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Uryson operators and Equimeasurable sets

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RIASSUNTO: *Si dimostra che ogni operatore di Uryson di ordine limitato trasforma gli intervalli d'ordine in insiemi equimisurabili. Utilizzando poi un risultato ottenuto in [11], si ottiene una caratterizzazione degli operatori di Uryson che è una versione non lineare di quella di W. Schachermayer [7].*

ABSTRACT: *In this paper we prove that every order bounded Uryson operator maps order intervals onto equimeasurable sets and "a fortiori" (by the main result of [11]) we obtain a criterion for recognizig Uryson operators. This criterion is a non-linear version of that proved by W. Schachermayer in [7].*

– Introduction

The concept of equimeasurable set goes back to A. GROTHENDIECK'S memoir [3, p. 20] where it is used for characterizing nuclear operators [3, p. 64] (see also [1, p. 258]). In 1979, W. SCHACHERMAYER characterizes linear integral operators by this concept [7, theorem 4.4]. Another characterization of linear integral operators had already been given by A.V. BUKHVALOV [2] (see also [15, theorems 96.5 and 96.8]). This one is based in the difference between ($*$)-convergence and almost everywhere convergence. Laterly, A.R. SCHEP [9] and W. SCHACHERMAYER [8] showed how to deduce one characterization from the other one (see also [14]).

KEY WORDS AND PHRASES: *Uryson operators – Equimeasurable sets – Ideal spaces of measurable functions – Banach function spaces*
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In a previous article [11] it was proved that a Bukhvalov type characterization enables us to recognize order bounded Uryson operators. Then a natural question arises, is it possible to give a criterion for Uryson operators that uses equimeasurable sets?. Theorem 2.1 below gives a positive answer to this question.

The paper is divided into two sections. Section 1 is on preliminaries. The main result and some consequences for operators defined between Banach function spaces are inclosed in section 2.

1 – Preliminaries

For terminology concerning Riesz spaces theory we shall follow [4, 13, 15]. We only recall that a subset H in a Riesz space E is said to be **order bounded** if there exists $g \in E^+$ such that $|h| \leq g$ for every $h \in H$.

Throughout this paper we shall write $\mathbf{1}_A$ to denote the characteristic function of the set A.

Let (X, μ) be a σ-finite and complete measure space. By $L^0(X, \mu)$ we denote the space of all real-valued measurable functions on X which are almost everywhere finite. Functions which are equal a.e. will be identified. The subspace of $L^0(X, \mu)$ consisting of essentially bounded functions will be denoted by $L^\infty(X, \mu)$. A subset $H \subset L^0(X, \mu)$ is said to be **equimeasurable** if for every $X_0 \subset X$ of finite measure and every $\epsilon > 0$ there is $X_\epsilon \subset X_0$ with $\mu(X_0 \backslash X_\epsilon) < \epsilon$ and such that the set $\{\mathbf{1}_{X_\epsilon} f \mid f \in H\}$ is relatively compact in $L^\infty(X_\epsilon, \mu_{|_{X_\epsilon}})$. The following characterization of equimeasurable sets will be used several times throughout this paper (see [10, theorem 1.2] and [8, proposition 2.4]).

Lemma 1.1. *A set $H \subset L^0(X, \mu)$ is equimeasurable if and only if there exists $g \in L^0(X, \mu)$ with $g > 0$ μ-a. e. such that the set $\frac{1}{g}H$ is relatively compact in $L^\infty(X, \mu)$.*

Thus, to prove that a set H is equimeasurable it is enough to see that the set $\frac{1}{g}H$ is so for some positive function $g \in L^0(X, \mu)$.

Recall that for sequences order convergence in $L^0(X, \mu)$ coincides with almost everywhere convergence [13, p. 65 and 4, theorem 71.3]. It is said that a sequence $(f_n)_{n\in\mathbb{N}}$ in $L^0(X, \mu)$ **(*)-converges** to f if an arbitrary subsequence $(f_{n_k})_{k\in\mathbf{N}}$ contains a subsequence $(f_{n_{k_i}})_{i\in\mathbb{N}}$ that order converges to f; i.e., $f_{n_{k_i}}(x) \to f(x)$ a.e. Thus, the sequence $(f_n)_{n\in\mathbb{N}}$ $(*)$-converges if and only if it converges in measure on every subset of finite measure. A. R. Schep [9, lemma 3.1] pointed out the following fact (see also [7, proposition 2.4]).

