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Second order nonautonomous systems with symmetric potential changing sign

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RIASSUNTO: *In questo lavoro si studia il problema della molteplicità di soluzioni periodiche per una classe di sistemi hamiltoniani non autonomi del secondo ordine aventi potenziale di segno variabile. Nel caso particolare in cui la parte quadratica del potenziale è definita negativa, si ottiene anche un risultato di soluzioni subarmoniche e omocline. La dimostrazione dei risultati di molteplicità si basa sul metodo della categoria di Ljusternik e Schnirelmann; le subarmoniche sono ottenute come punti di minimo del funzionale vincolati ad una opportuna varietà, e le omocline si ottengono con un procedimento di limite a partire dalla successione di subarmoniche.*

ABSTRACT: *In this paper we deal with the problem of multiplicity of periodic solutions for a class of nonautonomous second order Hamiltonian systems, having indefinite potential. In the particular case that the quadratic part of the potential is negative definite, one reaches a result of subharmonic and homoclinic solutions. The proof of the multiplicity results is based on the Ljusternik-Schnirelmam category theory; the subharmonic solutions are obtained through the constrained minima of the functional to a suitable manifold, and the homoclinics are obtained with a limit procedure starting by the sequence of subharmonics.*

1 – Introduzione

Let us consider the following second order Hamiltonian system

$$\ddot{x} + A(t)x + b(t)V'(x) = 0 \tag{P}$$

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where $A(t)$ is a continuous T-periodic (for some fixed $T > 0$) matrix valued function, $b(t)$ is a continuous T-periodic real function and $V \in C^2(\mathbb{R}^N, \mathbb{R})$. Recently many authors have studied the problem of existence and multiplicity for periodic and subharmonic solutions of (P) either in case that the quadratic term is identically zero (cfr. [5], [7], [9], [10]) or in case that it is definite in sign (see, e.g., [3], [6], [8], etc.). However there is no general result when $A(t)$ is indefinite in sign: as far as the authors know, the only existence results have been stated in [4], in case that the matrix satisfies the integral condition

$$\int_0^T \langle A(t)\xi, \xi\rangle \,\mathrm{d}t \;>\; 0 \quad \forall \xi \in \mathbb{R}^N,\ |\xi| \,=\, 1 \tag{1.1}$$

and $b(t)$ is such that $\int_0^T b(t)\,\mathrm{d}t > 0$.

In this paper we deal with a potential indefinite in sign that satisfies suitable evennes conditions. Working in H_0^1, we prove a multiplicity result for T-periodic solutions of (P). In the particular case that $A(t)$ is negative definite, we also find the existence of subharmonic solutions, which have some symmetry properties, and of an homoclinic solution, obtained by a limit procedure starting from a suitable translation of these subharmonics. The techniques of proofs are based on the consideration of the Nehari' s manifold M, suitably connected to the functional f associated with problem (P) (see also [2], where the manifold M was used for the first time in the variational approach to Hamiltonian systems). Precisely the symmetry assumptions on the potential allow to use some constrained minimum arguments and a result of the Ljusternik-Schnirelmann category theory, in order to find periodic solutions, subharmonic and homoclinic solutions to (P), starting by the critical points of f on M.

2 – The main results

Let us consider the following second order Hamiltonian system in $\mathbb{R}^N$

$$\ddot{x} + A(t)x + b(t)V'(x) \,=\, 0 \tag{P}$$

where

i) $A(t)$ is a symmetric continuous T-periodic $N \times N$ matrix valued function;

ii) $b(t)$ is a continuous T-periodic real function such that:

(b.1) $$\exists t_0 \in [0,T] \quad \text{such that} \quad b(t_0) > 0\,;$$

iii) $V \in C^2(\mathbb{R}^N, \mathbb{R})$ and there exists a constant $\beta > 2$ such that:
(V.1) $\exists a_1 > 0 \;:\quad V(x) \geq a_1|x|^\beta \quad \forall x \in \mathbb{R}^N$
(V.2) $\exists a_2 > 0 \;:\quad |V'(x)| \leq a_2|x|^{\beta-1} \quad \forall x \in \mathbb{R}^N$
(V.3) $V'(x) \cdot x \geq \beta V(x) \qquad \forall x \in \mathbb{R}^N$
(V.4) $V''(x)x \cdot x \geq (\beta-1)V'(x) \cdot x \qquad \forall x \in \mathbb{R}^N$
(V.5) $V(x) = V(-x) \qquad \forall x \in \mathbb{R}^N$.

