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On convergence of first derivatives of certain Szasz-Mirakyan type operators

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RIASSUNTO: *Si espongono quattro teoremi di convergenza delle derivate prime di certi operatori di tipo Szasz-Mirakyan per funzioni di una o due variabili. Alcune proprietà di approssimazione per questi operatori sono state date in lavori precedenti citati in bibliografia.*

ABSTRACT: *In this paper we present four theorems on the convergence of first derivatives of certain Szasz-Mirakyan type operators for functions of one and two variables. Some approximation properties of these operators are given in some previous papers quoted in bibliography.*

The approximation of function by linear positive operators is an important problem in many mathematical theories.

In the papers [1] and [2] were examined the Szasz-Mirakyan operators for functions of one variable with polynomial and exponential weighted spaces.

Recently, we can observe many published papers devoted various modified Szasz-Mirakyan operators of functions of one and several variables (e.g. [4], [8]). The authors study the degree of approximation, the

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Voronovskaya type theorem and the convergence of derivatives sequences of these operators.

In [3] and [5]-[7] we have introduced certain Szasz-Mirakyan type operators A_n, B_n, $A_{m,n}$ and $B_{m,n}$ for functions of one and two variables with polynomial and exponential weighted spaces. In these papers we proved theorems on the degree of approximation, the Vronovskaya type theorems and other approximation properties of considered operators.

In the present paper we study the convergence of first derivatives of these operators for functions with polynomial and exponential weighted spaces. In Section 1 we consider operators of function of one variable. Section 2 constains certain results for functions of two variables.

1 – The operators of functions of one variable

1.1 – Let us consider as in [1], for a fixed $p \in N_0 := \{0, 1, \dots\}$ and for all $x \in R_0 := [0, +\infty)$ the function

$$w_0(x) := 1, \quad w_p(x) := (1 + x^p)^{-1} \quad \text{if } p \geq 1\,,$$

and the polynomial weighted space $C_{1;p}$ of all real-valued functions f continuous on R_0 for which fw_p is uniformly continuous and bounded on R_0. The norm in $C_{1;p}$ is given by $||\,f\,||_{1;p} := \sup_{x\in R_0}\, w_p(x)\,|\,f(x)\,|$.

Moreover let $C^m_{1;p} := \{f \in C_{1;p} : f^{(k)} \in C_{1;p},\ k = 1, \dots, m\}$, for fixed $p \in N_0$ and $m \in N := \{1, 2, \dots\}$.

Analogously, by the function $\nu_q(x) := e^{-qx}$, $x \in R_0$, with a fixed $q \in R_+ := (0, +\infty)$, we define the exponential weighted space $C_{2;q}$ with the norm $||f||_{2;q} = \sup_{x\in R_0}\ \nu_q(x)\,|\,f(x)\,|$ and the class $C^m_{2;q}$, $m \in N$ (see [2]).

1.2 – In the papers [3], [5] and [6] we have introduced in $C_{1;p}$ and $C_{2;q}$ the following operators of the Szasz-Mirakyan type

$$(1)\ A_n(f(t); x) := (A_n(f))(x) := \sum_{k=0}^{\infty} a_{n,k}(x) f\left(\frac{2k}{n}\right)\,,$$

$$(2)\ B_n(f(t); x) := (B_n(f))(x) := f(0)d_n(x) + \sum_{k=0}^{\infty} b_{n,k}(x) f\left(\frac{2k+1}{n}\right)\,,$$

$x \in R_0$, $n \in N$, where

$$a_{n,k}(x) := \frac{1}{\cosh nx} \frac{(nx)^{2k}}{(2k)!}, \tag{3}$$

$$b_{n,k}(x) := \frac{1}{1+\sinh nx} \frac{(nx)^{2k+1}}{(2k+1)!}, \tag{4}$$

$$d_n(x) := \frac{1}{1+\sinh nx}, \tag{5}$$

for all $k \in N_0$, $n \in N$ and $x \in R_0$, and $\sinh x$, $\cosh x$ and $\tanh x$ are elementary hyperbolic functions. In these papers was proved that if $f \in C_{1;p}$ or $f \in C_{2;q}$ ($p \in N_0$, $q \in R_+$), then for every $x \in R_0$

$$\lim_{n\to\infty} A_n(f(t);x) = f(x) = \lim_{n\to\infty} B_n(f(t);x). \tag{6}$$

In the present paper we shall give the analogue of (6) for the first derivatives of these operators. We shall write $L_n \in \{A_n, B_n\}$ if the operator $L_n = A_n$ for all $n \in N$ or $L_n = B_n$ for all $n \in N$.

