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Existence and non-existence of radially symmetric non-negative solutions for a class of semi-positone problems in an annulus

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RIASSUNTO: *Si studia il seguente problema al contorno:*

$$\begin{cases} -\Delta u(x) = \lambda f(u(x)), & R < |x| < \hat{R} \\ u(x) = 0, & |x| \in \{R, \hat{R}\} \end{cases}$$

con $\lambda > 0$, $f(0) < 0$, essendo $f : [0, +\infty)$ una funzione superlineare. Si dimostra che per λ abbastanza piccolo esistono soluzioni non negative e dotate di simmetria radiale e che invece non esistono soluzioni di questo tipo se λ è troppo grande.

ABSTRACT: *We study the boundary value problem*

$$\begin{cases} -\Delta u(x) = \lambda f(u(x)), & R < |x| < \hat{R} \\ u(x) = 0, & |x| \in \{R, \hat{R}\} \end{cases}$$

where $\lambda > 0$, $f(0) < 0$ and f is superlinear. We prove existence of a radially symmetric non-negative solution for $\lambda > 0$ sufficiently small and nonexistence of such a solution for $\lambda > 0$ large.

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KEY WORDS AND PHRASES: *Superlinear semi-positone problems – Radial positive solution*

1 – Introduction

In this paper we study the existence and non-existence of radial positive solutions of the problem

$$(1) \qquad \begin{cases} -\Delta u(x) &= \lambda f(u(x)), \quad x \in \Omega \\ u(x) &= 0, \qquad\qquad x \in \partial\Omega \end{cases}$$

where $\lambda > 0$, $f : [0, +\infty) \to \mathrm{I\!R}$ is a continuous nonlinearity and $\Omega \subset \mathrm{I\!R}^N$, is the annulus $\Omega = C(0, R, \hat{R}) = \{x \in \mathrm{I\!R}^N / \; R < |x| < \hat{R}\}$ $(N \geq 3, \; 0 < R < \hat{R})$.

In the positone case, i.e. in the case that $f(0) \geq 0$, the study of positive solutions of (1) in a bounded domain Ω has been intensively studied by many authors (see [4] and [6] for excellents surveys and the references therein). Recently, the non-positone case (i.e. $f(0) < 0$) has been considered in [1, 2, 3, 8]. In particular the first two works study the existence [2] and non-existence [1] of positive solutions of (1) when $\Omega = B(0, R)$ is the ball of radius $R > 0$ centered at zero and f is a monotone non-decreasing nonlinearity satisfying $f(0) < 0$ and $\lim_{u \to +\infty} \frac{f(u)}{u} = +\infty$ (superlinear case). Observe that, since $f(0) < 0$, the constant 0 is an upper solution of (1) and as a consequence it is not possible, in general, to apply the usual techniques (for example: the method of upper and lower solutions, the fixed point index, etc...) to prove the existence of positive solutions of (1). However, it is well known [5] that all positive solutions u of (1) for $\Omega = B(0, R)$ are radial with $\frac{\partial u}{\partial r} < 0$ in Ω ($\frac{\partial}{\partial r}$ denotes the derivative of $u(r)$ $(r = |x|)$). This permits to the authors in [2] to apply the shooting method when f satisfies suitable hypotheses. We remark explicitely that the following fact: $u(0) = \max\{u(x) \,/\, |x| \leq 1\}$ is essential in the proofs in [1, 2].

In contrast with this, in the case $\Omega = C(0, R, \hat{R})$, it is not true [5] that the positive solutions of (1) are radial. In addition, even if $u(x) = u(r)$, $(r = |x|)$, is a radially positive solution of (1), we do not know what is the radius $r \in (R, \hat{R})$ in which u attains its maximum. These facts make our study more difficult and force us to apply the shooting method in a more careful way to extend the results in [1, 2] to the case in which Ω is an

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annulus (and $f(0) < 0$). In concrete, if we denote by $F(u) = \int_0^u f(s)ds$, a primitive of f, we shall prove the following theorem:

THEOREM I. *Let* $\Omega = C(0, R, \hat{R})$ $(0 < R < \hat{R})$ *and assume:*

(f_1) $f \in C^1([0, +\infty))$ *is such that there exists* $\beta > 0$ *with* $f|_{[0,\beta)} < 0$ *and* $f|_{(\beta,+\infty)} > 0$.

(f_2) $\lim\limits_{u \to +\infty} \dfrac{f(u)}{u} = +\infty$.

Then there exists $\lambda^* > 0$ *such that problem* (1) *has not any non-negative radially symmetric solution for all* $\lambda \geq \lambda^*$.

In addition, if f *satisfies also:*

(f_3) *The function* $h(u) = NF(u) - \frac{N-2}{2} uf(u)$ *is bounded from below in* $[0, +\infty)$

(f_4) f *is strictly increasing in* $(\beta, +\infty)$

then there exists $\lambda_* > 0$ *such that problem* (1) *has at least one positive radially symmetric solution for all* $\lambda \in (0, \lambda_*)$.

We must observe that our arguments also work in the case $\Omega = B(0, R)$, improving slightly the results in [1, 2]. In fact, in [1], moreover of imposing that f is increasing these authors need (f_1) and that

$$\liminf_{u \to +\infty} \frac{f(u)}{u^\alpha} > 0 \tag{2}$$

for some $\alpha > 1$. Observe that this hypothesis (2) is more restrictive that (f_2). On the other hand, their existence result in [2] requires $(f_1), (f_2)$, (f_4) and

$$\lim_{u \to +\infty} \left(\frac{u}{f(u)}\right)^{\frac{N}{2}} [F(ku) - \frac{N-2}{2N} uf(u)] = +\infty \tag{3}$$

for some $k \in (0, 1)$. By (f_2), it is clear that our hypothesis (f_3) is more general than (3).

The paper is organized as follows. The part of Theorem I about the existence of solutions will be proved in section 2 (see Theorem 2.4 below). The non-existence result will be given in section 3 (see Theorem 3.1 below). We reserve section 1 for some technical preliminaries.

