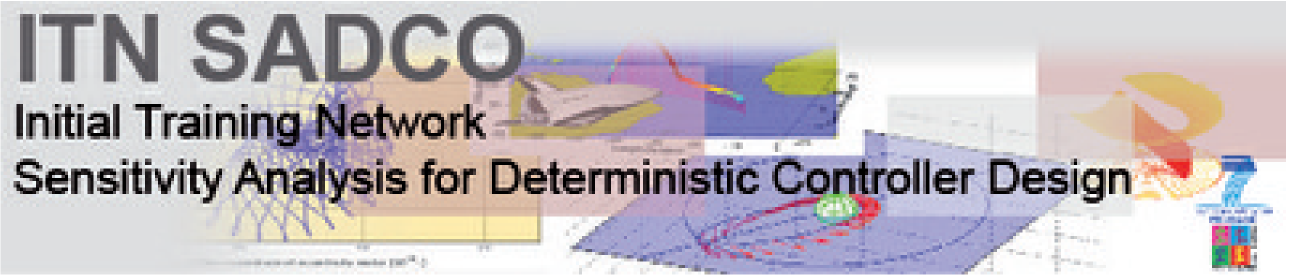

SADCO Internal Review Meeting 2014

Dipartimento di Matematica, SAPIENZA, Rome
November 10-12, 2014

New Perspectives in Optimal Control and Games

BOOK OF ABSTRACTS

This event is co-funded by the European Union under the 7th FP
Marie Curie Initial Training Network SADCO
Grant Agreement number 264735-SADCO

INVITED TALKS

Stochastic Dynamic Teams and Games with Asymmetric Information

M. T. Basar

Univ. of Illinois, Urbana-Champaign

In any real application of stochastic decision making, be it in the cooperative team framework or the non- cooperative game setting, asymmetry in the information acquired by different decision makers (synonymously agents or players) naturally arises. Presence of asymmetric information, particularly in dynamic (multi-stage) decision problems, creates challenges in the establishment of existence of optimal solutions (in teams) and non-cooperative equilibria (in games) as well as in their characterization and computation. No unified theory exists (such as dynamic programming or maximum principle) that would be applicable to such problems. In this talk, I will discuss our efforts toward developing such a unified theory with regard to existence of solutions. For stochastic dynamic teams, the framework will encompass problems with non-classical information, such as Witsenhausen counterexample (and its multi- dimensional extensions) and the Gaussian test channel (and its multi-relay versions with real-time information processing and transmission), among others, for which the existence of team-optimal solutions will be obtained. For stochastic dynamic games with asymmetric information, the existence of Nash equilibria will be established using the newly introduced refinement concept of common information based Markov perfect equilibrium. The approach for games entails establishing an equivalence between the original game and an appropriately constructed one in a higher dimensional space, with symmetric and perfect information, and with virtual players. For dynamic teams, the approach first lifts the analysis to the space of behavioral strategies, establishing existence in that richer space, and then brings the solution down to the original team problem while respecting the informational relationships. Several examples will be provided to illustrate the solution technique, the underlying caveats, and the conditions involved. Some open problems and future directions for research will be identified.

◇
◇◇◇◇◇
◇

A Level-set Crystalline Mean Curvature Flow

Y. Giga

University of Tokyo

It is by now standard that a level-set approach provides a global unique generalized solution (up to fattening) for mean curvature flow equations [G]. Even from the early stage of the theory, it is known that the method is very flexible to apply anisotropic curvature flow equations which correspond to anisotropic interfacial energy. The anisotropy is very important in materials sciences. However, if the anisotropy is very singular for example a crystalline mean curvature flow corresponding to crystalline interfacial energy, even local-in-time existence of a solution was not known except for convex initial data [BCCN]. A level set approach was not available except evolution of curves to which a foundation of the theory was established by M. H. Giga and Y. Giga more than ten years ago. In this talk we push forward a level-set approach to surface evolution by crystalline mean curvature. The main difficulty is that crystalline mean curvature is a nonlocal quantity and it may not be a constant on each flat potion of a surface. (In the case of curve evolution it is always a constant over a segment.) We overcome this difficulty by introducing a suitable notion of viscosity solutions so that a comparison principle holds. We further construct a global-in-time solution as a limit of smoother problem. A delicate analysis is necessary to achieve the goal. A similar but a simpler problem was studied in [MGP1], [MGP2]. We elaborate these approaches for our purpose.