1.3 – In the papers [3] and [6] was proved that A_n and B_n, $n \in N$, are linear positive operators from the space into $C_{1;p}$ into $C_{1;p}$, $p \in N_0$. Moreover in [5] was proved that A_n and B_n are operators from the space $C_{2;q}$ into $C_{2;r}$ with $r > q > 0$, provided that $n \ge n_0 > q(\ln \frac{r}{q})^{-1}$.

Let $L_n \in \{A_n, B_n\}$. In this paper we shall apply the following properties of the operator L_n proved in [3], [5] and [6]: for every $x \in R_0$

$$L_n(1;x) = 1, \qquad n \in N, \tag{7}$$

$$\lim_{n\to\infty} n^s L_n(t-x;x) = 0, \qquad s \ge 0, \tag{8}$$

$$\lim_{n\to\infty} n L_n((t-x)^2;x) = x, \tag{9}$$

$$\lim_{n\to\infty} n^2 L_n((t-x)^3;x) = x, \tag{10}$$

$$\lim_{n\to\infty} n^2 L_n((t-x)^4;x) = 3x^2. \tag{11}$$

Moreover, for every fixed $x \in R_0$ there exists a positive constant $M_1(x)$, depending only on x, such that

$$n^4 L_n((t-x)^8;x) \le M_1(x) \quad \text{for all } n \in N. \tag{12}$$

1.4 – Now we prove two main theorems.

THEOREM 1. *Let $L_n \in \{A_n, B_n\}$ and let $f \in C^1_{1;p}$ or $f \in C^1_{2;q}$ with fixed $p \in N_0$ or $q \in R_+$. Then*

$$\lim_{n\to\infty} (L_n(f))'(x) = f'(x) \quad \text{for every } x > 0 \,. \tag{13}$$

PROOF. a) First let $L_n \equiv A_n$, $n \in N$, and let $f \in C^1_{1;p}$. By (1), (3) and (7) we have $A_n(t-x;x) = x(\tanh nx - 1)$ for $x \in R_0$ and $n \in N$, and further

$$\begin{aligned}(A_n(f(t)))'(x) &= -(n \tanh nx)\, A_n(f(t);x) + \frac{n}{x} A_n(tf(t);x) = \\ &= -\frac{n}{x} A_n(t-x;x) A_n(f(t);x) + \frac{n}{x} A_n((t-x)f(t);x)\,,\end{aligned} \tag{14}$$

for $x > 0$ and $n \in N$. Fix $x > 0$. By the Taylor formula for $f \in C^1_{1;p}$ we have

$$f(t) = f(x) + f'(x)(t-x) + \varphi(t,x)(t-x)\,, \quad t \in R_0\,, \tag{15}$$

where the function $\varphi(t) \equiv \varphi(t,x)$ belongs to $C_{1;p}$ and $\lim_{t\to x} \varphi(t) = 0$. From (14), (15) and (7) we get for $n \in N$

$$\begin{aligned}&(A_n(f(t)))'(x) = \\ &= -\frac{n}{x} A_n(t-x;x)\, \{f(x) + f'(x) A_n(t-x;x) + A_n(\varphi(t)(t-x);x)\} + \\ &\quad + \frac{n}{x}\, \{f(x) A_n(t-x;x) + \\ &\quad + f'(x) A_n((t-x)^2;x) + A_n(\varphi(t)(t-x)^2;x)\}\ .\end{aligned} \tag{16}$$

By the properties of $\varphi(\cdot)$ and by (6) we have

$$\lim_{n\to\infty} A_n((t-x)\varphi(t);x) = 0, \quad \lim_{n\to\infty} A_n(\varphi^2(t);x) = 0\,. \tag{17}$$

