Statistical Mechanics and Evolution Equations

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A Workshop in Honor of Carlo Marchioro on the Occasion of his 70th Birthday

TITLES AND ABSTRACTS

- **Kazuo Aoki** (Kyoto University, Japan)

  Decay of a linear oscillator in a rarefied gas: Spatially one-dimensional case

  ABSTRACT: An infinitely wide plate, subject to an external force in its normal direction obeying Hooke's law, is placed in an infinite expanse of a rarefied gas. When the plate is displaced from its equilibrium position and released, it starts in general an oscillatory motion in its normal direction. This is the one-dimensional setting of a linear oscillator (or pendulum) considered previously for a collisionless gas and for a special Lorentz gas in our paper [T. Tsuji and K. Aoki, J. Stat. Phys. 146, 620 (2012)]. The motion decays as time proceeds because of the drag force on the plate exerted by the surrounding gas. The long-time behavior of the unsteady motion of the gas caused by the motion of the plate is investigated numerically on the basis of the BGK model of the Boltzmann equation, as well as the compressible Navier-Stokes equation (with the temperature-jump condition), with special interest in the rate of the decay of the oscillatory motion of the plate. The result provides numerical evidence that the displacement of the plate decays in proportion to an inverse power of time (power -3/2) for large time. This talk contains some results of the works in collaboration with Tetsuro Tsuji (Osaka University), Shingo Kosuge (Kyoto University, Japan), and Taiga Fujiwara (Kyoto University).

- **Paolo Buttà** (Sapienza Università di Roma, Italy)

  On the dynamics of infinite classical anharmonic crystals

  ABSTRACT: I consider the following system of anharmonic oscillators. At each point of the lattice $\mathbb{Z}^d$ there is an anharmonic oscillator that interacts with its first neighborhoods via a pair potential V and it is subjected to a restoring force of potential U. I assume that U and V be even nonnegative

polynomials of degree $2\sigma_U$ and $2\sigma_V$. I present a study on the time evolution of this system, based on a work in collaboration with Carlo Marchioro. I first discuss the existence and uniqueness of the dynamics, with a control of the growth in time of the local energy. Then, I present a nontrivial upper bound on the velocity of disturbances when the system is at thermal equilibrium. This is an extension to the case $\sigma_U < 2\sigma_V - 1$ of some already known results for $\sigma_U \geq 2\sigma_V - 1$.

- **Emanuele Caglioti** (Sapienza Università di Roma, Italy)

  Long time behavior of solutions of Vlasov-like equations

  ABSTRACT: Vlasov-like equations are self consistent 1-degree of fredoom (or d-degrees of freedom) Hamiltonian systems, like Vlasov Poisson Equation (VPE); 2D Euler; and the Hamiltonian mean field model (HMF). I'll make a short review of known rigorous results about asymptotic behavior of solutions of Vlasov-like equations: stationary stable solutions, BGK waves for VPE and rotating solutions for 2D Euler; Landau Damping. Finally, I'll discuss the possibility of building Eulerian periodic, and Lagrangian chaotic, solutions for Vlasov like equations in the spirit of Morita and Kaneco 2006. In particular, I enter a little bit in the details of the construction of these solutions.

- **Francesco Calogero** (Sapienza Università di Roma and INFN, Italy)

  Solvable many-body problems and generations of polynomials

  ABSTRACT: A novel class of dynamical systems — solvable by algebraic operations — will be reported, which include endless hierarchies of Newtonian (accelerations equal forces) problems describing an arbitrary number of nonlinearly interacting point-particles moving in the complex plane. And the related notion of generations of (monic) polynomials will be introduced. This is mainly work in collaboration with Oksana Bihun.

- **Silvia Caprino** (Università di Roma Tor Vergata, Italy)

  A review on recent results on a Vlasov-Poisson plasma with infinite charge and velocities

  ABSTRACT: In recent times we have studied a Vlasov-Poisson plasma having infinite charge. While the theory for finite charge is known since long time and quite exhaustive, in this case, besides the increasing difficulties, there is the preliminary problem of the good position of the evolution equation, since the electric field could be infinite at time $t = 0$. I will show some way to approach this problem and relative results obtained with Carlo Marchioro and Guido Cavallaro. Moreover, I will discuss the case of a plasma having infinite velocities together with some preliminary, recent results.