REMARK 2.1. It is well known that any given square symmetric matrix $A(t)$ can be written as the sum of two matrices, i.e.

(2.2) $$A = A^+ + A^-$$

where A^+ and A^- are positive semidefinite and negative semidefinite respectively, more precisely the positive (negative) eigenvalues of A coincide with the eigenvalues of A^+ (A^-) different from zero.

For any fixed $t \in \mathbb{R}$ let us put

(2.3) $$\Lambda^+ = \max_{t\in[0,T]}\left(\max_i \lambda_i^+(t)\right) \qquad \left(\Lambda^- = \max_{t\in[0,T]}\left(\max_i \lambda_i^-(t)\right)\right)$$

where $\lambda_i^+(t)$ ($\lambda_i^-(t)$) are the eigenvalues of the matrix $A^+(t)$ (resp. $A^-(t)$) ($i = 1,\dots,N$) for $T \in [0,T]$ (observe that, in general, we can't assure that Λ^+ and Λ^- are different from zero). We will prove the following theorem:

THEOREM 2.4. *Let $b(t) \in C^0([0,T],\mathbb{R})$ be a T-periodic function satisfying* (b.1), *$A(t) = [a_{ij}(t)]$ be a $N \times N$ symmetric matrix, where a_{ij} is a T-periodic continuous function, for $i,j = 1,\dots,n$, and such that:*

(A.1) $$\Lambda^+ < \frac{4}{T^2}$$

where Λ^+ is given by (2.2) *and $V \in C^2$ $(\mathbb{R}^N,\mathbb{R})$ satisfy* (V.1)-(V.5).

If we assume that $b(t)$, $A(t)$ *and* $V(x)$ *verify the following further conditions:*

$$\text{(b.2)} \qquad b(t) = b(T-t) \qquad \forall\, t \in \left[0, \frac{T}{2}\right],$$

$$\text{(A.2)} \qquad A(t) = A(T-t) \qquad \forall t \in \left[0, \frac{T}{2}\right];$$

$$\text{(V.6)} \qquad B^- \left[V'(x)\cdot x \;-\; \beta V(x)\right] \;\le\; c|x|^2, \quad \forall x \in \mathbb{R}^N$$

and

$$\text{(V.7)} \qquad B^-[V''(x)x\cdot x - (\beta-1)V'(x)\cdot x] \le\; d\,|x|^2 \quad \forall x \in \mathbb{R}^N$$

where the positive constants c *and* d *are such that*

$$\text{(2.5)} \qquad \max\{2c, d\} \;<\; (\beta-2)\left(1 - \frac{T^2}{4}\Lambda^+\right)\frac{4}{T^2},$$

and

$$B^- \;=\; \max_{t\in[0,T]} b^-(t) \quad \textit{with} \quad b^-(t) \;=\; -\min\{\,0, b(t)\,\}\,.$$

Then there exist infinitely many pairs of distinct T*-periodic solutions* $x(t)$ *of* (P), *which verifiy*

$$\text{(2.6)} \qquad x\left(t + \frac{T}{2}\right) \;=\; -x\left(\frac{T}{2} - t\right) \qquad \textit{for any} \quad t \in \left[0, \frac{T}{2}\right].$$

On the other hand, substituting (b.2), (A.2) *and* (2.5) *respectively with*

$$\text{(b.2}'\text{)} \qquad b(t) = b\Big(t + \frac{T}{2}\Big) \qquad \forall\, t \in \left[0, \frac{T}{2}\right],$$

$$\text{(A.2}'\text{)} \qquad A(t) = A\Big(t + \frac{T}{2}\Big) \qquad \forall\, t \in \left[0, \frac{T}{2}\right],$$

$$\max\{2c, d\} < (\beta - 2)\Big(1 - \frac{T^2}{4\pi}\Lambda^+\Big)\frac{4\pi}{T^2}\,, \tag{2.7}$$

the solutions $x(t)$ verify another kind of property, i.e.

$$x(t) = -x\Big(t + \frac{T}{2}\Big) \qquad \forall\, t \in \Big[0, \frac{T}{2}\Big]\,. \tag{2.8}$$

Remark 2.9. Let us observe that condition (2.5) implies (2.7), but they must be considered separately because they come from different symmetry conditions.