Applying the Hölder inequality, we get

$$|A_n(\varphi(t)(t-x)^2;x)| \le \left\{A_n(\varphi^{\,2}(t);x)\right\}^{1/2} \left\{A_n((t-x)^2;x)\right\}^{1/2} \ ,$$

which by (17) implies

$$\lim_{n\to\infty} nA_n(\varphi(t)(t-x)^2;x) = 0\,. \tag{18}$$

Using (8), (9), (17) and (18) to (16), we immediately obtain (13) for A_n.

b) Now let $L_n \equiv B_n$ and let $x > 0$ be a fixed point. From (2), (4), (5) and (7) it follows that $B_n(t-x;x) = x(d_n(x)\cosh nx - 1)$ for $x \in R_0$ and $n \in N$, and further

$$\begin{aligned}(B_n(f(t)))'(x) &= (-n\, d_n(x)\cosh nx)\, B_n(f(t);x) + \frac{n}{x} B_n(t\, f(t);x) = \\ &= -\frac{n}{x}\, B_n(t-x;x)\, B_n(f(t);x) + \frac{n}{x}\, B_n((t-x)\, f(t);x)\,,\end{aligned}$$

for $x \in R_+$ and $n \in N$, which by (15) and (7) and by elementary calculations yields

$$\begin{aligned}(B_n(f(t)))'(x) = f'(x) &\left\{-n(B_n(t-x;x))^2 + \frac{n}{x}\, B_n((t-x)^2;x)\right\} + \\ &+ \frac{n}{x} B_n(\varphi(t)(t-x)^2;x) - \\ &- \frac{n}{x} B_n(t-x;x) B_n((t-x)\varphi(t);x)\,.\end{aligned} \tag{19}$$

Arguing as in the case of A_n and applying (6)-(9), we derive (13) for B_n from (19).

The proof of (13) for $f \in C^1_{2;q}$ is identical. □

THEOREM 2. *Let $L_n \in \{A_n, B_n\}$ and let $f \in C^3_{1;p}$ or $f \in C^3_{2;q}$ with a fixed $p \in N_0$ or $q \in R_+$. Then for every $x > 0$*

$$\lim_{n\to\infty} n\,\{(L_n(f))'(x) - f'(x)\} = \frac{1}{2}\, f''(x) + \frac{x}{2} f'''(x)\,. \tag{20}$$

PROOF. Let $x > 0$ be a fixed point, $L_n \equiv A_n$ and let $f \in C^3_{1;p}$. By the Taylor formula we have

$$f(t) = \sum_{k=0}^{3} \frac{f^{(k)}(x)}{k!} + \psi(t,x)(t-x)^3, \quad t \in R_0, \tag{21}$$

where $\psi(t) \equiv \psi(t,x)$ is the function with the space $C_{1;p}$ and $\lim_{t\to x}\psi(t) = 0$. From (14), (21) and (7), we get for $n \in N$

$$\begin{aligned}
&(A_n(f(t)))'(x) = \\
&= -\frac{n}{x}A_n(t-x;x)\left\{\sum_{k=0}^{3}\frac{f^{(k)}(x)}{k!}A_n((t-x)^k;x)+A_n(\psi(t)(t-x)^3;x)\right\}+ \\
&\quad + \frac{n}{x}\left\{\sum_{k=0}^{3}\frac{f^{(k)}(x)}{k!}A_n((t-x)^{k+1};x)+A_n(\psi(t)(t-x)^4;x)\right\}.
\end{aligned}$$

Consequently,

$$\begin{aligned}
&n\{(A_n(f)'(x) - f'(x)\} = \\
&= f'(x)\,\frac{n^2}{x}\left\{-(A_n(t-x;x))^2 + +A_n((t-x)^2;x) - \frac{x}{n}\right\}+ \\
&\quad + f''(x)\frac{n^2}{2x}\{-A_n(t-x;x)\,A_n((t-x)^2;x) + A_n((t-x)^3;x)\}+ \\
&\quad + f'''(x)\frac{n^2}{6x}\{-A_n(t-x;x)A_n((t-x)^3;x) + A_n((t-x)^4;x)\}+ \\
&\quad + \frac{n^2}{x}\{-A_n(t-x;x)A_n(\psi(t)(t-x)^3;x)+A_n(\psi(t)(t-x)^4;x)\}.
\end{aligned} \tag{22}$$