References

1. G. Bellettini, V. Caselles, A. Chambolle, M. Novaga, Crystalline mean curvature flow of convex sets. Arch. Ration. Mech. Anal. 179 (2006), 109-152.
2. Y . Giga, Surface evolution equations. A level set approach. Monographs in Mathematics, 99. Birkhauser Verlag, Basel, 2006. xii+264 pp.
3. M. H. Giga, Y. Giga and N. Pozar, Periodic total variation flow of non-divergence type in R^n. J. Math. Pures Appl. (9) 102 (2014), 203-233.
4. M. H. Giga, Y. Giga and N. Pozar, Anisotropic total variation flow of non-divergence type on a higher dimensional torus. Adv. Math. Sci. Appl. 23 (2013), 235-266.

◊
◊◊◊◊◊
◊

Speeding Up Model Predictive Control and Model Predictive Regulation

Arthur Krener

Naval School, Monterey

Model Predictive Control (MPC) is an increasingly popular way of stabilizing a process to a set point. Recently we have offered a new way of tracking reference signals and/or rejecting disturbances called Model Predictive Regulation (MPR). MPC is widely used in the chemical processing industries where the dynamics is slow enough so that the time steps can be long enough to solve the nonlinear program in a small fraction of one time step. We will offer several suggestions about how to speed up MPC and MPR so that it can be applied to systems with faster dynamics. Our ultimate goal is to speed up

MPC and MPR enough so that they can be used to control constrained, fast and unstable systems such as a high performance airplane or helicopter.

◊
◊◊◊◊◊
◊

MFG and statistical learning

Jean-Michel Larsy

Univ. Paris Dauphine

Large set of data are often produced by large community of agents involved in strategic equilibrium contexts : ie, games with a large number of players. Then the modeling of this equilibrium and the algorithmic treatment of the data, is often an MFG issue.

◊
◊◊◊◊◊
◊

Optimal control of phase field systems involving dynamic boundary conditions and singular nonlinearities

Jurgen Sprekels

WIAS, Berlin

We study PDE systems of phase field type which are complemented by dynamic boundary conditions involving the Laplace-Beltrami operator. The evolution both in the bulk and on the boundary of the domain are driven by nonlinearities that exhibit a singular behaviour at the boundaries of their domains of existence. For such systems, distributed and boundary control problems are investigated, where we deal with both (differentiable) logarithmic nonlinearities and the (non-differentiable) case of subdifferentials of double obstacle potentials. First-order necessary and second-order sufficient optimality conditions are derived for the differentiable case, and a so-called "deep quench limit" in the differentiable case leads to first-order necessesary conditions in the nonsmooth double obstacle.

CONTRIBUTED TALKS

Necessary Conditions for Delayed Optimal Control Problems

Andrea Boccia, Richard B. Vinter

Control and Power research group, Imperial College London
London, UK, e-mail: `aboccia@mit.edu`

Necessary conditions are given for optimal control problems with time delay in the state variable. These include a new transversality condition for problems in which the end-time is included in the decision variables (free end-time variables). Examples are presented which illustrate the application of these conditions to the computation of optimal controls.

◊
◊◊◊◊◊
◊

An Hamilton-Jacobi-Bellman approach for stochastic optimal control problems with state constraints.

Olivier Bokanowski

A new approach to state-constrainted stochastic optimal control problems will be presented allowing to characterize the value function of a related problem as the unique solution of a particular Hamilton-Jacobi equation. This is a joint work with A. Picarelli and H. Zidani.