- **Guido Cavallaro** (Sapienza Università di Roma, Italy)

  Mathematical models of viscous friction

  ABSTRACT: I present a review on some recent papers in collaboration with K. Aoki, P. Buttà, S. Caprino, C. Marchioro, M. Pulvirenti, and T. Tsuji, concerning a mathematical description of viscous friction: a body moves in a medium of light particles (or in a fluid) in which it experiences a drag force given by the interaction with the medium. In many macroscopic situations the drag force can be modeled as $F_{\text{drag}}(t) = -\lambda V(t)$, $\lambda > 0$, being $V(t)$ the velocity of the body, obtaining an exponentially fast approach to the asymptotic velocity. From a microscopic point of view this is not always the case, if we take properly into account the recollisions of the body with the particles of the medium, which create a long memory effect. We will show how in this case the asymptotic behavior of $V(t)$ follows a power-law, considering a body/medium interaction given by elastic collisions or diffusive reflexions, for a medium constituted by a free gas. A similar result holds also in case the medium is constituted by a Stokes fluid, i.e., a Navier-Stokes fluid in the limit of zero Reynolds number.

- **Giovanni Gallavotti** (Sapienza Università di Roma, Italy)

  On some integrable systems: Brownian motion, normal forms and zeros of polynomials

  ABSTRACT: The Calogero-Marchioro-Moser model, and the Brownian motion derivation of the integral formula; action angle variables in CMM and the influence on certain conjectures on zeros of polynomials associated with normal forms.

- **Maxime Hauray** (Université d'Aix-Marseille, France)

  From vortices to 2D Navier-Stokes and "particles" to Landau

  ABSTRACT: I will expose results obtained in collaboration with Stphane Mischler and Nicolas Fournier. Our first result is about the propagation of chaos for stochastic vortices system, towards the NS2D equation. About the proof: a very important point is that the NS2D equation is well-posed in the class of dissipative solutions: i.e., among solutions that satisfy more or less the dissipation of entropy. Then, using the entropy dissipation on the particle system, we are able to show that the associated empirical measures form a tight sequence, and that possible limits concentrate on (the unique) dissipative solution of NS2D. The proof strongly relies on the Fisher information

and its properties that I will discuss shortly. The strategy also apply to particle systems converging to Keller-Segel like models (with sub-critical nonlinearity), and to Landau like equation (with the so-called moderately soft potentials).

- **Dragos Iftimie** (Université Claude Bernard Lyon 1, France)

  On stationary solutions of the incompressible Navier-Stokes equations

  ABSTRACT: We consider the stationary incompressible Navier-Stokes equations in $\mathbb{R}^2$ and $\mathbb{R}^3$. We discuss several results about the asymptotic behavior of the solutions at infinity.

- **Evelyne Miot** (Ecole Polytechnique, Paris, France)

  A uniqueness result for the Vlasov-Poisson system with unbounded density

  ABSTRACT: We establish a uniqueness criterion for the the Vlasov-Poisson system in two and three dimensions, allowing for densities that blow-up logarithmically. Moreover, we provide explicit examples of initial data for which the uniqueness condition is satisfied for all time.

- **Errico Presutti** (Gran Sasso Science Institute, L'Aquila, Italy)

  Latent heat and the Fourier law

  ABSTRACT: I consider the Kawasaki dynamics for a $d = 1$ lattice gas with interactions given by an attractive Kac potential with range γ^{-1}, $\gamma > 0$ the Kac scaling parameter. The temperature is fixed in a range where the system has a phase transition in the limit $\gamma \to 0$. The particles are confined in an interval of $\mathbb{Z}$ of length $\gamma^{-1}\ell$. At the endpoints we add birth-death processes so that the density at the left and right endpoints are kept at the values ρ_- and $\rho_+ > \rho_-$. The most interesting case is when ρ_+ is metastable and $\rho_- = 1 - \rho_+$ is (by symmetry) also metastable. In such a case we prove that after the limit $\gamma \to 0$ there is a stationary profile (for the $\gamma \to 0$ limit macroscopic equation) which for ℓ large has a non zero current which goes from the smaller to the larger density, i.e. the current goes in the direction of the gradient against what stated by the Fourier law. The phenomenon can be interpreted in terms of the latent heat associated to the phase transition.

  There is an apparent contradiction with the law of thermodynamics because using this result one could theoretically construct stationary motions with a non zero current without external forces, this is due to the fact that the theory describes what happens after the limit $\gamma \to 0$ and therefore does not take into account the large deviations which will eventually occur. The result therefore does not describe the behavior of the system for infinitely long times when $\gamma > 0$.

I will present some computer simulations of the system which give more details on the phenomenology. The talk is based on a paper in preparation that I have in collaboration with Matteo Colangeli and Anna De Masi.

- **Edriss S. Titi** (Texas A&M University, USA - Weizmann Institute of Science, Israel)

  Is Dispersion a Stabilizing or Destabilizing Mechanism?

  ABSTRACT: In this talk I will present a unified approach for the effect of fast rotation and dispersion as an averaging mechanism for, on the one hand, regularizing and stabilizing certain evolution equations, such as the Navier-Stokes and Burgers equations. On the other hand, I will also present some results in which large dispersion acts as a destabilizing mechanism for the long-time dynamics of certain dissipative evolution equations, such as the Kuramoto-Sivashinsky equation.