\mathcal{F}$ whose objects are $F \in Ob(\mathcal{F})$ s.t. $p(F) = U$ and whose morphisms are $\phi: F \to F'$ in $\mathcal{F}$ s.t. $p(\phi) = id_U$.

Def.: A category fibered in groupoids (CFG) ~~is a~~ over $\mathcal{C}$ is a category $p: \mathcal{F} \to \mathcal{C}$ over $\mathcal{C}$, s.t.:

1.) For every morphism $f: U \to V$ in $\mathcal{C}$ and $\check{F} \in \mathcal{F}(V)$ there is an $\overset{u}{F} \in \mathcal{F}(U)$ and a morphism $\varphi: \overset{u}{F} \to \check{F}$ in $\mathcal{F}$ such that $p(\varphi) = f$ ("pullbacks exist")

2.) Given a com. diag. in $\mathcal{C}$:

$$\begin{tikzcd} T \arrow[d, "g"'] \arrow[r, "h"] & V \\ U \arrow[ur, "f"'] & \end{tikzcd}$$

and $\varphi: \overset{u}{F} \to \check{F}$ in $\mathcal{F}$, $\psi: \overset{t}{F} \to \check{F}$ s.t. $p(\varphi) = f$, $p(\psi) = h$

there is a unique morphism $\gamma: \overset{t}{F} \to \overset{u}{F}$ in $\mathcal{F}$ s.t. $p(\gamma) = g$ and ~~ψ~~ $\psi = \varphi \circ \gamma$:

$$\begin{tikzcd} \overset{t}{F} \arrow[d, dashed, "\exists!\gamma"] \arrow[r, "\psi"] \arrow[dd, bend right] & \check{F} \arrow[dd] \\ \overset{u}{F} \arrow[ur, "\varphi"'] \arrow[dd, bend right] & \\ T \arrow[d, "g"'] \arrow[r, "h"] & V \\ U \arrow[ur, "f"'] & \end{tikzcd}$$

Rmk.: 1.) Applying 2.) with $U = T$, $h = f$ and $g = 1_T$ that the u in the ~~(first~~ 1.) is unique up to canonical iso.

2.) From ~~Applying~~ 2.) it follows, that every morphism in $\mathcal{F}(U)$ is an iso.
Indeed, ~~take~~ given $\varphi : t \to s$ in $\mathcal{F}(U)$, we get:

$$\begin{tikzcd}
S \arrow[d, dotted, "\gamma"'] \arrow[dr, "id_S"] & \\
t \arrow[r, "\varphi"] & S \\
U \arrow[d, "id"'] \arrow[dr, "id"] & \\
U \arrow[r, "id"] & U
\end{tikzcd}$$

and γ is an inverse.

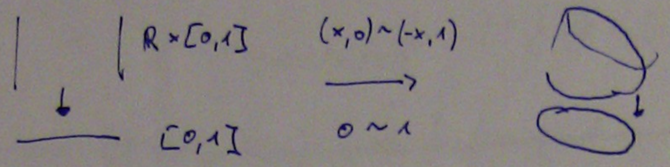
(Thus terminology)

~~Ex: $\mathcal{C}$ category, $X \in \mathcal{C}$. Define $\mathcal{C}/X$ with objects $Y \xrightarrow{f} X$ and morphisms~~

Ex.: Torsors. Let G be a top. group (alg., Lie) ~~and act on a top space (scheme, mfd.)~~ ~~X~~. A G-torsor or principal G-bundle is a continuous map $E \to S$ with a cont. action of G on E, ~~this action~~ ~~leaves~~ s.t. there is an open covering $\{U_\alpha\}$ of S s.t. $E|_{U_\alpha}$ is isomorphic to $U_\alpha \times G \to U_\alpha$. Define a category BG whose objects are G-torsors and a morphism ~~from~~ $E' \to S'$ to $E \to S$ is given by a pair of maps $E' \to E$, $S' \to S$ where the first is equivariant (i.e. commutes with the G-action) and the diagram
$$\begin{tikzcd}
E' \arrow[r] \arrow[d] & E \arrow[d] \\
S' \arrow[r] & S
\end{tikzcd}$$
is cartesian (i.e. commutative and the induced map $E' \to E \times_S S'$ is a homeom.).