2 – Preliminaries

The problem of the existence of positive radially symmetric solutions of (1) is equivalent to the existence of positive solutions of the problem

$$\text{(4)}\qquad \begin{cases} -u''(r) - \dfrac{N-1}{r}u'(r) = \lambda f(u(r)), \quad R < r < \hat{R} \\ u(R) = u(\hat{R}) = 0 \end{cases}.$$

In our study of (4), we apply the shooting method. This technique is based in considering the initial value problem

$$\text{(5)}\qquad \begin{cases} -u''(r) - \dfrac{N-1}{r}u'(r) = \lambda f(u(r)), \qquad R < r \\ u(R) = 0, u'(R) = d \end{cases}$$

to show that, for a convenient $d > 0$, this admits a solution $u(.) = u(., d, \lambda)$ (which depends on d and λ) such that $u > 0$ in $(R, \hat{R})$ and $u(\hat{R}) = 0$. So, such a solution u of (5) is also a positive solution of (4). In the sequel we suppose that the nonlinearity $f \in C^1([0, +\infty))$ is always extended to $\mathbb{R}$ by

$$f|_{(-\infty,0)} \equiv f(0).$$

In this section we present some technical results about the solutions of (5). These are standard in the literature. However, by definitiness of the reader, we include here the details. We begin with the following lemma which assure the existence of a unique solution $u(r, d, \lambda)$ of (5) in $[R, +\infty)$ for all $\lambda, d > 0$.

Lemma 1.1. *Let λ, $d > 0$ and $f \in C^1([0, +\infty))$ a function which is bounded from below. Then problem* (5) *has a unique solution $u(r, d, \lambda)$ defined in $[R, +\infty)$. In addition, for every $d > 0$ there exist $M = M(d) > 0$ and $\lambda(d) > 0$ such that*

$$\max_{r\in[R,\hat{R}]} |u(r, d, \lambda)| \le M, \ \forall \lambda \in (0, \lambda(d))$$

Proof. The proof of the first part of the lemma is given in two steps. First we show the existence and uniqueness of a local solution of (5), i.e, the existence of $\varepsilon = \varepsilon(d,\lambda) > 0$ such that (5) has a unique solution in $[R, R+\varepsilon]$. In the second step we prove that this unique solution can be extended to $[R, +\infty)$.

Step 1: (Local solution). Because in step 2 we need the local existence and uniqueness of solutions of the equation of (5) with more general initial conditions, we directly show the local existence and uniqueness for the problem

$$\begin{cases} -u''(r) - \dfrac{N-1}{r}u'(r) = \lambda f(u(r)), & r > R_1 \\ u(R_1) = a, u'(R_1) = b \end{cases} \tag{6}$$

with $R_1 \geq R$ fixed. Note that if u is a solution of (6), multiplying the equation by r^{N-1} and using the initial conditions, we obtain

$$u'(r) = \frac{1}{r^{N-1}}\left[R_1^{N-1}b - \lambda\int_{R_1}^r s^{N-1}f(u(s))ds\right] \tag{7}$$

from which u satisfies

$$\begin{aligned} u(r) = a + \frac{bR_1^{N-1}}{N-2}\left(\frac{1}{R_1^{N-2}} - \frac{1}{r^{N-2}}\right) + \\ -\lambda\int_{R_1}^r \frac{1}{t^{N-1}}\left[\int_{R_1}^t s^{N-1}f(u(s))ds\right]dt\,. \end{aligned} \tag{8}$$

Conversely, if u is a continuous function satisfying (8) then u is a solution of (6). Hence, in order to prove the existence and uniqueness of a solution u of (6) defined in some interval $[R_1, R_1+\varepsilon]$, it is sufficient to show the existence of a unique fixed point of the operator T defined on $X = C([R_1, R_1+\varepsilon], \mathbb{R})$ (the Banach space of the real continuous functions on $[R_1, R_1+\varepsilon]$ with the uniform norm) by

$$\begin{aligned} (Tv)(r) = a + \frac{bR_1^{N-1}}{N-2}\left(\frac{1}{R_1^{N-2}} - \frac{1}{r^{N-2}}\right) + \\ -\lambda\int_{R_1}^r \frac{1}{t^{N-1}}\left[\int_{R_1}^t s^{N-1}f(u(s))ds\right]dt, \end{aligned} \tag{9}$$

for all $r \in [R_1, R_1 + \varepsilon]$, and $v \in X$. To check this, let $\delta > 0$ such that $\delta > |a|$ and consider the closed ball $\bar{B}(0, \delta) = \{u \in X \ : \ \|u\| \leq \delta\}$. For all $u, v \in \overline{B}(0, \delta)$, we have:

$$\|Tu - Tv\| \leq \frac{\lambda}{N} \sup_{\xi \in (0,\delta]} |f'(\xi)| \ \varepsilon(R_1 + \varepsilon)\|u - v\| \tag{10}$$

and

$$\begin{aligned} \|Tu\| \leq |a| + \frac{|b|R_1^{N-1}}{N-2}\left(\frac{1}{R_1^{N-2}} - \frac{1}{(R_1+\varepsilon)^{N-2}}\right) + \\ + \frac{\lambda}{N}\varepsilon(R_1+\varepsilon) \sup_{\xi \in [0,\delta]} |f(\xi)| \end{aligned} \tag{11}$$

Now, by (10) and (11), we can choose $\varepsilon = \varepsilon(\delta) > 0$ (depending on δ) sufficiently small such that T is a contraction from $\overline{B}(0, \delta)$ to $\overline{B}(0, \delta)$. Hence T has a fixed point u in $\overline{B}(0, \delta)$. The fixed point u is unique in X because we can choose δ as large as we wanted.

