Conference on Nonlinear Evolution Problems

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ABSTRACTS

Catherine Bandle

University of Basel
Basel, Switzerland – c.bandle@gmx.ch

Quasilinear elliptic and parabolic problems with a Hardy potential

Abstract

We consider problems of the type $\Delta u + V(x)u = u^p$ in a bounded domain in $\mathbb{R}^n$ where V is a Hardy potential $\frac{\mu}{\delta^2(x)}$ and $\delta(x)$ is the distance from a point x to the boundary of the domain. We are interested in the existence of positive solutions, and the interplay between the nonlinearity and the boundary singularity. If $0 < p < 1$ the nonlinearity gives rise to dead cores and if $p > 1$ to boundary blowup. We give a fairly complete picture of the radial solutions and use those solutions as upper and lower solutions for general domains. At the end we discuss the corresponding parabolic problem. Some results are known in the literature for the linear problem. They are related to the Hardy constant. The talk reports on results obtained in collaboration with V. Moroz (Swansea), W. Reichel (KIT Karslruhe) and M. A. Pozio (Sapienza Rome).

Henri Berestycki

Centre d'Analyse et de Mathématiques Sociales, EHESS
Paris, France – hb⊙ehess.fr

Propagation in non homogeneous media - the effect of geometry

Abstract

I will discuss bi-stable reaction-diffusion equations in cylinders with varying cross-sections motivated by biology and medicine. The aim is to understand the effect of the non-homogeneous medium on propagation or blocking of advancing waves. The role played by the geometry of the domain of propagation is of particular interest for these models. I will report on joint work with Juliette Bouhours and Guillemette Chapuisat.

Michiel Bertsch

Dipartimento di Matematica, Università di Roma "Tor Vergata"
Rome, Italy – bertsch⊙mat.uniroma2.it

Travelling wave solutions of a system of PDE's

Abstract

We consider travelling wave solutions of a parabolic-hyperbolic system. For certain parameter values the structure of the travelling waves reminds that of the scalar Fisher-KPP equation. For other parameter regimes new phenomena occur.

Lorenzo Giacomelli

Dipartimento SBAI - Sezione di Matematica, Sapienza di Roma
Rome, Italy – giacomelli⊙dmmm.uniroma1.it

Well-posedness for the Navier-slip thin-film equation in complete wetting

Abstract

We are interested in the thin-film equation with quadratic mobility, modeling the spreading of a liquid film with a Navier-slip condition at the solid substrate. This degenerate fourth-order parabolic equation has the contact line (where liquid, solid, and vapor meet) as a free boundary. There, a zero-contact angle condition is imposed, modeling the so-called “complete wetting” regime.

We first argue that the self-similar source-type solution, once its leading order profile is factored-off, is analytic as a function of two variables (x, x^β) with β irrational, where x denotes the distance from the contact line. Motivated by this preliminary, we then argue that the full free-boundary problem is well-posed in weighted L^2-spaces whose norm captures the leading order terms of the (x, x^β)-expansion.

This is part of a joint project with Manuel V. Gnann, Hans Knüpfer, and Felix Otto.

Brian H. Gilding

Department of Mathematics, College of Science, Kuwait University
Kuwait City, Kuwait – gilding⊙mcs.sci.kuniv.edu.kw

One-dimensional free-boundary problems in inventory control

Abstract

In its simplest form, the problem to be studied concerns a warehouse holding a single perishable item. The number of items held in stock will decrease due to consumption upon demand, and, to losses caused by deterioration. When the costs involved are solely those of purchasing the item and storing it, the best policy of the inventory holder is to replenish the stock as and when the demand requires. However, when placing an order for replenishment of stock also incurs a cost; there is a play-off between the cost of these orders, and the cost of holding a large stock with its concurrent losses. The resulting optimization problem may be viewed as an evolution problem in which time is the only independent variable, and the moments at which the orders are to be made are free boundaries. Recent results and open questions will be reviewed.

Jesús Hernández

Departamento de Matemáticas, Universidad Autónoma de Madrid,
Madrid, Spain – jesus.hernandez⊙uam.es

On linearized stability results for positive solutions to semilinear singular elliptic problems

Abstract

Results concerning linearized stability for positive solutions to some semilinear singular elliptic problems on bounded domains have been obtained by Bertsch-Rostamian (working in weighted Sobolev spaces), Ambrosetti-Brezis-Cerami (usual Sobolev space), Hernàndez-Mancebo-Vega($C(\Omega)$) and Dhanya et al. (Sobolev spaces). Here we review this previous work and study the problem in the framework of very weak solutions using recent work by Díaz-Rakotoson.