Then we have $BG \to Top$, $(E \to S) \to S$, $((E \to S) \to (E' \to S')) \to S \to S'$.
~~This is a fibered~~ Then $BG \to Top$ is a CFG (By cartesian property).
Now suppose $G \curvearrowright X$ for a top. space (mfd., scheme). ~~Define~~ $[X/G]$ a category with objects G torsors $E \to S$ together with an equivariant map from $E \to X$ and morphisms $(S' \leftarrow E' \to X) \to (S \leftarrow E \to X)$ given by a morphism of torsors s.t. $E' \to E \to X$ is equal to $E' \to X$.
This again is a CFG over Top.

Ex. of G-torsor: A vector bundle in general is not a G-torsor, although all the fibers are isomorphic to lie groups, ~~[illegible]~~ since in general there is no global action of the group on the bundle.
For example: Consider the non-trivial bundle over S^1:



locally would need to have $(x, y) \cdot r = (x+r, y)$
for this to be globally defined, need $r = 0$, ~~which is no longer locally~~ otherwise the ~~transition~~ map wouldn't be well defined.

But: can associate to a vector bundle the GL_n-torsor called frame bundle consisting of ordered bases of the fibers.
(conversely, a repr. of G and a G-torsor ~~a GL_n bundle~~ gives a vector bundle of rank n)

Ex.: Fibered category associated to a groupoid object.

~~Denote~~ Let $X_1 \rightrightarrows X_0$ be a groupoid object in $\mathcal{C}$. For $U \in Ob(\mathcal{C})$, write $X_i(U) := \mathrm{Hom}_{\mathcal{C}}(U, X_i)$ and define for every U a category

$$\{X_0(U)/X_1(U)\} := \begin{cases} X_0(U) \text{ objects} \\ \mathrm{Hom}_{\{X_0(U)/X_1(U)\}}(u,u') = \{\varphi \in X_1(U) : s\circ\varphi = u,\ t\circ\varphi = u'\} \end{cases}$$

(*)

~~Define A morphism $f \in \mathrm{Hom}_{\mathcal{C}}(V,U)$ gives by composition a~~

~~functor $f^*: \{X_0(U)/X_1(U)\} \longrightarrow \{X_0(V)/X_1(V)\}$ and $g^* f^* = (gf)^*$~~

Define a category $\{X_0/X_1\}$ with objects pairs (U, u) where $U \in \mathcal{C}$ and $u \in \{X_0(U)/X_1(U)\}$ and morphisms $(V, v) \to (U, u)$ given ~~by~~ by $(f \in \mathrm{Hom}_{\mathcal{C}}(V,U),\ \alpha \in \mathrm{Hom}_{\{X_0(V)/X_1(V)\}}(v, u\circ f))$.

(**)

~~Then $p: \{X_0/X_1\} \longrightarrow \mathcal{C}$, makes gives a CFG over $\mathcal{C}$~~

~~$(U, u) \longmapsto U$~~

~~$(f, g) \longmapsto f$~~

Ex.:

$$\begin{tikzcd} (V,f^*u) \arrow[r, dashed] \arrow[d] & (U,u) \arrow[d] \\ V \arrow[r] & U \end{tikzcd}$$

$$(*) \quad \begin{tikzcd} U \arrow[r, "\varphi"] \arrow[dr, bend left, "u'"] \arrow[dr, bend right, "u"'] & X_1 \arrow[d, shift right, "s"'] \arrow[d, shift left, "t"] \\ & X_0 \end{tikzcd}$$

$t\circ\varphi = u'$

$s\circ\varphi = \mathrm{id}\, u$

~~$\mathrm{end}(\varphi)$~~

~~$\varphi\ldots = \alpha$~~

~~$\varphi \in (\varphi)$~~

~~$\varphi\circ\ldots = \alpha$~~

$$(**) \quad \begin{tikzcd} V \arrow[r, bend left, "\alpha"] \arrow[r, "v"'] \arrow[d, "f"] & X_1 \arrow[d, shift right, "s"'] \arrow[d, shift left, "t"] \\ U \arrow[r, "u"'] & X_0 \end{tikzcd}$$

with ~~fram~~ $v = s\circ\alpha$
$u\circ f = t\circ\alpha$

Composition is defined as follows:

$$\begin{tikzcd}
G \arrow[rr, "\alpha'"] \arrow[d, "f'"] \arrow[rrd] & & X_1 \arrow[d, shift left, "s"'] \arrow[d, shift right, "t"] \\
V \arrow[rru, "\alpha"] \arrow[rr, "v"] \arrow[d, "f"] & & X_0 \\
U \arrow[rru, "u"'] & &
\end{tikzcd}$$

now $t \circ \alpha' = v \circ f' = u \circ f \circ f' = s \circ \alpha \circ f'$

so have induced morphism

$(\alpha', \alpha \circ f') : G \longrightarrow X_1 \times_{t, X_0, s} X_1$

Define $\alpha \circ \alpha' = m(\alpha', \alpha \circ f')$ and $(f, \alpha) \circ (f', \alpha') = (f \circ f', \alpha \circ \alpha')$.