Lemma 1.2. *In an equimeasurable set every $(*)$-convergent sequence converges a.e.*

2 – Recognition of Uryson operators

Consider two σ-finite and complete measure spaces (Y, ν) and (X, μ). By E and F we shall denote order ideals in the spaces $L^0(Y, \nu)$ and $L^0(X, \mu)$ respectively. We shall assume that Y is the carrier of the ideal E; that is, the only sets $B \subset Y$ such that every $f \in E$ is vanished ν-a.e. on B, are the ν-null sets [15, S86].

Let $U : X \times Y \times \mathbb{R} \to \mathbb{R}$ be a function satisfying

- (C_0) $U(x, y, 0) = 0$ for $\mu \times \nu$-almost all $(x, y) \in X \times Y$.
- (C_1) The function $U(\cdot, \cdot, t)$ is $\mu \times \nu$-measurable for all $t \in \mathbb{R}$.
- (C_2) The function $U(x, y, \cdot)$ is continuous on $\mathbb{R}$ for $\mu \times \nu$-almost all $(x, y) \in X \times Y$.

It follows from the Carathéodory conditions (C_1 and C_2) that if $f \in L^0(Y, \nu)$, then the function $U(x, y, f(y))$ is $\mu \times \nu$-measurable and $\mu \times \nu$-a.e. finite. An operator $T : E \to F$ is called a **Uryson operator** with kernel U if for all $f \in E$ the following conditions hold.

1. The function $x \to \int_Y |U(x, y, f(y))| d\nu$ is μ-a.e. finite.
2. $(Tf)(x) = \int_Y U(x, y, f(y)) d\nu$ μ-a.e.

For a more detailed definition of Uryson operators we refer to [5, section 5]. As a consequence of (C_0) one has that $f, g \in E$ with disjoint supports implies $T(f + g) = Tf + Tg$. Operators satisfying this property are known as **orthogonally additive** operators. An orthogonally additive operator from E into F is **order bounded** if it transforms order bounded sets in E onto order bounded sets in F.

Our aim in this section is to obtain theorem 2.1 which characterizes those order bounded orthogonally additive operators which are Uryson operators. By the main result in [11], it is enough to prove that every order bounded Uryson operator maps order intervals onto equimeasurable sets, and we are able to do this using a kind of uniform order convergence. We point out that neither the similar result in the linear case nor its methods can be applied in a straightforward manner for Uryson operators (although we remark that this approach works for Hammerstein operators, see [12]). On the other hand, the technics developed in [8] to arrive to Schachermayer's condition from Bukhvalov's one, essentially depend on the fact that linear operators maps order intervals onto absolutely convex sets, and neither can be used in our case.

THEOREM 2.1. *Let $T : E \to F$ be an order bounded orthogonally additive operator. The following assertions are equivalent.*

1. *T is a Uryson operator with kernel satisfying the Carathéodory conditions.*
2. *If $(f_n)_{n=1}^{\infty}$ and $(g_n)_{n=1}^{\infty}$ are order bounded sequences in E such that*

$$f_n - g_n \to 0(*), \textit{then } Tf_n(x) - Tg_n(x) \to 0 \mu\textit{-a.e.}$$

3. *(a) T maps every order interval onto an equimeasurable set.*
   (b) For every $(f_n)_{n=1}^{\infty}$ and $(g_n)_{n=1}^{\infty}$ order bounded sequences in E,

$$f_n(y) - g_n(y) \to 0 \nu\textit{-a.e.implies } Tf_n(x) - Tg_n(x) \to 0 \mu\textit{-a.e.}$$

PROOF. (1) $\iff$ (2) This is already proved in [11].

(3) $\Rightarrow$ (2) Let $(f_n)_{n=1}^{\infty}$ and $(g_n)_{n=1}^{\infty}$ be order bounded sequences in E such that $f_n - g_n \to 0$ $(*)$. Condition (3) (b) implies that $Tf_n - Tg_n \to 0$ $(*)$, while it follows from (3) (a) that the set $\{Tf_n - Tg_n \mid n \in N\}$ is equimeasurable. Now (2) follows from lemma 1.2.

(2) $\Rightarrow$ (3) (b) This is straightforward.

(1) $\Rightarrow$ (3) (a) Denote by U the kernel of the operator T. Given $g \in E^+$ we have to prove that $T[-g, g]$ is a equimeasurable set. Changing of variable, if necessary, we may assume restrictions on the space E and on the function g. In fact, let $g \in E^+$ and define the function $V(x, y, t) := U(x, y, tg(y))$. It is routine to check that the function V satisfies the Carathéodory conditions. So the function V is the kernel of an order bounded Uryson operator from $L^\infty(Y, \nu)$ into F which maps the order interval $[-\mathbf{1}_Y, \mathbf{1}_Y]$ onto $T[-g, g]$. Hence we may suppose that

$E = L^\infty(Y,\nu)$ and then to prove that the set $T[-\mathbf{1}_Y, \mathbf{1}_Y]$ is equimeasurable. We prove this fact by several stages.