Step 2: Let $u(r) = u(r, d, \lambda)$ be the unique solution of (5) (we take $a = 0, b = d$ and $R_1 = R$ in (6)) and denote by $[R, R(d, \lambda))$ its maximal domain. We shall prove by contradiction that $R(d, \lambda) = +\infty$. For it, assume $R^* = R(d, \lambda) < +\infty$. Then we claim that u is bounded in $[R, R^*)$. In fact, using (8) and that f is bounded from below, we have

$$\begin{aligned} \frac{dR}{N-2} \geq \frac{dR^{N-1}}{N-2}\left(\frac{1}{R^{N-2}} - \frac{1}{r^{N-2}}\right) \\ = u(r) + \lambda \int_R^r \frac{1}{t^{N-1}}\left[\int_R^t s^{N-1} f(u(s))ds\right] dt \\ \geq u(r) + \lambda \inf_{\xi \in [0,+\infty)} f(\xi) \int_R^{R^*} \left[\frac{1}{t^{N-1}} \int_R^t s^{N-1} ds\right] dt, \end{aligned}$$

for all $r \in [R, R^*)$, and we deduce that there exists $K_1 > 0$ such that

$$u(r) \leq K_1, \quad \forall r \in [R, R^*)$$

On the other hand, using again (8) we obtain also

$$\begin{aligned} u(r) &\geq \frac{dR^{N-1}}{N-2}\left(\frac{1}{R^{N-2}}-\frac{1}{r^{N-2}}\right) \\ &\quad -\lambda \max_{\xi\in[0,K_1]} f(\xi)\int_R^{R^*}\left[\frac{1}{t^{N-1}}\int_R^t s^{N-1}ds\right]dt \\ &\geq -K_2\ , \quad \forall r\in[R,R^*) \end{aligned}$$

for convenient $K_2>0$. These last inequalities imply that u is bounded.

By using this and (7) and (8) we deduce that $\{u(r_n)\}$ and $\{u'(r_n)\}$ are Cauchy sequences for all sequence $\{r_n\}\subset[R,R^*)$ converging to R^*. This is equivalent to the existence of the finite limits

$$\lim_{r\to R^{*-}} u(r)=a, \qquad \lim_{r\to R^{*-}} u'(r)=b.$$

Considering the initial value problem

$$\text{(12)} \qquad \begin{cases} -v''(r)-\dfrac{N-1}{r}v'(r)=\lambda f(v(r)), & R^*<r \\ v(R^*)=a, v'(R^*)=b \end{cases}$$

and by step 1, we deduce the existence of a positive number $\varepsilon>0$ and a solution $v(r)$ of this problem in $[R^*,R^*+\varepsilon]$.

Now, it is easy to see that

$$\tilde{u}(r)=\begin{cases} u(r), & \text{if } R\leq r<R^* \\ v(r), & \text{if } R^*\leq r\leq R^*+\varepsilon \end{cases}$$

is a solution of (5) in $[R,R^*+\varepsilon]$, an interval which contains the maximal domain $[R,R_1)$. Clearly this is a contradiction, so $R^*=+\infty$ and we have finished the proof of the existence and uniqueness of a solution $u(r,d,\lambda)$ of (5) in $[R,+\infty)$.

In order to prove the second part of the lemma we consider again the operator T defined in (9) on $X_0=C([R,\hat{R}],\mathbb{R})$, with $R_1=R, a=0$ and $b=d$. Taking $M=\delta>\dfrac{2dR}{N-2}$ and

$$\lambda(d)=\min\left\{\frac{M}{2M_1\max_{\xi\in[0,M]}|f(\xi)|}\ ,\ \frac{1}{M_1\max_{\xi\in[0,M]}|f'(\xi)|}\right\}$$

with $M_1 = \int_R^{\hat{R}} \frac{1}{t^{N-1}}\{\int_R^t s^{N-1} ds\} dt$, we deduce from (10) and (11) that T is a contraction from $\bar{B}(0, M, X_0) = \{u \in X_0 : \max_{r\in[R,\hat{R}]} |u(r)| \le M\}$ into $\bar{B}(0, M, X_0)$. So, the unique corresponding fixed point of T, i.e., $u(r, d, \lambda)$, belongs to $\bar{B}(0, M, X_0)$ and the lemma is proved. ∎

Remark 1.2. The solution $u(., d, \lambda)$ depends continuously on (d, λ) in the sense that if $\{d_n\}$ converges to d and $\{\lambda_n\}$ to λ, then the sequence of the functions $u(., d_n, \lambda_n)$ converges uniformly to $u(., d, \lambda)$ on any bounded interval. A similar property is also true for $u'(., d, \lambda)$.

3 – Results of existence

In this section we are interested in giving sufficient conditions for the existence of positive solutions of (4). To do this, we prove the following lemmas about the behaviour of the solution $u(\cdot, d_0, \lambda)$ of (5).

Lemma 2.1. *Assume* (f_1, f_4) *and let* $d_0 > 0$*. Then there exists* $\lambda_1 = \lambda_1(d_0) > 0$ *such that the unique solution* $u(r, d_0, \lambda)$ *of* (5) *satisfies*

$$u(r, d_0, \lambda) > 0, \quad \forall r \in (R, \hat{R}],\ \forall \lambda \in (0, \lambda_1).$$

Proof. For $\lambda > 0$ we consider the set

$$\mathcal{E} = \{r \in (R, \hat{R})/\ u(.) = u(., d_0, \lambda) \text{ is nondecreasing in } (R, r)\}$$

Since $u'(R) = d_0 > 0$, $\mathcal{E}$ is nonempty and clearly bounded from above. Let $r_1 = \sup \mathcal{E}$ (which depends on λ). It may occur two cases:

1. $r_1 = \hat{R}$
2. $r_1 < \hat{R}$

In the first case, it is clear that $u(r) = u(r, d_0, \lambda) > 0$ for all $r \in (R, \hat{R}]$. In the second case, we shall also prove that $u(r, d_0, \lambda) > 0$, $\forall r \in (R, \hat{R}]$ for $\lambda > 0$ sufficiently small, which concludes the proof of the Lemma 2.1. In order to show it, assume that $r_1 < \hat{R}$. Then $u'(r_1) = 0$ and by (f_1) and (7), $u(r_1) > \beta$.