Then $\underbrace{[X_1 \rightrightarrows X_0]^{pre}}_{\{X_0 / X_1\}} \longrightarrow \mathcal{C}$ is a CFG over $\mathcal{C}$

$(U, u) \longrightarrow U$

$(f, \alpha) \longrightarrow f$

(by similar calculations as above).

write also $[X_1 \rightrightarrows X_0]^{pre}$ for $\{X_0 / X_1\}$.

pullback:

$$\begin{tikzcd}
 & & X_1 \arrow[d, shift left, "s"'] \arrow[d, shift right, "t"] \arrow[d, bend right, "\varepsilon"'] \\
V \arrow[rru, "u \circ f"] \arrow[rr, "u \circ f"] \arrow[d, "f"] & & X_0 \\
U \arrow[rru, "u"'] & &
\end{tikzcd}$$

$$V \xrightarrow{f} U$$

<u>Ex.:</u> $G \curvearrowright pt.$ (top. grp.) $\xrightarrow{\text{associate}}$ $X_0 = pt.$, $X_1 = pt. \times G = G$, $s, t : G \longrightarrow pt.$

$m(g, r) = g \cdot r$

$pt \rtimes G$ in $\mathcal{C} = \mathrm{Top}$. $\varepsilon(pt) = e$

$\xrightarrow[\text{fibred category}]{\text{associate}}$ $[G \rightrightarrows pt]^{pre}$ objects: $T \longrightarrow \begin{smallmatrix} G \\ \downarrow\downarrow \\ X_0 \end{smallmatrix}$ one object for every T

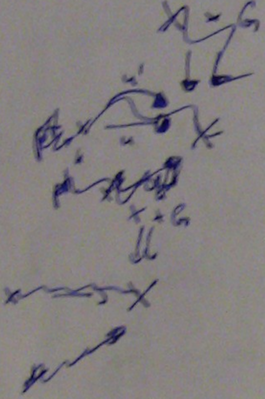
Suppose we have morphism of $T \longrightarrow X_0$:

$$\begin{tikzcd}
 & G \arrow[d, shift left, "s"'] \arrow[d, shift right, "t"] \\
T \arrow[ru, "\alpha"] \arrow[r] \arrow[d, "id"] & pt \\
T \arrow[ru] &
\end{tikzcd}
\quad \overset{bij}{\longleftrightarrow} \quad \mathrm{Mor}_{\mathrm{Top}}(T, G)$$

On the other hand have $T \times G$ trivial G-torsor over T.
Given $\alpha : T \to G$ get an automorphism by $(t, g) \mapsto (t, \alpha(t)\cdot g)$
and these are all automorphisms. Thus elements in $[G \rightrightarrows pt]^{pre}$ correspond
to trivial G-torsors.

Note: For $T = pt.$ we get an object with the whole group G as
isomorphisms.

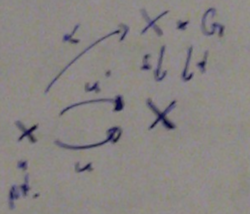
Ex.: $G \curvearrowright X$. $X_0 = X$, $X_1 = X \times G$, $s(x, g) = x$, $t(x, g) = x \cdot g$



~~then $x \xrightarrow{u'}$, $x \xrightarrow{u}$ isomorphic~~

~~$(\Leftrightarrow)$ $\exists\, \alpha : x \to X \times G$ s.t. $s \circ \alpha = x$~~

and



isomorphic $(\Leftrightarrow)$
$\exists\, \alpha : x \to X \times G$
s.t. $s \circ \alpha = u$, $t \circ \alpha = u'$
$(\Leftrightarrow)$ $\exists\, g \in G$ s.t. $u'(x) = g \cdot u(x)$
$\rightsquigarrow$ recover orbits

But more than that: An automorphism of $x \xrightarrow[u]{} X$ is $\alpha : x \to X \times G$
s.t. $t \circ \alpha = s \circ \alpha = u$, i.e. an element $g \in G$ s.t. $g \cdot u(x) = u(x)$.
Thus $\mathrm{Aut}(x \xrightarrow[u]{} X) = G_x$.

