Calabi-Yau and Geometry

"Sapienza" Università di Roma, 29 May - 1 June 2019

Abstract of talks

Jørgen Ellegard ANDERSEN (Aarhus University)

The Hitchin connection and Witten's interpretation of the BCOV-formulation

We will review the general setting for the Hitchin connection and discuss how fare this construction involving differential operators can be pushed in the context of Kähler quantisation. Towards the end of the talk we shall relate these latest development to Witten's interpretation of the BCOV-formalism.

Werner BALLMANN (Max Planck Institut Bonn)

Harmonic functions and covering spaces

For the pull back L of an elliptic diffusion operator on a closed manifold to a covering space M, I discuss specific Lyons-Sullivan discretizations of the L-diffusion along the fiber X of the covering and positive L-harmonic functions on M and of the associated random walk on X.
This is joint work with Panagiotis Polymerakis and builds upon the work of Lyons and Sullivan and earlier joint work of myself with François Ledrappier.

Robert BERMAN (Chalmers University)

An application of complex geometry to random matrices

In this talk I will explain how a circle of ideas surrounding the Yau-Tian-Donaldson conjecture can be applied to random matrix theory. More precisely, the talk is concerned with the fluctuations of the eigenvalues of a large Ginibre matrix, i.e. a complex matrix whose entries are taken as independent normal random variables.The main result is a sharp inequality which, in a nut shell, says that the fluctuations are sub-Gaussian with respect to the Laplacian of a Gaussian free field in the complex eigenvalue plane.

Sébastien BOUCKSOM (École polytechnique - CNRS)

Tropical limits of Calabi-Yau manifolds

The amoeba of an algebraic subvariety X of a complex torus is defined as its image under the log map, and one of the basic results in tropical geometry establishes the convergence of the rescaled amoeba to a piecewise linear space, the tropicalization of X. I will present a similar picture for the volume forms of a degenerating family of Calabi-Yau manifolds, in which the limit arises as the Lebesgue measure of a certain piecewise linear space known as the essential skeleton of the degeneration. This result can be viewed as a first step in a program of Kontsevich and Soibelman, aiming to describe Gromov-Hausdorff limits of maximal degenerations of CY manifolds, and is joint work with Mattias Jonsson.

Jean-Pierre BOURGUIGNON (Institut des Hautes Études Scientifiques - CNRS)
The history of Calabi-Yau revisited

Kähler Geometry has been an important tool to link differential and algebraic geometry. One of its key features is that the Ricci curvature of a Kähler metric gets a cohomological meaning. This led to the interesting question as to whether every closed 2-form in the first Chern class is, up to a constant, the Ricci curvature of a Kähler metric in a fixed cohomology class.

This question, an optimist view, was raised by Eugenio Calabi and known for a number of years as the "Calabi conjecture". It was solved by Shing-Tung Yau in 1976. This followed diverse attempts to solve it in the negative. I will report on the steps that have been followed including one attempt that Shing-Tung Yau and I made to disprove the conjecture.

Sylvian CAPPELL (New York University)
Manifolds with all $\mathbf{L^2}$ Betti numbers vanishing

We will show that for many infinite groups, there is a superabundace of closed smooth manifolds with that as fundamental group and all L^2 Betti numbers vanishing.
This is joint work with James Davis and Shmuel Weinberger.

Camillo DE LELLIS (Institute for Advanced Study)
Regularity for semicalibrated integral currents

Given a smooth closed *differential form ω with comass 1 on a smooth Riemannian manifold, a submanifold Σ is said to be calibrated by ω if the latter pulls back to the volume form on Σ. It is well known since the work of Federer that this notion can be extended to integral currents, which would then turn to be area minimizing in their homology class. We conclude therefore that they are rather regular, for instance a smooth submanifold outside a set of small size. The central topic of this talk is that the closeness of ω is not at all necessary to conclude the same regularity properties, as shown in a joint work with Emanuele Spadaro and Luca Spolaor in the 2-dimensional case, later extended by Luca Spolaor to any dimension. The challenging analytical aspect is that one needs to develop a suitable extension to "almost minimizers" of Almgren's regularity theory, despite its rather "nonperturbative" nature.*

Jean-Pierre DEMAILLY (Université de Grenoble Alpes)
Monge-Ampère equations, regularization of currents
and applications to geometry

In this talk, we will report about a number of techniques related to solutions of Monge-Ampère equations, in relation with regularization results for closed positive currents, that lead to existence and structure theorems in complex geometry.