◊
◊◊◊◊◊
◊

Some results on Optimal Control and MPC for the Fokker-Planck equation

A. Fleig, L. Grüne, and R. Guglielmi

Chair of Applied Mathematics

University of Bayreuth, Germany

e-mail: `roberto.guglielmi@uni-bayreuth.de`

In this talk we discuss the optimal control of the Fokker-Planck equation, in connection with the

optimal control of the Probability Density Function (PDF) associated to stochastic processes. Indeed, the PDF associated to a stochastic control system satisfies a Fokker-Planck equation with a control acting through the divergence term. This method is appealing because the state of a stochastic process can be completely characterized in many cases by the shape of its statistical distribution which is represented by the PDF. Therefore, a control methodology defined on the PDF provide an accurate and flexible control strategy that accommodates a wide class of objectives.
The numerical results that we present in this talk extend previous results by Annunziato and Borzì (2010, 2013) in various ways: on the one hand, we apply a Model Predictive Control (MPC) scheme implementing longer prediction horizons $N > 2$. Indeed, MPC computes a feedback law by iteratively solving optimal control problems on a finite time horizon, and it was applied by Annunziato and Borzì with the shortest possible horizon $N = 2$. Particularly, we investigate the interplay between N and the sampling time T and its impact on the quality of the solution. On the other hand, we investigate the improvements which can be achieved when the control is chosen time and space dependent. From a control point of view, this corresponds to a control structure which has both state dependent (i.e., feedback) character but may also vary with time. The additional dependence of the control on the state allows for a significant increase of the quality of the tracking of the MPC feedback.
Joint work with Arthur Fleig and Lars Grüne from the University of Bayreuth.

♢
♢♢♢♢♢
♢

From discrete microscopic models to macroscopic models and applications to traffic flow

Nicolas Forcadel

Normandie University, LMI, INSA of Rouen
Rouen, France, e-mail: nicolas.forcadel@insa-rouen.fr

The goal of this talk is to show how to derive macroscopic models for traffic flow problems from discrete microscopic models. At the microscopic scales, we will consider a Bando model, of the type following the leader, i.e. the acceleration of each vehicle depends on the distance to the vehicle in front of it. After rescaling, we will show that the solution of this system of ODEs converges to the solution of a macroscopic homogenized Hamilton-Jacobi equation which can be seen as a LWR (Lighthill-Whitham-Richards) model.
Joint work with W. Salazar (INSA of Rouen).

♢
♢♢♢♢♢
♢

Large deviation for some stochastic volatility models by viscosity methods

with applications in option pricing

Daria Ghilli

University of Padova
Padova, Italy, e-mail: ghilli@math.unipd.it

The topic of the talk is to present some recent results about short time behavior of stochastic systems affected by a stochastic volatility.
Our first motivation comes from financial models where the volatility is affected by a stochastic mean-reverting process often negatively correlated with the one driving the stock prices (the so-called leverage effect, i.e. prices go up when volatility goes down).
Moreover we put ourselves into the framework of multiple time scale systems and singular perturbation in order to model the typical *bursty* behavior of volatility, (as argued by Fouque, Papanicolau and Sircar in the book *Derivatives in financial markets with stochastic volatilty,*2000)
We study the asymptotics of a logarithmic functional of the log-price process by methods of the theory of homogenization and singular perturbations for fully nonlinear PDEs. We point out three regimes depending on how fast the volatility oscillates relative to the horizon length.
One of the main applications to our results is a large deviation principle for the log-price process (Roughly speaking, large deviation theory concern itself with the exponential decline of the probability measures of *tail events.*)
We then give some further applications in the context of financial mathematics, providing asymptotic estimate out-of-the-money option prices near maturity (small time) and an asymptotic formula for the Black-Scholes implied volatility.
Joint work with M. Bardi and A. Cesaroni (University of Padova).