By the properties of function ψ and by (6) we have

$$\lim_{n\to\infty} A_n(\psi(t)\,(t-x)^3;x) = 0, \tag{23}$$

$$\lim_{n\to\infty} A_n(\psi^2(t);x) = 0. \tag{24}$$

The inequality

$$|A_n(\psi(t)(t-x)^4;x)| \le \{A_n(\psi^2(t);x)\}^{1/2}\{A_n((t-x)^8;x)\}^{1/2}, \quad n \in N,$$

and (24) and (12) imply

$$\lim_{n\to\infty} n^2 A_n(\psi(t)(t-x)^4;x) = 0. \tag{25}$$

Applying the formula $A_n((t-x)^2;x) = \frac{x}{n} + (\frac{1}{n} - 2x)A_n(t-x;x)$ and (8)-(11), (23) and (25), we obtain from (22)

$$\lim_{n\to\infty} n \{(A_n(f))'(x) - f'(x)\} = \frac{1}{2} f''(x) + \frac{x}{2} f'''(x) .$$

The proof of (20) for B_n is analogous. ∎

2 – The operators of functions of two variables

2.1 – Using notation of Section 1, we define for fixed $p_1, p_2 \in N_0$ the function $w_{p_1,p_2}(x,y) := w_{p_1}(x)w_{p_2}(y)$, $(x,y) \in R_0^2 := R_0 \times R_0$, and the polynomial weighted space $C_{1;p_1,p_2}$ of all real-valued functions f continuous and bounded on R_0^2. The norm in $C_{1;p_1,p_2}$ is defined by $||f||_{1;p_1,p_2} := \sup_{(x,y)\in R_0^2} w_{p_1,p_2}(x,y)\,|\,f(x,y)\,|$. Similarly as in Section 1, for a fixed $m \in N$, let $C^m_{1;p_1,p_2}$ be the class of all $f \in C_{1;p_1,p_2}$ having partial derivatives of the order $\leq m$ and them belong to $C_{1;p_1,p_2}$ also.

Analogously we define the exponential weighted space $C_{2;q_1,q_2}$, $q_1, q_2 \in R_+$, of functions of two variables, with the norm $||f||_{2;q_1,q_2} := \sup_{(x,y)\in R_0^2} \nu_{q_1,q_2}(x,y)\,|\,f(x,y)|$ and the weighted function $\nu_{q_1,q_2}(x,y) := \nu_{q_1}(x)\nu_{q_2}(y)$, and the class $C^m_{2;q_1,q_2}$, $m \in N$.

2.2 – In [3], [5] and [7] were examined some approximation properties of the operators

$$\begin{aligned} A_{m,n}(f(t,z);x,y) :&= (A_{m,n}(f))(x,y) := \\ &= \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} a_{m,j}(x)a_{n,k}(y) f\left(\frac{2j}{m}, \frac{2k}{n}\right) , \end{aligned} \tag{26}$$

$$\begin{aligned} B_{m,n}(f(t,z);x,y) :&= (B_{m,n}(f))(x,y) := d_m(x)d_n(y)f(0,0) + \\ &+ d_m(x)\sum_{k=0}^{\infty} b_{n,k}(y) f\left(0, \frac{2k+1}{n}\right) + \\ &+ d_n(y)\sum_{j=0}^{\infty} a_{m,j}(x) f\left(\frac{2j+1}{m}, 0\right) + \\ &+ \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} b_{m,j}(x)b_{n,k}(y) f\left(\frac{2j+1}{m}, \frac{2k+1}{n}\right) , \end{aligned} \tag{27}$$

for $(x,y)\in R_0^2$, $m,n\in N$ and $f\in C_{1;p_1p_2}$ or $f\in C_{2;q_1,q_2}$, where $a_{n,k}(\cdot)$, $b_{n,k}(\cdot)$ and $d_n(\cdot)$ is defined by (3)-(5). Similarly as in Section 1 we write $L_{m,n}\in\{A_{m,n},B_{m,n}\}$ if $L_{m,n}=A_{m,n}$ for all $m,n\in N$ or $L_{m,n}=B_{m,n}$ for all $m,n\in N$. From (26), (27) and (3)-(5) it follows that

$$L_{m,n}(1;x,y)=1 \quad \text{for} \quad (x,y)\in R_0^2\,, \quad m,n\in N\,, \tag{28}$$

and if $f\in C_{1;p_1,p_2}$ or $f\in C_{2;q_1,q_2}$ and $f(x,y)=f_1(x)f_2(y)$ for all $(x,y)\in R_0^2$ then

$$\begin{aligned} L_{m,n}(f(t,z);x,y) &= L_m(f_1(t);x)L_n(f_2(z);y),\\ &\qquad (x,y)\in R_0^2, m,n\in N\,. \end{aligned} \tag{29}$$