Claim 1. *For every $n \in \mathbb{N}$, the set*

$$\{T\Big(\sum_{k=-n}^{n} \frac{k}{n}\mathbf{1}_{B_k}\Big) \mid \textit{the subsets } (B_k)_{k=-n}^{n} \textit{of } Y \textit{ are mutually disjoint}\}$$

is equimeasurable.

Fix $t \in [-1,1]$ and define a linear operator $S_t : L^\infty(Y,\nu) \to F$ by $S_t f(x) := \int_Y U(x,y,t)f(y)d\nu$. By Schachermayer's characterization of linear integral operators, the set $S_t[-\mathbf{1}_Y, \mathbf{1}_Y]$ is equimeasurable and hence the set

$$\{S_t(\mathbf{1}_B) \mid B \subset Y\} = \{T(t\mathbf{1}_B) \mid B \subset Y\}$$

is so. Since the number t is arbitrary, taking $t = \frac{k}{n}$, $-n \le k \le n$, and considering the sum of all them claim (1) follows.

Next we shall aproximate functions uniformly in $[-\mathbf{1}_Y, \mathbf{1}_Y]$ by simple ones. For each $f \in [-\mathbf{1}_Y, \mathbf{1}_Y]$ and each $n \in \mathbb{N}$ define $P_n f := \sum_{k=-n}^{n} \frac{k}{n}\mathbf{1}_{B_k}$ where

$B_k := \{y \in Y \mid \frac{k-1}{n} < f(y) \le \frac{k}{n}\}$ whenever $k > 0$;

$B_0 := \emptyset$ and

$B_k := \{y \in Y \mid \frac{k+1}{n} > f(y) \ge \frac{k}{n}\}$ whenever $k < 0$.

Claim 2. *For μ-almost all $x \in X$, $TP_n f(x) \to Tf(x)$ uniformly on $f \in [-\mathbf{1}_Y, \mathbf{1}_Y]$.*

Since the operator T is order bounded, it follows from [5, theorem 6.2] that there exists a $\mu \times \nu$-measurable function $M : X \times Y \to \mathbb{R}$ such that

(1) $|U(x,y,f(y))| \le M(x,y)$ $\mu \times \nu$-a.e. for all $f \in [-\mathbf{1}_Y, \mathbf{1}_Y]$.

(2) For μ-almost all $x \in X$, the function $M(x,\cdot)$ is ν-integrable.

(3) The function $x \to \int_Y M(x,y)d\nu$ belongs to F.

Consider $x \in X$ such that the function $M(x,\cdot)$ is ν-integrable and the set $\{y \in Y \mid$ the function $U(x,y,\cdot)$ is not continuous$\}$ is ν-null and let us see that

$$\int_Y U(x,y,P_n f(y))d\nu \to \int_Y U(x,y,f(y))d\nu \text{ uniformly for } f \text{ in } [-\mathbf{1}_Y, \mathbf{1}_Y].$$

Let $\epsilon > 0$, since the measure ν is σ-finite, there is an increasing sequence $(Y_n)_{n=1}^{\infty}$ of subsets of Y such that $\bigcup_{n=1}^{\infty} Y_n = Y$ and $0 < \nu(Y_n) < \infty$ for all $n \in \mathbb{N}$. Then $\lim_{n\to\infty} \int_{Y_n} M(x,y)d\nu = \int_Y M(x,y)d\nu$ and it follows that there is $n \in \mathbb{N}$ such that $\int_{Y\setminus Y_n} M(x,y)d\nu < \epsilon/6$. Let n be fixed. For each $k \in \mathbb{N}$ define the set

$$D_k := \{y \in Y_n \mid t,s \in [-1,1], |t-s| < \frac{1}{k} \implies \\ \implies |U(x,y,t) - U(x,y,s)| < \frac{\epsilon}{3\nu(Y_n)}\}$$

It is easy to see that $\lim_{k\to\infty} \nu(D_k) = \nu(Y_n)$, so given $\epsilon > 0$ it may be found $k_0 \in \mathbb{N}$ such that $k \geq k_0$ implies $\int_{Y_n\setminus D_k} M(x,y)d\nu < \epsilon/6$. Thus, fixed $k \geq k_0$, for every $f \in [-\mathbf{1}_Y, \mathbf{1}_Y]$ and for every $y \in D_k$ we have that $|U(x,y,P_kf(y)) - U(x,y,f(y))| < \frac{\epsilon}{3\nu(Y_n)}$ and it yields

$$\Big|\int_{D_k} U(x,y,P_kf(y))d\nu - \int_{D_k} U(x,y,f(y))d\nu\Big| < \frac{\epsilon}{3}.$$

Now compute to obtain that if $f \in [-\mathbf{1}_Y, \mathbf{1}_Y]$ and $k \geq k_0$, then

$$\Big|\int_Y U(x,y,P_kf(y))d\nu - \int_Y U(x,y,f(y))d\nu\Big| < \epsilon.$$

and so claim (2) is proved.

