Affine, Vertex and W-algebras

Rome, Italy, December 11–15, 2017

Organizers:

Dražen Adamović (Zagreb) and Paolo Papi (Rome)

ABSTRACTS OF TALKS

On realizations of simple affine vertex algebras and their modules

Dražen Adamović, University of Zagreb

We present some new explicit realizations of simple, non-rational affine vertex algebras, affine W–algebras and their modules. As an application, we discuss the existence of Whittaker, logarithmic and indecomposable modules for simple affine vertex algebras.

Higgs branch conjecture for Class S theories

Tomoyuki Arakawa, RIMS, Kyoto University/MIT

The Higgs branch conjecture states that the Higgs branch of a 4d N-2 SCFT is isomorphic to the associated variety of the corresponding vertex algebra. There is a distinguished class of 4d N-2 SCFTs called Class S theories, whose Higgs branches are described by the Moore-Tachikawa conjecture that was recently proved by Braverman, Finkelberg and Nakajima. In my talk I confirm the Higgs branch conjecture for Class S theories, by upgrading the Moore-Tachikawa conjecture to the setting of vertex algebras, as conjectured by Rastelli et al.

Some new logarithmic C_2-cofinite Vertex Operator Algebras

Jean Auger, University of Alberta

I will explain how we have obtained new examples of logarithmic C_2-cofinite Vertex Operator Algebras (VOAs). We consider infinite order extensions of parafermion algebras associated to $A_1^{(1)}$ at negative admissible rational level. We obtain the modular behaviours expected of C_2-cofinite VOAs with explicit S-transformation coefficients. Following an algorithmical approach, we have shown C_2-cofiniteness for a few new levels.

Twisted logarithmic modules of lattice vertex algebras

Bojko Bakalov, North Carolina State University

Twisted modules over vertex algebras formalize the relations among twisted vertex operators and have applications to conformal field theory and representation theory. A recent generalization, called twisted logarithmic module, involves the logarithm of the formal variable and is related to logarithmic conformal field theory. I will review this theory and then consider the case of lattice vertex algebras. For lattice vertex algebras, the classification of twisted logarithmic modules reduces to the classification of modules over a certain group, which is a semidirect product of a Heisenberg group and a central extension of the additive group of the lattice.

Higher level Zhu's algebras and indecomposable modules for vertex operator algebras.

Katrina Barron, University of Notre Dame

We discuss the relationship between indecomposable modules for a vertex operator algebra and indecomposable modules for the associative algebras defined by Dong, Li and Mason generalizing Zhu's associative algebra. In particular, we focus on the case of non simple modules.

Vertex algebra questions from physics and geometry

Thomas Creutzig, University of Alberta

Supersymmetric gauge theories in physics relate to vertex algebras and geometry. I want to introduce VOAs appearing in this context as well as properties of them predicted by physics/geometry. Most notable there are VOAs whose associated variety is the Higgs branch of a gauge theory and I would like to discuss a second variety attached to VOAs which should correspond to the Coulomb branch of the gauge theory.

Quantum finite and classical affine W-algebras for classical Lie algebras

Alberto De Sole, Sapienza Università di Roma

I will describe the construction of a Lax type operator L(z) with coefficients in the quantum finite W-algebra W(g,f). We show that for the classical linear Lie algebras gl_N, sl_N, so_N and sp_N, such operator $L(z)$ satisfies a generalized Yangian identity. In the case of classical affine W-algebras, the analogue of this operator is a Lax operator $L(\partial)$, which is used to construct an integrable Hamiltonian hierarchy of Lax type equations.

All the result presented are joint work with V. Kac and D. Valeri.

The Poisson Lie algebra, Rumin's complex and base change

Alessandro D'Andrea, Sapienza Università di Roma

Hopf actions on vertex operator algebras

Chongying Dong, Santa Cruz

We consider actions of a Hopf algebra H on a vertex operator algebra V. If H is semisimple and the Hopf action is inner faithful, we prove that H is a group algebra. Actually, we prove that the inner faithfulness is equivalent to faithfulness

Generators for quantum finite W-algebras in type A

Laura Fedele, Sapienza Università di Roma

For a quantum finite W-algebra in type A, associated to an arbitrary nilpotent element f, we construct a matrix of Yangian type $L(z)$ which encodes its generators and relations. The matrix $L(z)$, defined as a generalized quasideterminant, is exactly the quantum analogue of the matrix pseudodifferential operator of Adler type, defined by De Sole, Kac and Valeri, that encodes the W-algebra structure in the classical affine case. We moreover define a matrix $W(z)$ whose coefficients provide a finite set of generators for the W-algebra, satisfying Premet's conditions, from which the matrix $L(z)$ can be reconstructed as a generalized quasideterminant.