In [3] and [7] was proved that $L_{m,n}$ is a linear positive operator from the space $C_{1;p_1p_2}$ into $C_{1;p_1,p_2}$, $p_1,p_2\in N_0$.

In [5] and [7] was proved that $L_{m,n}$ is an operator from the space $C_{2;q_1,q_2}$ into $C_{2;r_1,r_2}$ with $r_1>q_1>0$, $r_2>q_2>0$, and $m\ge m_0>q_1(\ln\frac{r_1}{q_1})^{-1}$ and $n\ge n_0>q_2(\ln\frac{r_2}{q_2})^{-1}$. Moreover in these papers was proved that if $f\in C_{1;p_1,p_2}$ or $f\in C_{2;q_1,q_2}$ then for every $(x,y)\in R_0^2$

$$\lim_{m,n\to\infty} L_{m,n}(f(t,z);x,y)=f(x,y)\,. \tag{30}$$

2.3 – From (26) and (27) we derive the formulas for first partial derivatives of $L_{m,n}\in\{A_{m,n},B_{m,n}\}$

$$\begin{aligned} (L_{m,n}(f))'_x(x,y) &= -\frac{m}{x}L_m(t-x;x)L_{m,n}(f(t,z);x,y)+\\ &\quad+\frac{m}{x}L_{m,n}((t-x)f(t,z);x,y)\,, \end{aligned} \tag{31}$$

$$\begin{aligned} (L_{m,n}(f))'_y(x,y) &= -\frac{n}{y}L_n(z-y;y)L_{m,n}(f(t,z);x,y)+\\ &\quad+\frac{n}{y}L_{m,n}((z-y)f(t,z);x,y)\,, \end{aligned} \tag{32}$$

for all $(x,y)\in R_+^2:=R_+\times R_+$ and $m,n\in N$.

Now we shall prove the analogue of Theorem 1.

Theorem 3. *Let* $f \in C^1_{1;p_1,p_2}$ *or* $f \in C^1_{2;q_1,q_2}$ $(p_1, p_2 \in N_0,\ q_1, q_2 \in R_+)$ *and let* $L_{m,n} \in \{A_{m,n}, B_{m,n}\}$*. Then for every* $(x, y) \in R^2_+$

$$\lim_{n\to\infty} (L_{n,n}(f))'_x(x,y) = f'_x(x,y)\,, \tag{33}$$

$$\lim_{n\to\infty} (L_{n,n}(f))'_y(x,y) = f'_y(x,y)\,. \tag{34}$$

Proof. The formulas (31) and (32) show that the proofs of (33) and (34) are identical. We shall prove only (33) for $f \in C^1_{1;p_1,p_2}$ and a fixed $(x, y) \in R^2_+$. By the Taylor formula we have

$$\begin{aligned} f(t,z) = f(x,y) + f'_x(x,y)(t-x) + f'_y(x,y)(z-y) + \\ + \varphi(t,z;x,y)\sqrt{(t-x)^2+(z-y)^2} \end{aligned} \tag{35}$$

for $(t, z) \in R^2_0$, where $\varphi(t, z) \equiv \varphi(t, z; x, y)$ is function belonging to $C_{1;p_1,p_2}$ and $\varphi(x, y) = 0$. From (31) and (35) and by (28), (29) and (7) we get

$$\begin{aligned} (L_{n,n}(f(t,z)))'_x(x,y) = -\frac{n}{x} L_n(t-x;x)\Big\{ f'_x(x,y) L_n(t-x;x) + \\ + f'_y(x,y) L_n(z-y;y) + L_{n,n}(\varphi(t,z)\sqrt{(t-x)^2+(z-y)^2};x,y)\Big\} + \\ + \frac{n}{x}\Big\{ f'_x(x,y)\, L_n((t-x)^2;x) + f'_y(x,y)\, L_n(t-x;x)\, L_n(z-y;y) + \\ + L_{n,n}(\varphi(t,z)(t-x)\sqrt{(t-x)^2+(z-y)^2};x,y)\Big\}\,, \quad n \in N\,. \end{aligned} \tag{36}$$