Hence the set

$$\mathcal{F} = \{r \in [r_1, \hat{R}]/\ u(t) \ge \beta \ \ \forall t \in [r_1, r]\}$$

is nonempty and bounded. Take $r_2 = \sup \mathcal{F} > r_1$. The proof will be finished if we can show that, for λ sufficiently small, $r_2 = \hat{R}$. To do it, using again (f_1) and (7), we observe:

$$u'(r) = -\frac{\lambda}{r^{N-1}} \int_{r_1}^{r} t^{N-1} f(u(t))dt < 0, \quad \forall r \in \mathcal{F} \setminus \{r_1\} \tag{13}$$

and

$$u(r) \;\leq\; u(r_1), \quad \forall r \in [R, r_2]$$

Therefore, by the mean value theorem and (f_4), there exists $c \in (r_1, r_2)$ such that

$$\begin{aligned} u(r_2) &= u(r_1) + u'(c)(r_2 - r_1) \\ &\geq u(r_1) - \frac{\lambda \hat{R}}{N} f(u(r_1))(r_2 - r_1) \\ &> u(r_1) - \frac{\lambda \hat{R}}{N} f(u(r_1))(\hat{R} - R)\,. \end{aligned}$$

If $M = M(d_0) > 0$ and $\lambda(d_0) > 0$ are given by Lemma 1.1, we have that $\beta < u(r_1) \leq M$ for all $\lambda \in (0, \lambda(d_0))$. Taking $K = K(d_0) > 0$ such that

$$f(\xi) \;<\; K(\xi - \beta), \quad \forall \xi \in (\beta, M]$$

we deduce:

$$u(r_2) > u(r_1) - \frac{\lambda K \hat{R}}{N}(\hat{R} - R)(u(r_1) - \beta), \; \forall \lambda \in (0, \lambda(d_0)).$$

Thus, if $\lambda \in (0, \lambda_1)$ with $\lambda_1 = \min\Big\{\lambda(d_0), \dfrac{N}{\hat{R}(\hat{R} - R)K}\Big\}$, we have

$$u(r_2) \;>\; u(r_1) - (u(r_1) - \beta) = \beta$$

which implies that $r_2 = \hat{R}$. ∎

Our following result concerns the asymptotic behaviour of $r_1(d, \lambda) = Sup\{r \in (R, \hat{R}) / u(.) = u(., d, \lambda)$ is nondecreasing in $(R, r)\}$.

LEMMA 2.2. *Assume* (f_1, f_2, f_4). *Let* $\lambda > 0$. *Then*

(14) $$\lim_{d\to+\infty} r_1(d,\lambda) \;=\; R$$

and

(15) $$\lim_{d\to+\infty} u(r_1,d,\lambda) = +\infty$$

PROOF. We prove the first part of this lemma by contradiction. If (14) is not true, then there exist $R_0 \in (R,\hat{R})$ and a sequence $\{d_n\} \subset (0,+\infty)$ converging to infinity such that $u_n = u(.,d_n,\lambda)$ satisfies

(16) $$u_n(r) > 0,\ u_n'(r) \geq 0,\ \forall r \in (R, R_0],\ \forall n \in \mathbb{N}$$

Let $\bar{r} = \dfrac{R_0+R}{2}$. Observe that by (f_4) and (8) (with $R_1 = R, a = 0$ and $b = d_n$), the sequence $\{u_n(\bar{r})\}$ is unbounded. Passing to a subsequence of $\{d_n\}$, if it is necessary, we can suppose $\lim_{n\to+\infty} u_n(\bar{r}) = +\infty$. Consider

$$M_n = \inf\left\{\frac{f(u_n(r))}{u_n(r)}\ /r \in (\bar{r}, R_0)\right\}$$

In virtue of (f_2),

$$\lim_{n\to+\infty} M_n = +\infty,$$

and we may take $n_0 \in \mathbb{N}$ such that

$$\lambda M_{n_0} > \mu_3$$

where μ_3 is the third eigenvalue of $-\Big[\dfrac{d^2}{dr^2} + \dfrac{N-1}{r}\dfrac{d}{dr}\Big]$ in $(\bar{r}, R_0)$ with Dirichlet boundary conditions. Now we take a nonzero eigenfunction ϕ_3 asociated to this μ_3, i.e.,

$$\begin{cases} \phi_3''(r) + \dfrac{N-1}{r}\phi_3'(r) + \mu_3\phi_3(r) = 0, & \bar{r} < r < R_0 \\ \phi_3(\bar{r}) = 0 = \phi_3(R_0) & . \end{cases}$$

Since ϕ_3 has two zeros in $(\bar{r}, R_0)$, we deduce from the Sturm comparison theorem [7] that u_{n_0} has at least one zero in $(\bar{r}, R_0)$. But this is a contradiction with (16). Then (14) is true.

On the other hand, let r_2 be the same number as in the proof of Lemma 2.1 and taking into account (f_4) and (13), we deduce $u(r_1) = \max_{r\in[R,r_2]} u(r) > \beta$ and

$$0 > u'(r) = \frac{1}{r^{N-1}}[dR^{N-1} - \lambda \int_R^r t^{N-1} f(u(t))dt]$$

$$\geq \frac{dR^{N-1}}{\hat{R}^{N-1}} - \frac{\lambda r}{N} f(u(r_1)), \ \forall r \in (r_1, r_2)$$

from which we obtain

$$f(u(r_1)) \geq \frac{NdR^{N-1}}{\lambda \hat{R}^N}$$

and the proof of (15) is also concluded. □

Lemma 2.3. *Assume* $(f_1 - f_4)$ *and let* $\gamma_1 > 0$ *be a positive number. Then there exists* $\lambda_2 > 0$ *such that*

a) *For all* $\lambda \in (0, \lambda_2)$ *the unique solution* $u(r, d, \lambda)$ *of* (5) *satisfies*

$$u^2(r, d, \lambda) + u'^2(r, d, \lambda) > 0, \ \forall r \in [R, \hat{R}], \forall d \geq \gamma_1$$

b) *For all* $\lambda \in (0, \lambda_2)$ *there exists* $d > \gamma_1$ *such that* $u(r, d, \lambda) < 0$ *for some* $r \in (R, \hat{R}]$.