Claim 3. *There exists $h \in L^0(X,\mu)$, with $h(x) > 0$ μ-a.e., such that*

$$\|\cdot\|_\infty - \lim_{n\to\infty} \frac{1}{h} TP_nf = \frac{1}{h} Tf \text{ uniformly on } [-\mathbf{1}_Y, \mathbf{1}_Y].$$

For every $n \in \mathbb{N}$ define $g_n := \sup\{\sup_{k\geq n} |TP_kf - Tf| \mid f \in [-\mathbf{1}_Y, \mathbf{1}_Y]\}$, where the supremum is taken in the order of F. Observe that each function g_n lies in F since the operator T is order bounded. Furthermore, applying the order separability of F (see, for instance, [4, example 23.3 (iv)]) there exists a sequence $(f_{np})_{p=1}^{\infty}$ in $[-\mathbf{1}_Y, \mathbf{1}_Y]$ such that $g_n = \sup_{p\in\mathbb{N}} \sup_{k\geq n} |TP_kf_{np} - Tf_{np}|$ and so we may consider that each g_n is the pointwise supremum of $|TP_kf_{np} - Tf_{np}|$. It follows from claim (2) that $g_n(x) \downarrow 0$ μ-a.e.

Applying [4, theorems 71.4 and 16.3] there exists $h \in L^0(X,\mu)$, which we may take positive, such that $g_n \to 0$ h-uniformly; that is, for every $\epsilon > 0$ there is $n_0 \in \mathbb{N}$ such that $g_n(x) \le \epsilon h(x)$ μ-a.e. for all $n \ge n_0$. Therefore, if $f \in [-\mathbf{1}_Y, \mathbf{1}_Y]$ and $n \ge n_0$, then $\frac{1}{h(x)}|TP_n f(x) - Tf(x)| \le \epsilon$ μ-a.e. and consequently we have got claim (3).

Claim 4. *There exists $h \in L^0(X,\mu)$, with $h(x) > 0$ μ-a.e., such that the set $\{\frac{1}{h}Tf \mid f \in [-\mathbf{1}_Y, \mathbf{1}_Y]\}$ is equimeasurable.*

Let $X_0 \subset X$ be a set of finite measure and let $\epsilon > 0$. As a consequence of claim (1) and lemma 1.1, the set $\{\frac{1}{h}(TP_n f) \mid f \in [-\mathbf{1}_Y, \mathbf{1}_Y]\}$ is equimeasurable for all $n \in \mathbb{N}$. So there exists $X_n \subset X_0$ such that $\mu(X_0 \backslash X_n) < \epsilon/2^n$ and the set $\{\mathbf{1}_{X_n}\frac{1}{h}(TP_n f) \mid f \in [-\mathbf{1}_Y, \mathbf{1}_Y]\}$ is relatively compact in $L^\infty(X_n, \mu_{|_{X_n}})$. Define $X' := \bigcap_{n=1}^\infty X_n$, then $\mu(X_0 \backslash X') < \epsilon$ and each set $\{\mathbf{1}_{X'}\frac{1}{h}(TP_n f) \mid f \in [-\mathbf{1}_Y, \mathbf{1}_Y]\}$ is relatively compact in $L^\infty(X', \mu_{|_{X'}})$. Hence, it follows from claim (3) that the set $\{\mathbf{1}_{X'}\frac{1}{h}(Tf) \mid f \in [-\mathbf{1}_Y, \mathbf{1}_Y]\}$ is also relatively compact and this proves claim (4).

Finally, as a consequence of claim (4) and lemma 1.1, the desired statement follows.

