IN RICORDO DI BRUNO

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Stochastic Processes, Orderings and Dependence

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Bruno's Smile

Joint works with Bruno

- B.Bassan-C.Ceci: "An optimal stopping problem arising from a decision model with many agents", Probab. Engrg. Inform. Sci., 12, 3, 393-408, 1998.
- B.Bassan-C.Ceci: "Optimal stopping with discontinuous reward: regularity of the value function and viscosity solution", Stochastics and Stochastics Reports., 72, 55-77, 2002
- B.Bassan-C.Ceci: "Regularity of the value function and viscosity solutions in optimal stopping problems for general Markov processes", Stochastics, 74, 633-649, 2002.
- C.Ceci-B.Bassan: "Mixed optimal stopping and stochastic control problems with semicontinuous final reward for diffusion processes", Stochastics, 76 (4) 323-337, 2004.

Description of the Problem

- A Markov process $X = \{X_t\}$ of generator $\mathcal{L}$;
- A reward function $g : \mathbb{R}_+ \times \mathbb{R} \to [0, K]$ (positive, bounded)

The goal

- Find an optimal stopping time τ^* which realizes

$$\sup_{\tau} E[g(\tau, X_\tau)]$$

- Characterize the associated value function

$$w(t,x) = \sup_{\tau \geq t} E_{(t,x)}[g(\tau, X_\tau)].$$

($P_{(t,x)}$ denotes the unique solution of the martingale problem for $\mathcal{L}$).

MAIN RESULTS

Assumption

- *The martingale problem for $\mathcal{L}$ (MGP) is well-posed*
- *the solution to the MGP is Feller*
- *the reward g is bounded*
- *if $f \in C_b^{1,2}([0,T] \times \mathbb{R}^d)$ and f has compact support then $f \in \mathcal{D}$ and $\mathcal{L}f(t,x)$ is jointly continuous.*

Theorem (REGULARITY OF THE VALUE FUNCTION)

Under the previous assumptions

- *If g is l.s.c. then w is l.s.c.*
- *If g is u.s.c. then w is u.s.c.*
- *If g is continuous then w is continuous*

We introduced a new concept of viscosity solution of the following variational problem

$$min\{-\mathcal{L}w, w-g\} = 0.$$

$$(w \geq g; \quad \mathcal{L}w \leq 0; \quad \mathcal{L}w \times (w-g) = 0)$$

since g and w are not necessarily continuous but only semicontinuous.

Theorem (VISCOSITY SOLUTION)

- *If g is l.s.c. and $w \geq g^*$ (g^* denotes the u.s.c. envelope of g) then w is a viscosity solution of*

$$min\{-\mathcal{L}w, w-g\} = 0.$$

- *If g is continuous then w is a continuous viscosity solution of the above problem.*

- The previous results have been extended to mixed optimal stopping and stochastic control problems for diffusion processes in C.-Bassan 2004.

- The main results of the joint works with Bruno had been presented to the Symposium "Optimal Stopping with Applications ", University of Manchester (UK), January, 2006 (invitation by prof. S. Jacka and G. Perskin).