Proof. Let $\lambda, d > 0$ and consider $u(r) = u(r, d, \lambda)$ the unique solution of (5). We define the auxiliary function H on $[R, +\infty)$ by setting

$$H(r) = \frac{ru'^2(r)}{2} + \lambda r F(u(r)) + \frac{N-2}{2} u(r)u'(r), \quad \forall r \in [R, +\infty).$$

It can be proved, as in [2], the next identity of Pohozaev-type:

$$r^{N-1}H(r) = t^{N-1}H(t) + \lambda \int_t^r s^{N-1}[NF(u(s)) - \frac{N-2}{2} f(u(s))u(s)]ds$$

for all $R \le t \le r$. Taking $t = R$ in this identity we obtain
(17)

$$r^{N-1}H(r) = \frac{R^N d^2}{2} + \lambda \int_R^r s^{N-1}\Big[NF(u(s)) - \frac{N-2}{2} f(u(s))u(s)\Big] ds$$
$$\ge \frac{R^N d^2}{2} + \lambda m\Big(\frac{r^N}{N} - \frac{R^N}{N}\Big)$$
$$\ge \frac{R^N \gamma_1^2}{2} + \lambda m\Big(\frac{\hat{R}^N}{N} - \frac{R^N}{N}\Big), \quad \forall r \in [R, \hat{R}]$$

where m is a strictly negative number satisfying

$$NF(u) - \frac{N-2}{2} uf(u) \ge m, \quad \forall u \in \mathbb{R}$$

(remark that m exists by (f_3))
Hence there exists $\lambda_2 > 0$ such that

$$H(r) \ > \ 0, \quad \forall r \in [R, \hat{R}], \ \forall d \ge \gamma_1, \ \forall \lambda \in (0, \lambda_2) \tag{18}$$

As a consequence

$$u^2(r) + u'^2(r) > 0, \ \forall r \in [R, \hat{R}], \quad \forall d \ge \gamma_1, \quad \forall \lambda \in (0, \lambda_2),$$

and we have finished the proof of part a) of this lemma.

To prove the part b) we argue by contradiction: fix $\lambda \in (0, \lambda_2)$ and let us suppose that $u(r, d, \lambda) \ge 0$, $\forall r \in [R, \hat{R}]$, $\forall d \ge \gamma_1$. Choose $\rho > 0$ such that there exists a solution ω of

$$\omega'' + \frac{N-1}{r}\omega' + \rho\omega = 0$$

satisfying $\omega(0) = 1, \omega'(0) = 0$ and that $\dfrac{\hat{R} - R}{4}$ is the first zero of ω. Note that by [5], $\omega(r) \ge 0$ and $\omega'(r) < 0$ for all $r \in \Big(0, \dfrac{\hat{R} - R}{4}\Big]$. Since (f_2) is satisfied, there exists $d_0 = d_0(\lambda) > \gamma_1$ such that

$$\frac{f(u)}{u} \ \ge \ \frac{\rho}{\lambda}, \ \forall u \ge d_0\,. \tag{19}$$

On the other hand, let $r_1 = r_1(d,\lambda)$ and $r_2 = r_2(d,\lambda)$ be the same numbers as in the proof of Lemma 2.1. By Lemma 2.2 we can suppose that $r_1(d,\lambda) < R + \dfrac{\hat{R}-R}{4} < \hat{R}$ with $u(r_1,d,\lambda) > d_0$ for all $d \geq d_0$.

Let $r \in [r_1, \hat{R}]$ such that $u'(r,d,\lambda) = 0$ $(d \geq d_0)$. Observe that by (18), $H(r) = \lambda r F(u(r)) > 0$. Then, if, by (f_1), θ denotes the unique zero of F, we obtain $u(r) > \theta$ and by (5) we have $u''(r,d,\lambda) = -\lambda f(u(r,d,\lambda)) < 0$. Thus, every critical point $r \in [r_1,\hat{R}]$ of $u(.,d,\lambda)$ is a strict local maximum of this function. So, the definitions of r_1 and r_2 imply

$$u'(r,d,\lambda) \; < \; 0, \quad \forall r \in (r_1,\hat{R}], \quad \forall\, d \geq d_0 \tag{20}$$

Define $v(r) = u(r_1)w(r - r_1)$ and observe that v satisfies

$$v''(r) + \frac{N-1}{r-r_1}v'(r) + \rho v(r) = 0, \quad r_1 < r < r_1 + \frac{\hat{R}-R}{4}$$

with $v(r_1) = u(r_1), v'(r_1) = 0, v\Big(r_1 + \dfrac{\hat{R}-R}{4}\Big) = 0$ and

$$v(r) > 0,\ v'(r) \leq 0,\ \forall r \in \Bigg(r_1, r_1 + \frac{\hat{R}-R}{4}\Bigg).$$

Then

$$v''(r) + \frac{N-1}{r}v'(r) + \rho v(r) \geq 0,\ \forall r \in \Bigg(r_1, r_1 + \frac{\hat{R}-R}{4}\Bigg)$$

and by (19), the Sturm comparison theorem [7] implies that if $u(r) \geq d_0$, for all $r \in \Big(r_1, r_1 + \dfrac{\hat{R}-R}{4}\Big)$, then u will have a zero in $\Big(r_1, r_1 + \dfrac{\hat{R}-R}{4}\Big)$. Since this is impossible we obtain that for any $d \geq d_0$ there exists $r^* \in \Big(r_1, r_1 + \dfrac{\hat{R}-R}{4}\Big)$ satisfying $u(r^*,d,\lambda) = d_0$ (remind that $u(r_1,d,\lambda) > d_0,\ \ \forall d \geq d_0$).

Consider now the *energy* function

$$E(r,d,\lambda) = \frac{u'(r,d,\lambda)^2}{2} + \lambda F(u(r,d,\lambda)), \quad \forall r \geq R$$

By (17), (20) and the equality $H(r) = rE(r) + \dfrac{N-2}{2}u(r)u'(r)$ we have

$$\begin{aligned} r^N E(r,d,\lambda) &\geq r^{N-1} H(r,d,\lambda) \\ &\geq \frac{R^N d^2}{2} + \lambda m\Big(\frac{\hat{R}^N}{N} - \frac{R^N}{N}\Big), \quad \forall\, r \in [r_1, \hat{R}] \end{aligned}$$

and so there exists $d_1 = d_1(\lambda) \geq d_0$ such that

$$E(r,d,\lambda) \geq \lambda F(d_0) + \frac{2}{(\hat{R}-R)^2} d_0^2, \quad \forall r \in [r_1, \hat{R}], \ \forall\, d \geq d_1.$$

Using that $E'(r) = -\dfrac{N-1}{r}u'(r)^2 \leq 0$, $\forall r \in [R, \hat{R}]$, we deduce

$$\frac{u'(r)^2}{2} \geq \frac{2d_0^2}{(\hat{R}-R)^2}, \quad \forall r \in [r^*, \hat{R}], \ \forall d \geq d_1$$

i.e., by (20),

$$u'(r) \ \leq \ -\frac{2d_0}{(\hat{R}-R)}, \quad \forall r \in [r^*, \hat{R}], \ \forall d \geq d_1 \, .$$

This bound and the mean value theorem imply

$$u\Big(r^* + \frac{\hat{R}-R}{2}\Big) \leq u(r^*) - d_0 = 0$$

with $u'\Big(r^* + \dfrac{\hat{R}-R}{2}\Big) < 0$. This is a contradiction with the fact that u is non-negative in $[R, \hat{R}]$. Therefore, the second part of the lemma is also proved. ∎

Now as a consequence of Lemmas 2.1 and 2.3, we have the following result of the existence of a positive solution of (4) and so a radial positive solution of (1).