The properties of φ and (30) imply

$$\lim_{n\to\infty} L_{n,n}(\varphi(t,z)\sqrt{(t-x)^2+(z-y)^2};x,y) = 0\,, \tag{37}$$

$$\lim_{n\to\infty} L_{n,n}(\varphi^2(t,z);x,y) = 0\,. \tag{38}$$

By the Hölder inequality and by (29) and (7) we have for $n \in N$

$$\left|L_{n,n}(\varphi(t,z)(t-x)\sqrt{(t-x)^2+(z-y)^2};x,y)\right| \leq$$

$$\leq \left\{L_{n,n}(\varphi^2(t,z);x,y)\right\}^{1/2} \times$$

$$\times \left\{L_n((t-x)^4;x) + L_n((t-x)^2;x)\, L_n((z-y)^2;y)\right\}^{1/2},$$

which by (9), (11) and (38) implies

$$\lim_{n\to\infty} nL_{n,n}(\varphi(t,z)(t-x)\sqrt{(t-x)^2+(z-y)^2};x,y) = 0\,. \tag{39}$$

Using (8), (9), (37) and (39) to (36), we immediately obtain (33). Thus the proof is completed. □

Arguing similarly as in the proofs of Theorem 2 and Theorem 3, we can prove the following

Theorem 4. *Suppose that $f \in C^3_{1;p_1,p_2}$ or $f \in C^3_{2;q_1,q_2}$ ($p_1, p_2 \in N_0$, $q_1, q_2 \in R_+$) and $L_{m,n} \in \{A_{n,n}, B_{n,n}\}$. Then for every $(x,y) \in R^2_+$*

$$\lim_{n\to\infty} n\{(L_{n,n}(f))'_x(x,y) - f'_x(x,y)\} = \frac{1}{2}f''_{x^2}(x,y) + \frac{x}{2}f'''_{x^3}(x,y) + \frac{y}{2}f'''_{xy^2}(x,y),$$

$$\lim_{n\to\infty} n\{(L_{n,n}(f))'_y(x,y) - f'_x(x,y)\} = \frac{1}{2}f''_{y^2}(x,y) + \frac{y}{2}f'''_{y^3}(x,y) + \frac{x}{2}f'''_{x^2y}(x,y).$$

REFERENCES

[1] M. Becker: *Global approximation theorems for Szasz-Mirakjan and Baskakov operators in polynomial weight spaces*, Indiana Univ. Math. J., **27** (1) (1978), 127-142.

[2] M. Becker – D. Kucharski – R. J. Nessel: *Global approximation theorems for the Szasz-Mirakjan operators in exponential weight spaces*, In: Linear Spaces and Approximation (Proc. Conf. Oberwolfach, 1977) Birkhäuser Verlag, Basel ISNM, Vol. 40 (1978), 319-333.

[3] B. Firlej – L. Rempulska: *Approximation of functions by some operators of the Szasz-Mirakjan type*, Fasc. Math., **27** (1997), 15-27.

[4] H. S. Kasana: *On approximation of unbounded functions by linear combinations of modified Szasz-Mirakyan operators*, Acta Math. Acad. Sci. Hungary, **61** (3-4) (1993), 283-290.

[5] M. Leśniewicz – L. Rempulska: *Approximation by some operators of the Szasz-Mirakjan type in exponential weight spaces*, Glasnik Mat., **32** (52) (1997), 57-69.

[6] L. Rempulska – M. Skorupka: *On approximation of functions by some operators of the Szasz-Mirakjan type*, Fasc. Math., **26** (1996), 125-137.

[7] M. Skorupka: *Approximation of functions of two variables by some linear positive operators*, Le Matematiche, **50** (2) (1995), 323-336.

[8] Sun Xiehuna: *On the convergence of the modified Szasz-Mirakjan operators*, Approx. Theory and its Appl., **10** (1) (1994), 20-25.

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