◊
◊◊◊◊◊
◊

**Exact Penalization applied to
First Order State Constrained Problems**

Igor Kornienko

University of Porto, Faculdade de Engenharia
Porto, Portugal, e-mail: igor@fe.up.pt

We apply exact penalization techniques to optimal control problems with first order state constraints and impose a hypothetical condition ensuring that a set of necessary conditions with absolutely continuous multipliers holds for such class of problems. The hypothetical condition is hard to verify. We show by an example that known conditions asserting regularity of the multipliers do not prevent the appearance of atoms in the multiplier measure

Our accompanying example is treated both numerically and analytically. An extension to cover problems with additional mixed state constraints is also discussed.
Joint work with Maria do Rosário de Pinho (University of Porto).

◊
◊◊◊◊◊
◊

Observability inequalities and their applications to control problems

Paola Loreti

Sapienza, Università di Roma
Roma, Italy, e-mail: paola.loreti@uniroma1.it

In this talk we consider observability and controllability results for vibrating strings and beams, by using Fourier series.
We use well-known results in nonharmonic Fourier series to illustrate the evolutionary problems and to discuss the applications: we focus on the observability of strings and beams.
Then we show new theorems and their applications to solve new observability problems, as observability on small sets.
The talk is based on joint works with V. Komornik

◊
◊◊◊◊◊
◊

Local regularity of the minimum time function

Nguyen Van Luong

Department of Mathematics, University of Padova
Padova, Italy, e-mail: luonghdu@gmail.com

We consider the minimum time problem of optimal control theory. It is well known that under appropriate controllability type conditions the minimum time function $\mathcal{T}$ is locally Lipschitz and, thus, differentiable almost everywhere. Furthermore, in general, $\mathcal{T}$ fails to be differentiable at points where there are multiple time optimal trajectories and differentiability of $\mathcal{T}$ at a point x does not guarantee its continuous differentiability around x. In this work, however, we show that, under

some regularity assumptions, the nonemptiness of proximal subdifferential of $\mathcal{T}$ at a point x implies continuous continuous differentiability of $\mathcal{T}$ on a neighborhood of x..
Joint work with H. Frankowska (University of Paris 6).

◊
◊◊◊◊◊
◊

Singular Perturbations of Stochastic Control Problems with Unbounded Fast Variables

Joao Meireles

University of Padova
Padova, Italy

In this talk we consider singular perturbation problems of a class of optimal stochastic control problems with super linear cost with respect to the control acting on the fast state variables. We will present a convergence result of the value function associated to it to the solution of a limit (effective) Cauchy problem. The methods used are of the theory of viscosity solutions, homogenisation of fully nonlinear PDEs and ergodic stochastic control problems in the whole space $|-R^m$. Work still in progress.

This is a joint work with Martino Bardi (University of Padova) and Guy Barles (Univeristy François Rabelais of Tours).

◊
◊◊◊◊◊
◊

When does relaxation create a strict reduction of cost in optimal control?

Michele Palladino

EEE Department, Imperial College London
London, United Kingdom, e-mail: `m.palladino@imperial.ac.uk`

Relaxation is a regularization procedure used in optimal control, involving the replacement of velocity sets by their convex hulls, to ensure the existence of a minimizer. It can be an important step in the

construction of sub-optimal controls for the original, unrelaxed, optimal control problem (which may not have a minimizer), based on obtaining a minimizer for the relaxed problem and approximating it. In some cases the infimum cost of the unrelaxed problem is strictly greater than the infimum cost over relaxed state trajectories; there is a need to identify such situations because then the above procedure fails. Following on from earlier work by Warga, we explore the relation between, on the one hand, non-coincidence of the minimum cost of the optimal control and its relaxation and, on the other, abnormality of necessary conditions (in the sense that they take a degenerate form in which the cost multiplier set to zero). For optimal control problems in which the dynamic constraint is formulated as a differential inclusion, we show that a local minimizer which is not also a relaxed local minimizer is an abnormal extremal, in the sense that it satisfies an abnormal form of the Hamiltonian inclusion in which the cost multiplier is zero. We also show that a relaxed local minimizer that is not also a local minimizer is a relaxed abnormal extremal. We discuss the extent to which the existence of an infimum gap is also manifested through the existence of abnormal extremals, also for optimal control problems in which the dynamic constraint is formulated as a differential equation with control.
Joint work with R. Vinter (Imperial College London).