This is joint work with J.I.Díaz.

Danielle Hilhorst

Département de Mathématiques, Université Paris-Sud
Orsay, France – Danielle.Hilhorst⊙math.u-psud.fr

On the large time behavior of solutions of a nonlocal evolution equation

Abstract

We consider an initial value problem for a nonlocal differential equation with a bistable nonlinearity in several space dimensions and discuss the large time behavior of the solution. The proof that the solution orbits are relative compact is based upon rearrangement theory. This is joint work with Hiroshi Matano, Thanh Nam Nguyen and Hendrik Weber.

Mimmo Iannelli

Dipartimento di Matematica, Università di Trento
Trento, Italy – iannelli⊙science.unitn.it

A model for describing the structure and growth of epidermis

Abstract

In this talk we propose a model with age and space structure for the evolution of the supra–basal epidermis. The model will include different categories of cells: proliferating cells, differentiated cells, corneous cells, and apoptotic cells. We assume that all cells move with the same velocity and that the local volume fraction, occupied by the cells is constant in space and time. This hypothesis, based on experimental evidence, allows to determine a constitutive equation for the cell velocity.

We investigate the well–posedness of the problem determining conditions for the existence of a moving boundary representing the surface of the skin. We also consider stationary case of the problem, that takes the form of a quasi–linear evolution problem of first order, and investigate the conditions under which we have a solution.

Some numerical simulations will also be shown.

Shoshana Kamin

Department of Mathematical Sciences, Tel Aviv University
Tel Aviv, Israel – kamin⊙post.tau.ac.il

Prescribed conditions at infinity for parabolic equations

Abstract

We are concerned with existence and uniqueness of the solutions for linear and nonlinear parabolic equations with time–dependent coefficients, in the class of bounded solutions satisfying appropriate conditions at infinity. That is a joint work with Fabio Punzo.

Robert Kersner

Pollack Mihály Faculty of Engineering, University of Pécs
Pécs, Hungary – kersner⊙pmmk.pte.hu

On a cross-diffusion PDE system

Abstract

The natural generalization of parabolic systems includes the use of cross-diffusion, for which the flows ("fluxes") are written as

$$J_1 = -(uu_x + e_1 uv_x + e_3 vu_x),$$

$$J_2 = -(dvv_x + e_4 uv_x + e_2 vu_x).$$

Cross-diffusion models have been considered in a number of studies in physical (plasma physics), chemical (dynamics of electrolyte solutions), and biological (cross-diffusion transport) systems. The same refers to population dynamics and ecological (forest age- structure dynamics) studies. The Burridge – Knopoff model was used to describe interactions between tectonic plates in seismology. Mathematical models with cross- diffusion have been extensively employed in the past decade to gain insight into the mechanisms of tumor growth and development.

I will consider a two–equation system with "fluxes" J_1, J_2 and the reaction terms are of competition type. I shall speak mainly on existence and stability of nonconstant stationary solutions, on some limit cases when $e_i \to 0$ and mention some recent numerical results too.

Corrado Mascia

Dipartimento di Matematica "Guido Castelnuovo", Sapienza di Roma
Rome, Italy – mascia⊙mat.uniroma1.it

Hyperbolic variations of the Allen-Cahn equation

Abstract

A well–estabilished prototype in the area of evolutive nonlinear partial differential equations is the Allen–Cahn equation (also known as bistable reaction diffusion or Nagumo or, occasionally, Ginzburg–Landau equation) consisting of the combination of a transport mechanism of diffusive type and a zero-order reactive term which determines the presence of two stable constant states. The fundamental question is to understand and describe the interactions of such attractive states.

The main concern of the talk is to discuss a class of hyperbolic equation in the presence of a reaction term of Allen-Cahn type, motivated by the assumption that the alignment of the flux term with the gradient of the unknown function is not istantaneous but delayed by the presence of a relaxation time. After a brief overview on the derivation of such class of equations starting from some appropriate modelling assumptions, emphasis will be given to the topic of front propagation in one dimension and (hopefully) in several dimensions. Some rigorous results concerning existence and stability of planar fronts will be presented, comparing it with the corresponding results for the standard parabolic Allen–Cahn equation.