THEOREM 2.4. *Assume* $(f_1 - f_4)$. *Then there exists* $\lambda_* > 0$ *such that problem* (1) *has at least one positive radially symmetric solution for all* $\lambda \in (0, \lambda_*)$.

Proof. Let $d_0 > 0$. By Lemmas 2.1 and 2.3, there exist $\lambda * > 0$ such that if $\lambda \in (0, \lambda_*)$ then

i) $u(r, d, \lambda) > 0$, $\forall r \in (R, \hat{R}]$,

ii) $u'^2(r, d, \lambda) + u^2(r, d, \lambda) > 0$, $\forall r \in [R, \hat{R}]$, $\forall d \geq d_0$,

iii) There exist $d_1 > d_0$ and $r \in (R, \hat{R}]$ such that $u(r, d_1, \lambda) < 0$.

Define $\mathcal{G} = \{d \geq d_0 \;/\; u(r, \overline{d}, \lambda) > 0, \forall r \in (R, \hat{R}), \forall \overline{d} \in [d_0, d]\}$. Observe that $d_0 \in \mathcal{G}$ (by i)) and so $\mathcal{G} \neq \emptyset$. In addition, by iii), $\mathcal{G}$ is bounded from above by d_1. Take $d^* = \sup \mathcal{G}$.

By Remark 1.2 it is clear that:

$$u(r, d^*, \lambda) \;\geq\; 0, \;\forall r \in [R, \hat{R}]$$

from which $d^* < d_1$ and using ii), we deduce

$$u(r, d^*, \lambda) \;>\; 0, \quad \forall r \in (R, \hat{R})\,. \tag{21}$$

The lemma will be proved (with $u(., d^*, \lambda)$ as a solution) if we can show that $u(\hat{R}, d^*, \lambda) = 0$. This will be done by contradiction. Assume that $u(\hat{R}, d^*, \lambda) > 0$. Then by Remark 1.2, by (21) and the fact that $u'(R, d^*, \lambda) = d^* > 0$ we have that

$$u(r, d, \lambda) \;>\; 0, \;\forall r \in (R, \hat{R}], \;\forall d \in [d^*, d^* + \delta]$$

where $\delta > 0$ is sufficiently small. Hence $d^* + \delta \in \mathcal{G}$ which is a contradiction with the definition of d^*. So $u(\hat{R}, d^*, \lambda) = 0$ and the proof is finished. □

4 – Results of non-existence

In this section we give sufficient conditions to assure the non-existence of non-negative radially symmetric solution of (1), for $\lambda > 0$ sufficiently greater. Our main result is the following:

Theorem 3.1. *Assume (f_1, f_2). Then there exists $\lambda^* > 0$ such that* (1) *has no non-negative radially symmetric solution for all $\lambda \geq \lambda^*$.*

In order to prove Theorem 3.1, we observe that it is equivalent to show the non-existence of non-negative solutions for (4) where $\lambda > 0$ is sufficiently greater. We shall need some previous lemmas. Denote by $u_\lambda(r)$ a non-negative solution of (4) (if there exists) and let $R_0 = \dfrac{R+\hat{R}}{2}$.

LEMMA 3.2. *Let $f \in C^1([0,+\infty))$ satisfying (f_2) and consider $\lambda > 2$. If $u_\lambda(r)$ is a non-negative solution of (4), then for every $r \in (R_0, \hat{R}]$ there exists a positive number $M = M(r) > 0$ (which is independent on $\lambda > 2$) such that*

$$u_\lambda(r) \ \le \ M\,.$$

PROOF. Let ϕ_1 be a positive eigenfunction associated to the first eigenvalue $\mu_1 > 0$ of the eigenvalue problem

$$\begin{cases} -(r^{N-1}v')' = \mu r^{N-1}v, & R < r < \hat{R} \\ v(R) = v(\hat{R}) = 0 & . \end{cases}$$

Multiplying the equation in (4) by $r^{N-1}\phi_1(r)$ and integrating from R to $\hat{R}$, we obtain

$$\begin{aligned} \int_R^{\hat{R}} r^{N-1}u_\lambda'(r)\phi_1'(r)dr &= -\int_R^{\hat{R}} (r^{N-1}u_\lambda'(r))'\phi_1(r)dr \\ &= \lambda \int_R^{\hat{R}} r^{N-1}f(u_\lambda(r))\phi_1(r)dr\,. \end{aligned} \tag{22}$$

On the other hand, multiplying the equation $-(r^{N-1}\phi_1'(r))' = \mu_1 r^{N-1}\phi_1(r)$, $(R < r < \hat{R})$ by u_λ and integrating from R to $\hat{R}$ again, we obtain

$$\begin{aligned} \int_R^{\hat{R}} r^{N-1}u_\lambda'(r)\phi_1'(r)dr &= -\int_R^{\hat{R}} (r^{N-1}\phi_1')'u_\lambda(r)dr \\ &= \mu_1 \int_R^{\hat{R}} r^{N-1}\phi_1(r)u_\lambda(r)dr\,. \end{aligned}$$

Now, combining this with (22) and choosing $\mu > \dfrac{\mu_1}{2}$, $c > 0$ such that

$$f(\xi) \ \ge \ \mu\xi - c, \quad \forall \xi \ge 0$$

(remind that f is superlinear), we deduce

$$\mu_1 \int_R^{\hat{R}} r^{N-1}\phi_1(r)u_\lambda(r)dr = \lambda \int_R^{\hat{R}} r^{N-1} f(u_\lambda(r))\phi_1(r)dr$$
$$\geq \lambda\mu\int_R^{\hat{R}} r^{N-1}u_\lambda(r)\phi_1(r)dr - \lambda c\int_R^{\hat{R}} r^{N-1}\phi_1(r)dr$$

from which

$$\int_R^{\hat{R}} r^{N-1}u_\lambda(r)\phi_1(r)dr \leq \frac{\lambda k}{\lambda\mu - \mu_1} \leq \frac{k}{\mu - \frac{\mu_1}{2}} \equiv A,\ \forall\lambda > 2$$

with $k = c\int_R^{\hat{R}} r^{N-1}\phi_1(r)dr > 0$ and $A > 0$ independent on $\lambda > 2$.