Remark 2.10. If $A(t) \equiv 0$, Theorem 2.1 yields Theorem 3 of [7] as a particular case.

Starting from the critical points of the functional $f\colon H_0^1\,[0, \frac{T}{2}] \longmapsto \mathbb{R}$ defined by

$$f(x) = \frac{1}{2}\int_0^{\frac{T}{2}} |\dot{x}|^2 - \frac{1}{2}\int_0^{\frac{T}{2}} \langle A(t)x, x\rangle - \int_0^{\frac{T}{2}} b(t)V(x),$$

we get the solutions of the problem

$$\text{(P}'\text{)} \qquad \begin{cases} \ddot{x} + A(t)x + b(t)V'(x) = 0 \\ x(0) = x\Big(\dfrac{T}{2}\Big) = 0\,. \end{cases}$$

Precisely, putting

$$\tilde{x}(t) = \begin{cases} x(t) & t \in \Big[0, \dfrac{T}{2}\Big] \\ -x(T-t) & t \in \Big[\dfrac{T}{2}, T\Big] \end{cases}$$

one can easily check that if x solves (P$'$), $\tilde{x}$ solves (P). Observe that the expression

$$\|x\|_A = \left[\int_0^{T/2} |\dot{x}|^2 - \int_0^{T/2} \langle A^-(t)x, x\rangle\right]^{\frac{1}{2}} \quad \forall x \in H_0^1 = H_0^1\left(\Big[0, \frac{T}{2}\Big]; \mathbb{R}^N\right)$$

defines a norm in H_0^1 which is equivalent to the usual norm

$$\|x\|_{H_0^1} = \left(\int_0^{\frac{T}{2}} |\dot{x}|^2 \right)^{\frac{1}{2}}.$$

Thus the functional f can be written in the form:

$$f(x) = \frac{1}{2}\, \|x\|_A^2 \,-\, \frac{1}{2}\int_0^{\frac{T}{2}} \langle A^+(t)x, x\rangle \,-\, \int_0^{\frac{T}{2}} b(t)V(x).$$

On the other hand, if we consider the space $H^1_{0,k} = H_0^1([0,kT];\mathbb{R}^N)$ endowed with the norm of $H_k^1 = \{\, x \in H^1(0,kT;\mathbb{R}^N) : x(0) = x(kT)\}$, provided Λ^- defined in (2.3) is strictly negative, we obtain that

$$\|x\|_{L_k^2} \leq \frac{1}{\lambda}\|x\|_A, \qquad \text{with} \quad \lambda = \min\{1, -\Lambda^-\}, \tag{2.11}$$

where $\|\cdot\|_{L_k^2}$ is the usual $L^2([0,kT])$-norm. Taking these notations one can state the following existence result for subharmonic solutions:

THEOREM 2.12. *Let $b(t) \in C^0([0,T],\mathbb{R})$ be a T-periodic real function satisfying* (b.1) *and* (b.2) *and $A(t) \equiv A^-(t)$, with $A^-(t)$ negative definite, satisfying* (A.2). *Moreover let $V \in C^2(\mathbb{R}^N,\mathbb{R})$ verify* (V.1)-(V.7) *where*

$$\max\{2c, d\} < (\beta - 2)\lambda\,,$$

with λ given by (2.11). *Then for all $k \in \mathbb{N}$ there exists a kT-periodic solution x_k, having minimal period kT and verifying*

$$x_k\left(t + \frac{kT}{2}\right) = -x_k\left(\frac{kT}{2} - t\right) \quad \textit{for any} \quad t \in \left[0, \frac{kT}{2}\right].$$

In particular, for each couple of numbers k, h, with $k \neq h$, the solutions x_h and x_k are geometrically distinct.

Starting from Theorem 2.12, the following result holds:

THEOREM 2.13. *Under the same hypothesis of Theorem* 2.12 *there exists at least one homoclinic solution of* (P), *i.e. a nontrivial solution $x \in C^2(\mathbb{R},\mathbb{R}^N)$ of* (P) *which satisfies*

$$\lim_{|t|\to+\infty} x(t) = 0 \qquad \textit{and} \qquad \lim_{|t|\to+\infty} \dot{x}(t) = 0.$$

3 – Proof of the Theorems

Theorems 2.4 is obtained as an application of the following proposition, based on the theory of Ljusternik-Schnirelmann category:

PROPOSITION 3.1 [1]. *Let X be a closed C^1 manifold of a Hilbert space E, symmetric with respect to the origin 0 of E and such that $0 \notin X$. Let there exist a closed infinite dimensional subspace $\tilde{E}$ of E such that the manifold $\tilde{X} = X \cap \tilde{E}$ is homeomorphic to the unit sphere S_∞ of $\tilde{E}$, through an even homeomorphism. Suppose $I \in C^1(X, \mathbb{R})$ satisfies the Palais-Smale condition and*

(I.1) *I is bounded from below on X;*

(I.2) *I is even.*

Then I has infinitely many pairs of critical points.