◊
◊◊◊◊◊
◊

Robustness of performance and stability
for multistep and updated multistep MPC schemes

Vryan Gil Palma

Mathematical Institute, University of Bayreuth
Bayreuth, Germany, e-mail: vryan.palma@uni-bayreuth.de

We consider a model predictive control approach to approximate the solution of infinite horizon optimal control problems for perturbed nonlinear discrete time systems. By reducing the number of re-optimizations, the computational load can be lowered considerably at the expense of reduced robustness of the closed-loop solution against perturbations. We propose and analyze an update strategy based on re-optimizations on shrinking horizons which is computationally less expensive than that based on full horizon re-optimization, and at the same time allowing for rigorously quantifiable robust performance estimates.
Joint work with Lars Grüne (University of Bayreuth).

◊
◊◊◊◊◊
◊

Zubov's method for controlled diffusions under state-constraints

Athena Picarelli
ENSTA ParisTech and INRIA Saclay Ile de France

We consider a controlled stochastic system in presence of state-constraints. Under the assumption of state constrained exponential stabilizability of the system near an attractor, we aim to characterize the set of points which can be driven by an admissible control to the attractor with positive probability. This will lead us to introduce a suitable unconstrained optimal control problem and to study the associated Zubov equation.

Joint work with Lars Grüne (University of Bayreuth).

◊
◊◊◊◊◊
◊

Optimal Control Problems for the control of Epidemiology

Maria Rosario de Pinho

University of Porto, Portugal
e-mail: mrpinho@fe.up.pt

Optimal control can help to determine vaccination policies for infectious diseases. Based on a SEIR compartment model we introduce an L1 cost and we treat such problems analytically and numerically. This is a work done with the collaboration of Helmut Maurer.

◊
◊◊◊◊◊
◊

Optimal Control with Heterogeneous Agents in Continuous Time

Galo Nuno Barrau

Bank of Spain

Madrid, Spain, e-mail: galonunobarrau@gmail.com

In this talk we consider optimal control problems in which there is an infinite number of ex-ante identical agents. The evolution of the idiosyncratic state variables follows a controllable It process, that is, there exists a vector of controls that allows the planner to modify the individual state of each agent. In addition there are some aggregate states. In this problem, the aim of the planner is to maximize an optimality criterion over the full distribution (across agents) of state variables. The evolution of the state distribution across agents can be characterized by the Kolmogorov forward (KF) equation In this talk we discuss a particular case of interest: the analysis of constrained efficiency in heterogeneous agents economies. The constrained efficient allo- cation is defined as the one of a social planner who maximizes a utilitarian social welfare function (SWF) subject to the same equilibrium budget constraints and competitive price setting as the individual agents. The planner cannot complete markets or use any transfers between agents. We employ the techniques developed here to compare the solution of this problem with the case of a competitive equilibrium in which each agent maximizes its own discounted utility subject to its state dynamics taking the aggregate conditions and the dynamics of the other agents as given. Joint work with B. Moll (Princeton University).

◊
◊◊◊◊◊
◊

Filtered Scheme and error estimates for first order Hamilton-Jacobi Bellman equations.

Smita Sahu

Dipartmento di Matematica, SAPIENZA - Università di Roma.

Rome, Italy, e-mail: sahu@mat.uniroma1.it

We introduce a new class of "Filtered" scheme for some first order non-linear Hamilton-Jacobi-Bellman equations. The work follows an idea of a recent work of Froese and Oberman [1]. The proposed scheme is not monotone but still satisfies some ϵ-monotone property. We establish a convergence result for filtered scheme with precise error estimate. This allows us to construct finite difference discretizations and easy to implement that behave like a monotone scheme in the singular region and as a high order scheme where the solution is smooth.

Joint work with Olivier Bokanowski, and Maurizio Falcone.

References
[1] B. D. Froese and A. M. Oberman, Convergent filtered schemes for the monge- ampeère partial differential equation. SIAM J. Numer. Anal., 51: 423-444, 2013.