Hiroshi Matano

Graduate School of Mathematical Sciences, University of Tokyo
Komaba, Tokyo, Japan – matano⊙ms.u-tokyo.ac.jp

Propagating terrace for multi-stable nonlinear diffusion equations

Abstract

In this talk we will consider semilinear diffusion equations on $\mathbf{R}^N$ with multi-stable nonlinearities and discuss the long-time behavior of solutions with compactly supported non-negative intial data. We will show that every such solution behaves like what we call a "radially symmetric propagating terrace". This is joint work with Yihong Du.

Masayasu Mimura

Meiji Institute for Advanced Study of Mathematical Sciences, Meiji University
Nakanoku, Tokyo, Japan – mimura⊙math.meiji.ac.jp

Spatio-temporal oscillations in the Keller-Segel system with logistic growth

Abstract

The Keller-Segel system with logistic growth is discussed from spatio-temporal oscillation point of view. It is already reported in [1] that this system exhibits spatio–temporal regular and irregular patterns in the certain distinct parameter regimes. In this talk, we show the occurrence of a new type of spatio-temporal patterns. Furthermore, we demonstrate that the onset of such spatio–temporal patterns is an infinitely dimensional relaxation oscillation [2] and reveal the mechanism of occurrence of the relaxation oscillation. This result is obtained as a joint work with S.-I. Ei and H. Izuhara.

[1] K. J. Painter and T. Hillen, Spatio-temporal chaos in a chemotaxis model, *Physica D*, **240** (2011) 363–375.

[2] S.-I. Ei, H. Izuhara and M. Mimura, Infinite dimensional relaxation oscillation in aggregation-growth systems, *Discrete Contin. Dyn. Syst. Ser. B* **17** (2012), 1859–1887.

Roberto Natalini

Istituto per le Applicazioni del Calcolo "Mauro Picone", CNR
Rome, Italy – roberto.natalini⊙cnr.it

Hyperbolic perturbations of scalar laws: from oil recovery to vasculogenesis

Abstract

In this talk I will review some problems where hyperbolic non–equilibrium perturbations of scalar equations are considered to improve the predictive accuracy of some models, both in the transitory and in the asymptotic behavior of the solutions. Such models arise in a number of frameworks. Here I just focus on problems in filtration, stem cells organization, and vasculogenesis, and I will show as the output result can greatly vary according to the considered specific problem.

Wei–Ming Ni

Center for PDE, East China Normal University, Shanghai, P. R. China,
and
School of Mathematics, University of Minnesota, Minneapolis, U. S. A.
wmni⊙umn.edu

Global dynamics of heterogeneous Lotka-Volterra competition– diffusion systems

Abstract

In this talk, we will discuss the joint effects of diffusion and spatial variation on the global dynamics of a classical Lotka– Volterra competition system. A complete understanding of the change in dynamics is obtained in terms of diffusion rates, and various special cases will be discussed to illustrate the understandings.

Adriano Pisante

Dipartimento di Matematica "Guido Castelnuovo", Sapienza di Roma
Rome, Italy – pisante⊙mat.uniroma1.it

Allen-Cahn approximation of mean curvature flow on Riemannian manifolds

Abstract

For a general class of initial data, we prove convergence of solutions to the parabolic Allen-Cahn equation to Brakke's motion of generalized hypersurfaces by mean curvature in Riemannian manifolds with Ricci curvature bounded from below, extending previous results of Ilmanen in the Euclidean space. In analyzing the evolution of the limit energy density in the framework of rectifiable varifolds a key role is played by a local monotonicity formula which is available once we can control the energy discrepancy. Application to the mean curvature flow of complete hypersurfaces with prescribed boundary at infinity on hyperbolic space will be also discussed (joint with Fabio Punzo).