PROOF OF THEOREM 2.4. Let us introduce the set

$$(3.2)\quad M = \left\{ x \in H_0^1 \setminus \{0\} : \|x\|_A^2 = \int_0^{\frac{T}{2}} \langle A^+(t)x, x\rangle + \int_0^{\frac{T}{2}} b(t)V'(x)\cdot x \right\}.$$

Taking

$$h(x) \;=\; \|x\|_A^2 \;-\; \int_0^{\frac{T}{2}} \langle A^+(t)x, x\rangle \;-\; \int_0^{\frac{T}{2}} b(t)V'(x)\cdot x,$$

we can write

$$M = \left\{ x \in H_0^1 \setminus \{0\} : h(x) \;=\; 0 \right\}.$$

Let us prove that M has the following properties:

(m.1) $M \neq \emptyset$

(m.2) M is a closed C^1-manifold.

Indeed if one considers an element $\overline{x} \in H_0^1 \setminus \{0\}$, with $\|\overline{x}\|_A = 1$ such that $\operatorname{supp}(\overline{x}) \subset \operatorname{supp}(b^+)$, (observe that (b.1) implies $\operatorname{supp}(b^+) \neq \emptyset$), then from (V.2) it follows that for any constant $r > 0$

$$h(r\overline{x}) \geq r^2\|\overline{x}\|_A^2 - \Lambda^+ r^2\|\overline{x}\|_{L^2}^2 - a_2 r^\beta \int_0^{\frac{T}{2}} b^+(t)|\overline{x}|^\beta .$$

Putting $B^+ = \max_{t\in[0,T]} b^+(t)$, by the continuous embedding of H_0^1 in L^β we obtain

$$h(r\overline{x}) \geq r^2\left(1-\Lambda^+\frac{T^2}{4}\right) - a_2 B^+ r^\beta \cdot \text{const}\,. \tag{3.3}$$

As (A.1) holds and $\beta > 2$, $h(r\overline{x}) \geq 0$ for r sufficiently small.

On the other hand, by (V.1), (V.3) and the positivity property of A^+, we have:

$$h(r\overline{x}) \leq r^2 - \beta\int_0^{\frac{T}{2}} b^+(t)V(r\overline{x}) \;\leq\; r^2 - a_1\beta r^\beta \int_0^{\frac{T}{2}} b^+(t)|\overline{x}|^\beta := \alpha.$$

As $\beta > 2$, for r sufficiently large, α is negative, so $h(r\overline{x}) \leq 0$, and, together with (3.3), this proves (m.1). Let us show now (m.2). By the continuous embedding of H_0^1 in L^β and (V.2), for any x in M we have:

$$\begin{aligned}0 = \|x\|_A^2 \;-\int_0^{\frac{T}{2}}\langle A^+(t)x,x\rangle \;-\int_0^{\frac{T}{2}} b(t)V'(x)\cdot x \geq\\ \geq \|x\|_A^2\left(1-\Lambda^+\frac{T^2}{4}\right) \;-\; \text{const}\cdot B^+ a_2\|x\|_A^\beta.\end{aligned}$$

So

$$\|x\|_A^{\beta-2} \geq \left(1-\Lambda^+\frac{T^2}{4}\right)\cdot \text{const}\cdot\frac{1}{a_2 B^+} := \gamma, \tag{3.4}$$

where γ is positive as (A.1) holds.

