Summer School “Optimal Control of PDEs“; Cortona (Italy), July 12-17, 2010-02-09

Mini-Course “Introduction to Optimal Control Problems for PDEs”

Jürgen Sprekels, Weierstrass Institute for Applied Analysis and Stochastics (WIAS), Mohrenstr. 39, 10117 Berlin, Germany

This series of lectures will provide an introduction into the theory of optimal control problems involving elliptic and parabolic partial differential equations. We will discuss basic concepts such as admissibility, existence and uniqueness of optimal controls, necessary and sufficient optimality conditions, Lagrange multipliers, and adjoint state equations. While this series of introductory lectures will mainly focus on linear-quadratic problems, we will also touch more advanced topics like nonlinear state equations, state constraints and general optimization theory in Banach spaces. Numerical aspects will not be discussed, since they will be covered by the other lecturers.

Recent monographs in the field:

F. Tröltzsch: *Optimal Control of Partial Differential Equations: Theory, Methods and Applications.* To appear spring 2010 in Graduate Studies in Mathematics, American Mathematical Society

P. Neittaanmaki, J. Sprekels, D. Tiba: *Optimization of Elliptic Systems: Theory and Applications*. Springer Monographs in Mathematics, Springer, New York 2006.

Contents

Lecture 1: Basic concepts

1. General problem statement and examples for optimal control problems involving elliptic and parabolic state equations
2. Motivation of basic concepts by reviewing finite-dimensional theory
3. General existence results

Lecture 2: Elliptic control problems

1. Existence, uniqueness and regularity of weak solutions to elliptic boundary value problems
2. Differentiation in normed spaces, adjoint operators
3. First-order necessary optimality conditions
4. The formal Lagrange method
5. Regularity of optimal controls

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Lecture 3: Linear-quadratic parabolic control problems

1. General functional analytic setting in spaces of vector-valued functions
2. Well-posedness of parabolic initial-boundary value problems
3. First-order necessary optimality conditions

Lecture 4: Necessary optimality conditions for general optimization problems in Banach spaces

1. General setting for problems involving control and state constraints
2. The convex case: subdifferential calculus and mathematical programming
3. The nonconvex differentiable case
4. Applications

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