Now, taking an arbitrary $r \in (R_0, \hat{R}]$ and choosing $\delta > 0$ such that $R_0 < r - \delta$, the fact that $u_\lambda(r)$ is nonincreasing in $(R_0, \hat{R})$ (see [5]) implies

$$u_\lambda(r) \leq \frac{\int_{r-\delta}^{r} t^{N-1}u_\lambda(t)\phi_1(t)dt}{\int_{r-\delta}^{r} t^{N-1}\phi_1(t)dt}$$
$$\leq \frac{\int_R^{\hat{R}} t^{N-1}u_\lambda(t)\phi_1(t)dt}{\int_{r-\delta}^{r} t^{N-1}\phi_1(t)dt}$$
$$\leq \frac{A}{\int_{r-\delta}^{r} t^{N-1}\phi_1(t)dt} \equiv M,\quad \forall\lambda > 2$$

and the proof is concluded. ∎

Lemma 3.3. *Assume* (f_1, f_2) *and let* $R_1 \in (R_0, \hat{R}]$, $c \in (\beta, \theta)$, *where* θ *denotes the unique zero of the primitive* F *of* f *(by* (f_1)*). Then there exists* $\lambda_1 > 0$ *such that for all non-negative solution* u_λ *of* (4) *with* $\lambda \geq \lambda_1$, *there is* $t_1 = t_1(\lambda) \in (R_0, R_1]$ *satisfying* $u_\lambda(t_1) < c$.

Proof. We argue by contradiction. Suppose that there exists a sequence $\{\lambda_n\} \subset (0, +\infty)$ converging to $+\infty$ such that

$$u_{\lambda_n}(r) \geq c,\quad \forall r \in (R_0, R_1],\ \forall n \in \mathbb{N}.$$

Consider

$$\bar{t}_n = Max\{r \in (R, \hat{R})\ :\ u'_{\lambda_n}(r) = 0\ \}.$$

By [5], $\bar{t}_n \leq R_0$ and u_{λ_n} is decreasing in $(\bar{t}_n, \hat{R})$. Using (7) (with $R_1 = R$, $a = 0$ and $b = d$) and the fact that $u'_{\lambda_n}(\bar{t}_n) = 0$, we deduce

$$\begin{aligned} u'_{\lambda_n}(r) &= -\frac{\lambda_n}{r^{N-1}} \int_{\bar{t}_n}^{r} s^{N-1} f(u_{\lambda_n}(s))ds \\ &\leq -\frac{\lambda_n}{r^{N-1}} m \int_{\bar{t}_n}^{r} s^{N-1} ds \\ &\leq -\frac{\lambda_n}{r^{N-1}} m \int_{R_0}^{r} s^{N-1} ds \\ &= -\frac{\lambda_n}{r^{N-1}} m [\frac{r^N}{N} - \frac{R_0^N}{N}], \ \forall n \in \mathbb{N}, \ \forall r \in (R_0, R_1] \end{aligned} \tag{23}$$

where

$$m = \min\{f(\xi)/ \ \xi \geq c\} > 0.$$

Let $r_1, r_2 \in (R_0.R_1]$ be such that $R_0 < r_1 < r_2 \leq R_1$. By the mean value theorem, there exists $s_n = s(\lambda_n) \in (r_1, r_2)$ satisfying

$$u_{\lambda_n}(r_2) = u_{\lambda_n}(r_1) + u'_{\lambda_n}(s_n)(r_2 - r_1)\,.$$

Observe that by Lemma 3.2 the first summand in the last equality satisfies

$$u_{\lambda_n}(r_1) \ \leq \ M, \quad \forall n \in \mathbb{N}$$

for some $M = M(r_1) > 0$. In addition, by (23), the second summand tends to $-\infty$. Hence we deduce

$$\lim_{n \to +\infty} u_{\lambda_n}(r_2) \ = \ -\infty$$

which clearly is a contradiction with $u_{\lambda_n} \geq 0, \ \forall n \in \mathbb{N}$. Then Lemma 3.3 is proved. □

LEMMA 3.4. *Assume* (f_1). *Let* $R_2 \in (R_0, \hat{R}]$ *and* $\bar{c} > 1$. *Then there exists* $\lambda_2 > 0$ *such that every non-negative solution* u_λ *of* (4) *with* $\lambda \geq \lambda_2$ *satisfies*

$$\frac{\beta}{\bar{c}} \in u_\lambda([R_2, \hat{R}]).$$

Proof. If $u = u_\lambda$ is a non-negative solution of (4), we denote by

$$b_\lambda = \max\{r \in (R, \hat{R}) \ / \ u(r) = \frac{\beta}{\bar{c}}\}.$$

The lemma will be proved if we can show

$$\lim_{\lambda \to +\infty} b_\lambda = \hat{R}. \tag{24}$$

To do it, we multiply the equation in (4) by r^{N-1}, integrate it from b_λ to $\hat{R}$ and use that

$$u(r) < \frac{\beta}{\bar{c}}, \ \forall r \in (b_\lambda, \hat{R}]$$

to deduce:

$$\int_{b_\lambda}^{\hat{R}} (r^{N-1}u'(r))' dr = -\int_{b_\lambda}^{\hat{R}} \lambda r^{N-1} f(u(r)) dr$$

$$\geq \int_{b_\lambda}^{\hat{R}} \lambda r^{N-1} K dr$$

where

$$K = -\max\{f(\xi) / \ \xi \in [0, \beta]\} > 0$$

Hence

$$\hat{R}^{N-1}u'(\hat{R}) - b_\lambda^{N-1}u'(b_\lambda) \geq \frac{\lambda}{N} K(\hat{R}^N - b_\lambda^N) > 0 \tag{25}$$

