Reduced Invariant Sets

Let K be a compact Lie group, W a K-module and $X \subset W$ real algebraic and K-stable. Let $I(X)$ be the ideal of X in $\mathbb{R}[W]$ and let $I_K(X)$ be generated by $I(X) \cap \mathbb{R}[W]^K$. For which X do we have that $I(X) = I_K(X)$? We give necessary conditions and sufficient conditions for this to be true.

Complexifying one is forced to consider the following question. Let G be complex reductive and V a G-module. Let $I(\mathcal{N}(V))$ be the ideal of the null cone $\mathcal{N}(V)$ (as a subset of V) and let $I_G(\mathcal{N}(V))$ be generated by $I(\mathcal{N}(V)) \cap \mathbb{C}[V]^G$. When do we have that $I(\mathcal{N}(V)) = I_G(\mathcal{N}(V))$? We give classifications of V which have (or do not have) this property.