3. Stacks

<u>Recall</u>: • (Grothendieck top.) Let $\mathcal{C}$ be a category. A Grothendieck top. on $\mathcal{C}$ consists of a collection $\mathrm{Cov}(X)$ of sets $\{X_i \to X\}$ of morphisms for every object X in $\mathcal{C}$, called coverings of X, s.t.:

1. If $V \to X$ is an isomorphism, then $\{V \to X\} \in \mathrm{Cov}(X)$
2. If $\{X_i \to X\} \in \mathrm{Cov}(X)$ ~~and~~, then for every ~~an~~ morphism $Y \to X$ the fiber products $X_i \times_X Y$ exist and $\{X_i \times_X Y \to Y\} \in \mathrm{Cov}(Y)$
3. If $\{X_i \to X\} \in \mathrm{Cov}(X)$ and $\{V_{ij} \to X_i\} \in \mathrm{Cov}(X_i)$ for each i, then $\{V_{ij} \to X_i \to X\} \in \mathrm{Cov}(X)$

A category together with such a topology is called a site.
(For this talk can think of families of open covers in Top)

• (presheaf) A $\mathcal{T}$-valued presheaf on ~~the~~ a category $\mathcal{C}$ is a functor $F: \mathcal{C}^{op} \to \mathcal{T}$

From now on let $p: \mathcal{F} \to \mathcal{C}$ be a CFG over a site $\mathcal{C}$.

<u>Def.</u>: ~~Let $x, y \in \mathcal{F}(U)$ for some~~ $U \in \mathcal{C}$. Denote by $\mathcal{C}/U$ the category of morphisms $\{C \to U\}$ for all $C \in \mathcal{C}$*. It inherits a topology from $\mathcal{C}$ by taking $\left\{\begin{smallmatrix} X_i & \to & X \\ & \searrow U \swarrow & \end{smallmatrix}\right\}$ to be a cover, if $\{X_i \to X\}$ is one.

<u>Def.</u>: ~~Define a presheaf~~ For $x, y \in \mathcal{F}(U)$ ~~for some~~ and $U \in \mathcal{C}$, define a presheaf ~~$\mathrm{Isom}_{\mathcal{F}}(x,y)$~~ $\mathcal{C}/U \to \mathrm{Sets}$ by setting

$$\mathrm{Isom}_{\mathcal{F}}(x,y)(T \xrightarrow{f} U) = \{\text{isomorphisms from } f^*(x) \text{ to } f^*(y) \text{ in } \mathcal{F}(T)\}$$

* with morphisms of the form $C \to C'$ over U

⑤

<u>Def.:</u> • $\mathcal{F}$ is called a prestack, if for all $\{T_i : \emptyset \to T\} \in \mathrm{Cov}(\mathcal{C}/\mathcal{U})$ the following is exact:

$$\mathrm{hom}_{\mathcal{F}}(x,y)(T) \longrightarrow \prod_{i} \mathrm{hom}_{\mathcal{F}}(x,y)(T_i) \rightrightarrows \prod_{i,j} \mathrm{hom}_{\mathcal{F}}(x,y)(T_i \times_T T_j)$$

(A sequence of sets $A \to B \rightrightarrows C$ is called exact, if A maps bijectively to the set of ~~images~~ elements in B that have the same image under both maps to C)

(i.e. it is a sheaf)

• Let $\mathcal{F}$ be a prestack. Then it is a stack if the following holds: ~~Given~~ Suppose we have $\{T_i \to T\} \in \mathrm{Cov}(\mathcal{C}/\mathcal{U})$, ~~then given~~ and any collection of objects $t_i \in \mathcal{F}(T_i)$ ~~and isomorphisms φ_{ij}~~. Write $p_1 : T_i \times_T T_j \to T_i$, $p_2 : T_i \times_T T_j \to T_j$. Suppose furthermore we have isomorphisms $\varphi_{ij} : p_1^* t_i \to p_2^* t_j$ over $T_i \times_T T_j$ satisfying the cocycle condition, then there is an object $x \in \mathcal{F}(T)$ and for each i an isomorphism $\lambda_i : x_i \to t_i$, where x_i denotes the/a pullback to T_i. These isomorphisms are required to satisfy the natural compatibility condition on $T_i \times_T T_j$.