Moreover (V.4) and (V.7) imply that, for any $x \in M$:

$$\begin{aligned}\langle h'(x),x\rangle &= \|x\|_A^2 \;-\int_0^{\frac{T}{2}}\langle A^+(t)x,x\rangle \;-\int_0^{\frac{T}{2}} b(t)V''(x)x\cdot x \leq\\ &\leq \|x\|_A^2 - \int_0^{\frac{T}{2}}\langle A^+(t)x,x\rangle \;-(\beta-1)\int_0^{\frac{T}{2}} b^+(t)V'(x)\cdot x+\\ &+\int_0^{\frac{T}{2}} b^-(t)V''(x)x\cdot x \leq \left[2-\beta+(\beta-2)\Lambda^+\frac{T^2}{4}+d\frac{T^2}{4}\right]\|x\|_A^2.\end{aligned} \tag{3.5}$$

Therefore, by (2.5), one has $\langle h'(x), x\rangle \leq c_1\|x\|_A^2$, with

$$c_1 = \left[2 - \beta + (\beta - 2)\Lambda^+\frac{T^2}{4} + d\frac{T^2}{4}\right] < 0,$$

so (3.4) and (3.5) imply the C^1-regularity of M.

At this point, taking $X = M$ and $\tilde{E} = \{u \in E\colon \mathrm{supp}(u) \subset \mathrm{supp}(b^+)\}$, we can apply Proposition 3.1 to the functional f after verifying that it is bounded from below and verifies Palais-Smale condition on M. If we denote with f_M the restriction of f to M, (V.3) and (V.6) imply that, for any $x \in M$,

$$\begin{aligned} f_M(x) &= \frac{1}{2}\|x\|_A^2 - \frac{1}{2}\int_0^{\frac{T}{2}}\langle A^+(t)x, x\rangle - \int_0^{\frac{T}{2}} b(t)V(x) \geq \\ &\geq \frac{1}{2}\|x\|_A^2 - \frac{1}{2}\int_0^{\frac{T}{2}}\langle A^+(t)x, x\rangle + \\ &\quad - \frac{1}{\beta}\int_0^{\frac{T}{2}} b(t)V'(x)\cdot x - \frac{1}{\beta}\int_0^{\frac{T}{2}} B^-[V'(x)\cdot x - \beta V(x)] \geq \\ &\geq \frac{\beta-2}{2\beta}\|x\|_A^2 - \frac{\beta-2}{2\beta}\int_0^{\frac{T}{2}}\langle A^+(t)x, x\rangle - \frac{c}{\beta}\frac{T^2}{4}\|x\|_A^2 \geq \\ &\geq \left[\frac{\beta-2}{2\beta}\left(1 - \Lambda^+\frac{T^2}{4}\right) - \frac{c}{\beta}\frac{T^2}{4}\right]\|x\|_A^2, \end{aligned} \tag{3.6}$$

where, by hypothesis (2.5),

$$c < (\beta - 2)\left(1 - \Lambda^+\frac{T^2}{4}\right)\frac{2}{T^2}.$$

Therefore

$$f_M(x) \geq \mathrm{cost}\|x\|_A^2 > 0 \tag{3.7}$$

i.e f_M is coercive; in particular it is bounded from below.

Moreover, arguing as in [7], we can state that f_M verifies Palais-Smale condition, so we obtain the first statement of the theorem. For the second part we will search the solutions of (P) in the space

$$H^1_{\text{odd}} = \Big\{ v \in H^1(0,T;\mathbb{R}^N) \colon v = \sum_{k=2h-1} v_k^{(1)} \cos\Big(\frac{2k\pi t}{T}\Big) + v_k^{(2)} \sin\Big(\frac{2k\pi t}{T}\Big), \quad v_k^{(1)}, v_k^{(2)} \in \mathbb{R}^N, h \in \mathbb{Z} \Big\}.$$

In fact the functions belonging to H^1_{odd} are T-periodic, $\frac{T}{2}$ antiperiodic (i.e. satisfy a property of type (2.8)), with zero mean and verify the Wirtinger inequality.

Moreover they don't necessarily satisfy (2.6), but in some particular cases, if b, A verify a suitable condition of symmetry, the solutions found in H^1_{odd} could concide with those found in H^1_0.