Andrea Terracina

Dipartimento di Matematica "Guido Castelnuovo", Sapienza di Roma
Rome, Italy – terracin⊙mat.uniroma1.it

Uniqueness and non-uniqueness results for entropy solutions of forward–backward parabolic problems

Abstract

We consider an entropy formulation for a forward–backward parabolic problem. More precisely we study a parabolic problem where the diffusion function is of cubic type. Obviously standard Cauchy-Dirichlet boundary problems are ill posed. Then we introduce an entropy formulation by considering a Sobolev approximation of the problem. We analyze existence uniqueness and some qualitative behavior for this kind of solutions. In particular we study the "two phase problem" in which initial datum takes values only in two of the three monotone regions of the diffusion function. This allows to simplify the problem showing that in the stable–stable case it is possible to have a good formulation of the problem. We present some recent results regarding the stable–unstable case. Moreover we give a counter-example of non-uniqueness in the general context of entropy solutions.

1) A. Terracina: *Qualitative behaviour of the two-phase entropy solution of a forward-backward parabolic problem*, SIAM J. Math. Anal. 43 (2011), 228–252.

2) A. Terracina: *Two–phase entropy solutions of forward–backward parabolic problems with unstable phase*, preprint arXiv: 1310.7728 (2013).

3) A. Terracina: *Non-uniqueness results for entropy two-phase solutions of forward-backward parabolic problems with unstable phase*, J. Math. Anal. Appl. 413 (2014), 963–975.

Laurent Véron

Laboratoire de Mathématiques et Physique Théorique, Université "François Rabelais"
Tours, France – veronl⊙univ-tours.fr

Initial trace of positive solutions of weakly superlinear parabolic equations

Abstract

We study the initial trace of positive solutions of (E)

$$\text{(E)} \quad \partial_t u - \Delta u + g(u) = 0 \quad \text{in } \mathbb{R}_+ \times \mathbb{R}^N$$

in the particular case where g is a continuous nondecreasing function vanishing at 0 which satisfies the following properties

$$\text{(I)} \qquad \int^{\infty} \frac{ds}{g(s)} < \infty,$$

and

$$\text{(II)} \qquad \int^{\infty} \frac{ds}{\sqrt{\int_0^s g(t)dt}} = \infty.$$

The first condition implies that there exists a *flat maximal solution* ϕ_∞ of (E) i.e. the solution of

$$\begin{array}{ll} \phi' + g(\phi) = 0 & \text{on } \mathbb{R}_+ \\ \lim_{t\to\infty} \phi(t) = \infty. & \end{array}$$

The second condition implies that for any $a >$, the maximal solution v_a of

$$\begin{array}{rl} -v'' - \frac{N-1}{r}v' + g(v) = 0 & \text{on } (0, r_{max}) \\ v(0) = a & \\ v'(0) = 0 & \end{array}$$

is defined on whole $\mathbb{R}_+$. These solutions are called the *stationnary solutions* and the mapping $a \mapsto v_a$ is increasing. Typical examples $g(r) = r(\ln r)^\alpha$ with $1 < \alpha \le 2$.

Theorem 1 *The initial trace of a positive solution of (E) exists and is a locally bounded Borel measure.*

The minimal solution with initial data u_0 is the increasing limit of the solutions u^k ($k \in \mathbb{N}^*$) which satisfy $u^k(.,0) = \min\{u_0, k\}$. However other solutions may exist.

Theorem 2 *For any $u_0 \in C(\mathbb{R}^N)$ satisfying $v_a(x) \leq u_0(x) \leq v_b(x)$ there exist at least two distinct solutions $\underline{u}$ and $\overline{u}$ of (E) with initial data u_0 wich satisfy*

$$\underline{u}(x,t) \leq \min\{\phi_\infty(t), v_b(x)\} \quad \text{and} \quad v_a(x) \leq \overline{u}(x,t) \leq v_b(x)\,,$$

$$\forall (x,t) \in \mathbb{R}_+ \times \mathbb{R}^N.$$

When u_0 grows to much at infinity, the minimal solution is the flat maximal solution and the maximal one is infinity. This establishes a maximal growth of the initial trace of the positive solutions of (E).

Theorem 3 *Assume that $g(r) = o(r \ln^2 r)$ when $r \to \infty$, that is $g(r) = rh(r)$ with $h(r) = (\frac{\ln r}{\ln \eta(r)})^2$ and $\lim_{s\to\infty} \eta(s) = \infty$. If $u_0(x) = e^{\eta^{-1}(\ell|x|)}$ at ∞ for some $\ell > 1$, there holds $\underline{u} = \phi_\infty$ and $\overline{u} = \infty$.*