Criteria in the above result must be seen as complementary. Next we shall make clearer this fact.

remark 2.2. Observe that condition (2) suggests a kind of uniform continuity on order bounded sets but it is not obvious how to precise this idea since in general almost everywhere convergence does not came from any metric (or uniform structure). However, it follows from condition (3) (a) that the above statement can be precised. On the one hand, take $B \subset Y$ with $\nu(B) < \infty$ and for each $g \ge 0$ consider in the interval $[-g, g]$ the metric $d_{g,B}$ defined by $d_{g,B}(f_1, f_2) := \int_{\mathrm{supp}(g)\cap B} (|f_1 - f_2|/g)d\nu$. Note that by the dominated convergence theorem, convergence with respect to this metric is equal to convergence in measure. On the other hand, if $H \subset L^0(X,\mu)$ is an equimeasurable set, then there is a positive function $v \in L^0(X,\mu)$ such that $\frac{1}{v}H$ is relatively compact in $L^\infty(Y,\nu)$. Observe that the formula $\delta_v(h_1,h_2) := \|\mathbf{1}_{\mathrm{supp}(v)}(h_1 - h_2)/v\|_\infty$ defines a metric on $[-v, v]$. With these preliminaries the following result is straightforward.

Corollary 2.3. *A necessary and sufficient condition for an orthogonally additive operator $T : E \to L^0(X,\mu)$ to be an order bounded*

Uryson operator is the following. For each $g \in E^+$ there is $v \in L^0(X,\mu)^+$ such that $T : ([-g,g], d_{g,B}) \to ([-v,v], \delta_v)$ is uniformly continuous for all $B \subset Y$ of finite measure.

We point out that uniform continuity may be changed by continuity when T is a Hammerstein operator (see [12, theorem 2.4]).

We next consider operators defined between Banach function spaces. Recall that an ideal of measurable functions is a **Banach function space** if it is also a Banach space and its norm is a Riesz norm; that is, $|f| \leq |g|$ implies $\|f\| \leq \|g\|$. A Banach function space is said to have **order continuous norm** if for every net $(f_\alpha)_{\alpha \in \mathrm{A}}$ in the space, it follows from $f_\alpha \to 0$ (o) that $\|f_\alpha\| \to 0$. It is easy to see that $L^p(X,\mu)$ is a Banach function space for every $p, 1 \leq p \leq \infty$; moreover, the space $L^p(X,\mu)$ has order continuous norm whenever $1 \leq p < \infty$. An order bounded orthogonally additive operator is called **AM-compact** if it maps order bounded sets onto relatively compact ones. In the next result we show that theorem 2.1 gives a new proof of [6, theorem 3.5]. To see it, note that, as a consequence of theorem 2.1, it is enough to check that every equimeasurable set in a Banach function space with order continuous norm is relatively compact and this fact is a consequence of [10, theorem 2.2].

Corollary 2.4. *Let E be an ideal in $L^0(Y,\nu)$ and let F be a Banach function space in $L^0(X,\mu)$ having order continuous norm. Then every order bounded Uryson operator from E to F is AM-compact.*

Finally, we state several other consequences of theorem 2.1, which are non linear versions of results in [9]. We shall omit the proofs since they are similar to those in the linear case.

Corollary 2.5. *Let E be an ideal in $L^0(Y,\nu)$ and let F be a Banach function space in $L^0(X,\mu)$. Suppose that $T : E \to F$ is an order bounded Uryson operator. Then the following property holds.*

(†) *If $(f_n)_{n=1}^\infty$ and $(g_n)_{n=1}^\infty$ are order bounded sequences in E, then $\|Tf_n - Tg_n\| \to 0$ implies $Tf_n(x) - Tg_n(x) \to 0$ μ-a.e.*

Property (†) characterizes order bounded Uryson operators under some conditions.

1. If F has besides order continuous norm and T is an order bounded orthogonally additive operator which satisfies that given two order

bounded sequences $(f_n)_{n=1}^{\infty}$ and $(g_n)_{n=1}^{\infty}$ in E, $f_n(y)-g_n(y) \to 0$ ν-a.e. implies $Tf_n(x) - Tg_n(x) \to 0$ μ-a.e., then T is a Uryson operator.

2. If E is a Banach function space with order continuous norm and T is an order bounded orthogonally additive operator which is uniformly continuous on the order bounded sets of E, then T is a Uryson operator.

In particular one deduces that if E has order continuous norm, then every order bounded orthogonally additive operator $T : E \to L^{\infty}(X, \mu)$ which is uniformly continuous on the order bounded sets is a Uryson operator.

Corollary 2.6. *Let E and F be Banach function spaces and assume that $T : E \to F$ is an order bounded orthogonally additive operator. Consider the following assertions.*

1. T is a Uryson operator.

2. (a) T maps order bounded sets in E onto equimeasurable sets.
(b) T is uniformly continuous on the order bounded sets of E.

Then, if E has order continuous norm, (2) ⇒ (1) and if F has order continuous norm, (1) ⇒ (2).

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