◊
◊◊◊◊◊
◊

Second-order sensitivity relations in optimal control theory

Teresa Scarinci

Dipartimento di Matematica, Università di Roma Tor Vergata and
CNRS, IMJ-PRG,Univ. UPMC, Univ. Paris Diderot
e-mail: teresa.scarinci@gmail.com

In this talk I will discuss some new second-order sensitivity relations for the value function in optimal control problems, that is, relations between the couple given by the co-state of the maximum principle and the solution of a Riccati equation and super/sub jets of the value function along a given optimal trajectory. We will also discuss the presence of a finite-time blow-up for such Riccati equation. Among the other things, this analysis allows to provide sufficient conditions for the local C2 regularity of the value function on tubular neighborhoods of optimal trajectories.
Joint work with P. Cannarsa and H. Frankowska.

◊
◊◊◊◊◊
◊

High order discrete controllability and the approximation of the minimum time function

Thuy T. T. Le

Department of Mathematics,University of Padova
Padova, Italy, e-mail: lttthuy@math.unipd.it

In this talk, we give sufficient conditions to reach a target for a suitable discretization of a control affine nonlinear dynamics. Such conditions involve higher order Lie brackets of the vector fields driving the state and so the discretization method needs to be of a suitably high order as well. As a result, the discrete minimal time function is bounded by a fractional power of the distance to the

target of the initial point. This allows to use methods based on Hamilton-Jacobi theory to prove the convergence of the solution of a fully discrete scheme to the (true) minimum time function, together with error estimates. Finally, we design an approximate suboptimal discrete feedback and provide an error estimate for the time to reach the target through the discrete dynamics generated by this feedback. Our results make use of ideas appearing for the first time in M. Bardi M. Falcone (An approximation scheme for the minimum time function SIAM J. Control Optim. 28 (1990)) and now extensively described in M. Falcone R. Ferretti, (Semi-Lagrangian Approximation Schemes for Linear and Hamilton-Jacobi Equations , SIAM (2014)).
Joint work with Giovanni Colombo.

◊
◊◊◊◊◊
◊

Asymptotic Behavior of Singularly Perturbed Control System: non-periodic setting

Thuong Nguyen

Department of Mathematics, University of Rome La Sapienza
Rome, Italy, e-mail: thuong@mat.uniroma1.it

In this talk we are interested in asymptotic behavior of singularly perturbed control system in the non-periodic setting. More precisely, we consider the value function of finite horizon optimal control problem (Bolza form) associated with singularly perturbed control system, and aim at characterizing its limit as the unique viscosity solution of a limiting Hamilton-Jacobi-Bellman equation (also called effective HJB equation). This approach is extensively studied in a recent series of papers by Alvarez and Bardi in the periodic setting [1, 2, 3]. Our contribution is to extend the results of Alvarez and Bardi to the non-periodic case. The idea is to replace the periodicity (in fast variables) on the datum by coercivity on the running cost, and we only need the local version of bounded-time controllability used in [3]. The remarkable novelty of our work is to develop some basic tools of Aubry-Mather theory (see [4]) into the non-compact case, and we then obtained some similar results as those of Alvarez and Bardi.
Joint work with Antonio Siconolfi.

References

1. O. Alvarez, M. Bardi, Viscosity solutions methods for singular perturbations in deterministic and stochastic control. SIAM J. Control Optim. 40 (2001/02), no. 4, 1159-1188 (electronic)
2. O. Alvarez, M. Bardi, Singular perturbations of nonlinear degenerate parabolic PDEs: a general convergence result. Arch. Ration. Mech. Anal. 170 (2003), no. 1, 17-61.
3. O. Alvarez, M. Bardi, Ergodicity, stabilization, and singular perturbations for Bellman-Isaacs equations. Mem. Amer. Math. Soc. 204 (2010), no. 960.
4. A. Fathi, A. Siconolfi (2005), PDE aspects of Aubry-Mather theory for quasiconvex Hamiltonian, Calc. Var. 22, 185-228.