The first chiral homology group

Reimundo Heluani, IMPA

We prove under some finiteness condition that the first chiral homology group of an elliptic curve (in the nodal limit) with coefficients on vertex algebra is determined by the first Hochchild homology group of its Zhu algebra.

Integrable Hamiltonian hierarchies of PDE, associated to classical affine W-algebras, attached to classical Lie algebras and their nilpotent elements

Victor Kac, MIT

In my talk I will explain first the basics of the theory of Hamiltonian integrable systems in the framework of Poisson vertex algebras. Then Ill explain how to construct such a system for each classical Affine W-algebra, attached to a classical Lie algebra and its nilpotent orbit.

The characters of relaxed highest-weight modules over affine Kac-Moody algebras

Kazuya Kawasetsu, The University of Melbourne

The relaxed highest-weight modules over affine Kac-Moody algebras play an important role in the Creutzig-Ridout Verlinde formula for admissible affine vertex algebras. In this talk, we compute the characters of the irreducible relaxed highest-weight modules over the affine Kac-Moody algebra $\widehat{sl}_2$ induced from the dense irreducible modules over sl_2, using Mathieu's coherent families. We show that the characters are "coherent", that is, they are the product of a q-series and a formal delta function in z. If time allows, we will also consider the characters of relaxed highest-weight modules over $\widehat{osp}(1|2)$ and $\widehat{sl}_3$. This is a joint work with David Ridout.

Quantum groups and Nichols algebras acting by screening operators

Simon Lentner, University of Hamburg

In recent work I prove that (short) screening operators acting on a general lattice VOA fulfill the relations of an associated Nichols algebra. As a special case, for rescaled root lattices this proves quantum group relations, which was conjectured for some time.

The key step is to see that certain analytic functions (generalized Selberg integrals) have zeroes, wherever the Nichols algebra has a relation. This fact is proven by an analytic quantum symmetrizer formula, and is of independent interest.

The intended application is that the kernel of screenings produces interesting W-algebras, and the algebra of screenings appears in their representation category.

Iterated integrals and (mock) modular forms

Antun Milas, SUNY Albany

We present a generalization of Eichler and Mordell integrals (joint work with Bringmann and Kaszian). This concept is useful for understanding modularity of characters of certain vertex algebras.

Arc spaces and chiral symplectic cores

Anne Moreau, University of Lille

We introduce the notion of chiral symplectic cores which can be viewed as chiral analogs of symplectic leaves. As an application we show that any quasi-lisse vertex algebra is a quantization of the arc space of its associated variety, in the sense that its reduced singular support coincides with the arc space of its associated variety. We also show that the coordinate ring of the arc space of Slodowy slices is free over its vertex Poisson center, and the latter coincides with the vertex Poisson center of the coordinate ring of the arc space of the corresponding simple Lie algebra. This is a joint work with Tomoyuki Arakawa.

Classification of conformal embeddings in affine and W algebras

Pierluigi Möseneder Frajria, Politecnico di Milano

An embedding of vertex operator algebras is said conformal if the corresponding Virasoro vectors coincide. We classify conformal embeddings in various special cases, starting from the classical results on embeddings in affine vertex algebras at integrable level to embeddings in the affine Lie superalgebra case as well as in W algebras corresponding to minimal nilpotent orbits. This is joint work with Adamovic, Kac, Papi and Perse.

Tensor Categories for Vertex Operator (Super)-Algebra Extensions

Shashank Kanade, University of Alberta

Let V be a vertex operator algebra and let $\mathcal{C}$ be a suitable category of modules for V that can be endowed with a vertex tensor category structure in the sense of Huang-Lepowsky. (For instance, by a theorem of Huang, V may be C_2 cofinite CFT-type with $\mathcal{C}$ being the category of finite length modules.) Huang-Kirillov-Lepowsky showed that certain algebra objects in the tensor category $\mathcal{C}$ are in one-to-one correspondance with conformal embeddings $V \hookrightarrow A$ and that a certain category $Rep^0(A)$ (whose construction is completely categorical) related to $\mathcal{C}$ is equivalent as an abelian category to the category of untwisted modules for the VOA A. In this talk, I'll explain how this is equivalence is far richer: it is in fact a braided tensor equivalence. Consequently, there exists a natural induction functor for conformal embeddings that is actually monoidal and also preserves the vertex tensor structure. This is joint work with Thomas Creutzig and Robert McRae. Time permitting, I'll mention an application (joint work with Thomas Creutzig and Jesse Frohlich) to the representation theory of vertex operator algebras based on $\widehat{\mathfrak{osp}(1|2)}$.