Arguing as in the proof of Theorem 2.1, we can apply Proposition 3.1 and find out that there exist infinitely many pairs of distinct T-periodic solutions in H^1_{odd}. In order to end the proof we need to show that the critical points of the functional f on H^1_{odd} are critical on H^1_T too. Indeed, let us consider the decomposition of H^1_T given by

$$H^1_T \;=\; H^1_{\text{odd}} \oplus H^1_{\text{even}}$$

where H^1_{even} is the subspace of H^1_T of functions which have only even terms in their Fourier expantion. Since (V.5) implies that $V'(x) = -V'(-x)$ it is easy to check that, if $x \in H^1_{\text{odd}}$,

$$\langle f'(x), y\rangle = 0 \qquad \forall y \in H^1_{\text{even}}. \qquad \square$$

In order to prove Theorem 2.12 we will use a standard minimizing argument, i.e. the following

PROPOSITION 3.8. *Let X be a reflexive space and $K \subset X$ a closed subset. Let F be a continuous functional on X, bounded from below on K and satisfying the Palais-Smale condition on K. Then F admits a minimum value on K.*

PROOF OF THEOREM 2.12. Putting

$$M_k = \Big\{ x \in H^1_0\Big(0, \frac{kT}{2}; \mathbb{R}^n\Big) \setminus \{0\} \colon \|x\|^2_A + \int_0^{\frac{kT}{2}} b(t) V'(x)\cdot x = 0 \Big\}$$

we can verify that $F = f_{M_k}$ is bounded from below and satisfies the Palais-Smale condition on M_k. Moreover, since the H^1-convergence implies the uniform convergence we can take the limit under the integral sign and obtain that f_{M_k} is continuous. So we can apply Proposition 3.8 and find out that f_{M_k} has a minimum u_k.

Now one has to show that the solution corresponding to u_k has minimal period kT. Suppose, by contradiction, that for some $h \in \mathbb{N}$, $h \geq 2$, $\frac{kT}{h}$ is the minimal period of x_k. Then there would exists $t_0 \in [0, \frac{kT}{2}]$ such that $x_k(t_0) = 0$.

If we consider the function

$$\overline{x}_k(t) = \begin{cases} x_k(t) & t \in [0, t_0] \\ 0 & t \in \left[t_0, \dfrac{kT}{2}\right], \end{cases}$$

arguing as in [7],it is easy to verify that $\overline{x}_k$ belongs to M_k, and

$$f_M(x_k) > f_M(\overline{x}_k),$$

which contradicts the minimality property of x_k. □

Proof of Theorem 2.13. First of all observe that condition (b.1) implies that there exists an interval $[t_1, t_2] \subset [0, \frac{T}{2}]$ such that $b(t) > 0$, for any $t \in [t_1, t_2]$. Let us consider the function

$$\tilde{\phi}(t) = \begin{cases} \eta \sin\left[\dfrac{2\pi}{(t_2 - t_1)}(t - t_1)\right] & \text{if } t \in [t_1, t_2] \\ 0 & \text{if } t \in [0, T] \setminus [t_1, t_2], \end{cases}$$

where $\eta \in \mathbb{R}^N$, and $|\eta| = 1$.

By construction $\tilde{\phi}$ belongs to $H_0^1(0, \frac{T}{2}; \mathbb{R}^N)$. Arguing as in the proof of the nonemptyness of M, we claim that there exists a constant $r > 0$ such that $\phi = r\frac{\tilde{\phi}}{\|\tilde{\phi}\|}$ belongs to M.

If we consider the element $\psi \in H_0^1(0, kT; \mathbb{R}^N)$ given by:

$$\psi(t) = \begin{cases} \phi(t) & \text{if } t \in [0, T] \\ 0 & \text{if } t \in [T, kT], \end{cases}$$

then, by Theorem 2.12 and (3.6), we can construct a sequence $\{x_k\}$ of subharmonics of (P) (corresponding to the minima of f_M) such that

$$0 < \alpha\|x_k\|_A^2 \le f(x_k) \le \frac{1}{2}\|\psi\|_A^2 - \int_{t_1}^{t_2} b(t)V(\psi) = L,$$

where L is a constant independent of k.

Therefore, using the same arguments of [3], and [6], we find the estimates from below and from above on $\{x_k\}$ independent of k. Moreover we can construct a sequence $\{\tilde{x}_k\} \subset H_k^1$ which verifies $\tilde{x}_k(t) = x_k(t + r_k T)$, where the sequence $r_k \subset \mathbb{N}$ is such that

$$-\infty < \max_{t\in[0,T]} |x_k(t + r_k T)| = \max_{t\in\mathbb{R}} |x_k(t)| < +\infty.$$

By construction the functions $\tilde{x}_k$ satisfy the same estimates of x_k, so, applying Ascoli's Theorem and arguing again as in [3] and [6], we obtain the homoclinic solution of (P) as a limit of a subsequence of $\{x_k\}$ in C^2_{loc}. □

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