On the other hand, multiplying now the same equation by $r^{2(N-1)}u'(r)$ and integrating from b_λ to $\hat{R}$, we have

$$-\int_{b_\lambda}^{\hat{R}} [u''(r)u'(r)r^{2(N-1)} - (N-1)r^{2N-3}u'(r)^2] dr =$$

$$= \lambda \int_{b_\lambda}^{\hat{R}} f(u(r)) r^{2(N-1)} u'(r) dr\,.$$

That is

$$-\int_{b_\lambda}^{\hat{R}} [r^{N-1}u'(r)]' u'(r) r^{N-1} dr = \lambda \int_{b_\lambda}^{\hat{R}} [F(u(r))]' r^{2(N-1)} dr$$

Computing the two integrals by parts, we obtain

$$\frac{1}{2}[b_\lambda^{2(N-1)}u'(b_\lambda)^2 - \hat{R}^{2(N-1)}u'(\hat{R})^2] = -\lambda b_\lambda^{2(N-1)}F(\frac{\beta}{\bar{c}})$$

$$- 2(N-1)\lambda\int_{b_\lambda}^{\hat{R}} F(u(r))r^{2N-3}dr$$

Since $u(r) < \dfrac{\beta}{\bar{c}}$, $\forall r \in (b_\lambda, \hat{R}]$ and F is decreasing in $(0,\beta)$ by (f_1), we deduce

$$\frac{1}{2}[b_\lambda^{2(N-1)}u'(b_\lambda)^2 - \hat{R}^{2(N-1)}u'(\hat{R})^2] \leq -\lambda b_\lambda^{2(N-1)}F\Big(\frac{\beta}{\bar{c}}\Big)$$

$$- 2(N-1)F\Big(\frac{\beta}{\bar{c}}\Big)\lambda\int_{b_\lambda}^{\hat{R}} r^{2N-3}dr$$

$$= \quad -\lambda\hat{R}^{2(N-1)}F\Big(\frac{\beta}{\bar{c}}\Big)$$

Observe that (25) implies that the left hand of this inequality is positive (because $u'(b_\lambda) \leq 0$ by the definition of b_λ and $u'(\hat{R}) \leq 0$ by [5]). Consequently we can take square roots:

$$\frac{1}{\sqrt{2}}\sqrt{[{b_\lambda}^{N-1}u'(b_\lambda)]^2 - [\hat{R}^{N-1}u'(\hat{R}]^2} \leq \hat{R}^{N-1}\sqrt{-\lambda F\Big(\frac{\beta}{\bar{c}}\Big)}$$

and using that $A - B \leq \sqrt{A^2 - B^2}$, $\forall A \geq B \geq 0$ we get

$$\frac{1}{\sqrt{2}}[|b_\lambda^{N-1}u'(b_\lambda)| - |\hat{R}^{N-1}u'(\hat{R})|] \leq \hat{R}^{N-1}\sqrt{-\lambda F\Big(\frac{\beta}{\bar{c}}\Big)}$$

which, by (25) again, implies

$$\frac{\lambda}{N\sqrt{2}}K(\hat{R}^N - b_\lambda^N) \leq \hat{R}^{N-1}\sqrt{-\lambda F\Big(\frac{\beta}{\bar{c}}\Big)}$$

i.e.,

$$-\frac{1}{N\sqrt{2}}K\frac{1}{\sqrt{-F(\frac{\beta}{\bar{c}})}}\sqrt{\lambda}(\hat{R}^N - b_\lambda^N) \leq \hat{R}^{N-1}$$

and as a consequence (24) is satisfied. So the proof is finished. □

Proof of Theorem 3.1: Let us consider $c \in (\beta, \theta), \bar{c} > 1$ and $R_1, R_2 \in (R_0, \hat{R})$ such that $R_1 < R_2$. Let $\lambda_1, \lambda_2 > 0$ be given, respectively, by Lemmas 3.3 and 3.4. Take $\mu = \left(\frac{\beta}{\bar{c}} + c\right)(R_2 - R_1)^{-1}$ and choose

$$\lambda^* \;\geq\; \max\{\lambda_1, \lambda_2\}$$

such that

$$\lambda^* M + \frac{\mu^2}{2} < 0$$

where

$$M = \max\left\{F(\xi) : \frac{\beta}{\bar{c}} \leq \xi \leq c\right\} < 0\,.$$

We claim that (4) has no non-negative solutions for $\lambda \geq \lambda^*$. Otherwise there exists $\lambda \geq \lambda^*$ such that (4) has at least one non-negative solution u_λ. Since $\lambda \geq \lambda_i, i = 1, 2$, we deduce from Lemmas 3.3 and 3.4 the existence of $t_1 = t_1(\lambda) \in (R_0, R_1]$ and $t_2 = t_2(\lambda) \in [R_2, \hat{R}]$ satisfying $u_\lambda(t_1) < c$ and $u_\lambda(t_2) = \frac{\beta}{\bar{c}}$. Then, by the mean value theorem there is $t_3 = t_3(\lambda) \in (t_1, t_2)$ such that

$$u_\lambda(t_2) - u_\lambda(t_1) \;=\; u'_\lambda(t_3)(t_2 - t_1)\,.$$

Hence

$$|u'_\lambda(t_3)| \;=\; \frac{|u_\lambda(t_2) - u_\lambda(t_1)|}{t_2 - t_1} \;\leq\; \mu \tag{26}$$

On the other hand, by [5] , u_λ is strictly decreasing in $[R_0, \hat{R})$. Then

$$u_\lambda(t_3) \in [\frac{\beta}{\bar{c}}, c]\,. \tag{27}$$

The expressions (26) and (27) imply that the energy function $E(r) = \lambda F(u_\lambda(r)) + \frac{u'_\lambda(r)^2}{2}$ (considered in the proof of Lemma 2.3) satisfies

$$E(t_3) \leq \lambda M + \frac{\mu^2}{2} \leq \lambda^* M + \frac{\mu^2}{2} < 0$$

But this is impossible because E is a nonincreasing function (remind that $E'(r) = -\dfrac{N-1}{r}u'(r)^2 \leq 0$) with $E(\hat{R}) = \dfrac{u'(\hat{R})^2}{2} \geq 0$. Therefore the claim is true. ∎

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