The cocycle condition is for $p_{12} : T_i \times_T T_j \times_T T_k \to T_i \times_T T_j$ etc., the diagram

pullback of the morphism

can. iso. (question is only, to which of the three comp. we pull it back)

$$\begin{tikzcd}
p_{12}^* p_1^* t_i \arrow[r, "p_{12}^* \varphi_{ij}"] \arrow[d, phantom, "\wr\!\!\|"] \arrow[dr, phantom, "\circlearrowleft"] & p_{12}^* p_2^* t_j \cong p_{23}^* p_1^* t_j \arrow[d, "p_{23}^* \varphi_{jk}"] \\
p_{13}^* p_1^* t_i \arrow[r, "p_{13}^* \varphi_{ik}"'] & p_{13}^* p_2^* t_k = p_{23}^* p_2^* t_k
\end{tikzcd}$$

The natural compatibility condition on $T_i \times_T T_j$ is:

$$\begin{tikzcd}
p_1^* x_i \arrow[r, "p_1^* \lambda_i"] \arrow[d, phantom, "\wr\!\!\|"] \arrow[dr, phantom, "\circlearrowleft"] & p_1^* t_i \arrow[d, "\varphi_{ij}"] \\
p_2^* x_j \arrow[r, "p_2^* \lambda_j"] & p_2^* t_j
\end{tikzcd}$$

Ex.: $[G \rightrightarrows pt.]^{pre} = \mathcal{F}$, open coverings top.

$\mathcal{F}$ is ~~not~~ a prestack:

$$x = y = \{U \xrightarrow{u} pt.\}$$

$$f^*(x) = V \xrightarrow{u \circ f} pt.$$

$$\hom_{\mathcal{F}}(x,y)(V) = \cancel{V \to G}$$

$$\text{but } (V \xrightarrow{u \circ f} pt.) = \{V \to G\}$$

$$\begin{tikzcd} & & G \arrow[d] \\ V \arrow[r, "f"] \arrow[rr, bend left, "u \circ f"] & U \arrow[r, "u"] & pt. \end{tikzcd}$$

Now, given an open cover $\{V_i \to V\}$, the sequence

$$\{V \to G\} \longrightarrow \prod_i \{V_i \to G\} \rightrightarrows \prod_{i,j} \{V_i \cap V_j \to G\}$$

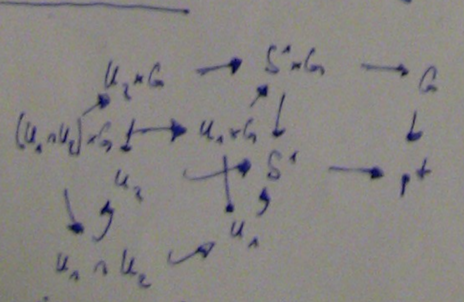
$$(V \to G) \mapsto \prod_i (V_i \to V \to G)$$

$$\prod_i (V_i \to G) \to \prod_{i,j} (V_i \cap V_j \to V_i \to G)$$

$$\prod_i (V_i \to G) \to \prod_{i,j} (V_i \cap V_j \to V_j \to G)$$

The last two images coincide, iff all the restrictions of the $V_i \to G$ and $V_j \to G$ on $V_i \cap V_j$ coincide, which in turn is exactly the case, if they glue to give a function $V \to G$. Thus the sequence is exact and we get a prestack.

$\mathcal{F}$ is not a stack: Recall, may view it as trivial torsors. Consider



$S^1 = U_2 \bigcirc U_1$ with $U_1 \cap U_2 = V_1 \sqcup V_2$

take $V_1 \to e$
$u_{12}: V_2 \to g$
$u_{21}: V_1 \to e$
$V_2 \to g^{-1}$

this fulfills cocycle condition, but the isomorphism cannot come from isos. of U_1 and U_2

→ <u>Fact</u>: Can associate to any prestack a stack.

The stack associated to $[G\times G\times X \;\substack{\to\\\to\\\to}\; G\times X \rightrightarrows X]^{\mathrm{pre}}$ is the category we defined earlier: $[X/G]$ of G torsors + equiv. map.