LOTHAT GÖTTSCHE (Internation Centre for Theoretical Physics)
Vafa-Witten formulas and generalizations

This is joint work with Martijn Kool.
25 years ago Vafa and Witten predicted generating functions for the Euler numbers of the moduli spaces of sheaves on surfaces. In this talk I review joint work with Martijn Kool to interpret and check these predictions in terms of virtual Euler numbers, and to extend them to finer invariants like χ_y genus and elliptic genus. Time premitting I will also also mention recent results on Chern numbers of tautological sheaves, and Verlinde type formulas.

RICHARD HAMILTON (Columbia University)
The nonconic estimate for the Ricci flow

Abstract TBA.

STANISŁAW JANECZKO (Polish Academy of Sciences)
Residual algebraic restrictions of differential forms

For a smooth manifold M and the space $\Lambda^p(M)$ of all differential p-forms on M the restriction $\omega|_N$ of $\omega \in \Lambda^p(M)$ to a smooth submanifold $N \subset M$ is well-defined by the geometry of N. If N is any subset of M then the forms $\alpha + d\beta$, $\alpha \in \Lambda^p(M)$, $\beta \in \Lambda^{p-1}(M)$, where α and β annihilates any p-tuple (and $(p-1)$-tuple respectively) of vectors in T_xM, $x \in N$, are called algebraically vanishing on N or having zero algebraic restriction to N. Now the restriction (algebraic restriction) of $\omega \in \Lambda^p(M)$ to N is defined as an equivalence class of ω modulo forms with zero algebraic restriction to N. We study germs of differential forms over singular varieties. The geometric restriction of differential forms to singular varieties is introduced and algebraic restrictions of differential forms with vanishing geometric restrictions, called residual algebraic restrictions, are investigated. Residues of plane curve-germs, hypersurfaces, Lagrangian varieties as well as the geometric and algebraic restriction via a mapping were calculated.
This is a joint work with Goo Ishikawa.

JÜRGEN JOST (Max Planck Institut Leipzig)
Variational problems from quantum field theory

The supersymmetric nonlinear sigma model of quantum field theory has a very rich mathematical structure, from the perspectives of supergeometry, Riemannian geometry and nonlinear PDEs.

DIETER LÜST (Ludwig-Maximillians-Univesität München)
Mirror symmetry and non-geometric fluxes in string theory

As shown by Strominger, Yau and Zaslow, Calabi-Yau Mirror symmetry can be regarded as T-duality on a three-dimensional torus. Starting with a 3-torus equipped with H-flux and possibly with D0-branes, T-duality transformations will eventually lead to a non-geometric torus with R-flux and D3-branes. As we will discuss this non-geometric background possesses an interesting non-associative geometric structure.

KIERAN O'GRADY ("Sapienza" Università di Roma)
Natural sheaves on hyperkähler manifolds

By Yau's solution of Calabi's conjecture, a compact irreducible hyperkähler manifold may be viewed as a compact simply connected complex Kähler manifold with a holomorphic symplectic form spanning the space of global holomorphic 2 *forms. Thus, in complex dimension* 2*, a compact hyperkähler manifold is nothing else than a K3 surface. Holomorphic vector bundles on K3's, and more generally coherent sheaves, play a key role in many non-trivial results. We do not expect that general holomorphic vector bundles on higher dimensional hyperkähler manifolds behave as well as they do in dimension* 2*. In the talk I will describe a special class of vector bundles on hyperkähler's, and I will motivate the expectation that they behave (almost) as well as vector bundles on K3's.*