Dirac index and twisted characters

Pavle Pandžić , University of Zagreb

Dirac operators have played an important role in representation theory of real reductive Lie groups since the work of Parthasarathy and Atiyah-Schmid on the construction of discrete series representations in the 1970s.

One of the important invariants of representations is the Dirac index. An algebraic way to define the Dirac index is as the Euler characteristic of the Dirac cohomology of the associated Harish-Chandra module. The concept of Dirac cohomology was introduced by Vogan and subsequently studied by Huang-Pandžić and others. One of the important properties of the Dirac index of a representation in the equal rank case is its close relationship with the character on the compact Cartan subgroup.

In the unequal rank case, the Dirac index of all representations is zero and therefore it is a useless notion. We have however introduced a new invariant, twisted Dirac index, which is a good substitute for the classical notion in the unequal rank cases. In this lecture I will first review some basic facts about representations, Harish-Chandra modules, Dirac cohomology and index. I will then explain the notion of twisted Dirac index and present some examples and applications.

This is joint work with Dan Barbasch and Peter Trapa.

Conformal embeddings and associated vertex algebras

Ozren Perše, University of Zagreb

In this talk we will review some recent results on conformal embeddings of affine vertex algebras, obtained in joint works with D. Adamović, V. G. Kac, P. Möseneder Frajria and P. Papi. It turns out that in many examples of conformal embeddings, the structure theory and representation theory of associated vertex algebras is not very well understood so far. Such cases include affine vertex algebras of negative integer level. A particular emphasis of the talk will be given on the understanding of properties of such vertex algebras.

Mishchenko-Fomenko subalgebras for centralizers and affine $W-$algebras

Alexander Premet, University of Manchester

Let L be the centralizer of a nilpotent element in a finite dimensional semisimple Lie algebra. In my talk, based on a joint work with Arakawa, I will explain how to lift Mishchenko-Fomenko subalgebras of of S(L) to commutative subalgebras of $U(L)$. The construction relies in a crucial way on affine W-algebras at the critical level.

Some combinatorial coincidences for standard representations of affine Lie algebras

Mirko Primc, University of Zagreb

In some cases the combinatorial parametrizations of bases of standard modules for affine Lie algebras coincide with the combinatorial parametrizations of bases of Feigin-Stoyanovsky-type subspaces of (some other) standard modules. A particular case is the correspondence of the combinatorial bases of $A_1^{(1)}$–vacuum module $L(k\Lambda_0)$ and the combinatorial bases of Feigin-Stoyanovsky-type subspaces of $B_2^{(1)}$–vacuum modules.

Generalised Galilean W-algebras

Chris Raymond

The Galilean contraction procedure presents a method for generating new vertex operator algebras from existing ones, similar to Inonu-Wigner contractions of Lie algebras. We discuss generalising this procedure from tensor products of pairs of VOAs to higher-order products. This gives rise to a new infinite family of W-algebras. This is joint work with Jrgen Rasmussen.

Parafermion vertex operator algebras

Li Ren, Sichuan University, China

Associated to finite dimensional simple Lie algebras and positive integers, parafermion vertex operator algebras have origins in the Z-algebras for constructing irreducible modules for affine Kac-Moody algebras in mathematics and parafermion conformal field theory in physics. This talk will report our recent work on the representation theory for the parafermion vertex operator algebras. There are three main results: (1) The parafermion vertex operator algebras are rational, (2) The irreducible modules are classified, (3) The quantum dimensions and the fusion rules are determined.

Towards higher-rank logarithmic CFTs

David Ridout, University of Melbourne

Rational CFTs are generally constructed from irreducible highest-weight modules for some VOA. On the other hand, logarithmic CFTs are constructed from modules that are not all irreducible but also need not be highest-weight.

Non-highest-weight modules appear, in particular, in the LCFTs corresponding to admissible level affine VOAs. These models are fairly well understood for $sl(2)$, but not at all otherwise. I will discuss current work that aims to extend our understanding to $sl(3)$.

This is a joint work with K. Kawasetsu and S. Wood.

Generalised Galilean W-algebras

Christopher Raymond, University of Queensland

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Construction and classification of holomorphic vertex operator algebras

Nils Scheithauer, University of Darmstadt

Dimension Formulae and Uniqueness of Vertex Algebras

Jethro van Ekeren, Universidade Federal Fluminense, Brazil

The component of degree 1 of a regular vertex algebra carries the natural structure of a reductive Lie algebra. The question of completeness of this invariant naturally arises, i.e., "To what extent does the Lie algebra determine the vertex algebra up to isomorphism?". Combining tools from the theory of modular curves of genus zero and finite automorphism groups of simple Lie algebras, we show completeness of the invariant in certain cases. Joint work with Sven Möller and Nils Scheithauer.