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Automorphisms and derivations in prime rings

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Riassunto: *Siano R un anello non commutativo, I un ideale bilatero non nullo di R e f una mappa in R tale che $f([x, y]) - [x, y]$ sia zero o invertibile, per ogni scelta di $x, y \in I$. Se R é un anello primo e f un automorfismo non banale o una derivazione non nulla di R allora $R = D$ oppure $R = M_2(D)$, anello di matrici 2×2 su D, dove D é un corpo. Esamineremo inoltre il caso in cui R sia un anello semiprimo e f una derivazione non nulla di R.*

Abstract: *Let R be a non-commutative ring, I a non-zero two-sided ideal of R and f a mapping on R such that $f([x, y]) - [x, y]$ is zero or invertible for every $x, y \in I$. If R is a prime ring and f a non-trivial automorphism or a non-zero derivation on R then either $R = D$ or $R = M_2(D)$, the ring of all 2×2 matrices over D, where D is a division ring. Moreover we will examine the case when R is a semiprime ring and f a non-zero derivation of R.*

Many authors have studied the relationship between the structure of a ring R and the behaviour of a special mapping on R (see for example [2], [12]). In particular in [2] Bergen and Herstein proved that if a ring R with unit has an automorphism $f \neq 1$ such that $f(x) - x$ is zero or invertible, for any $x \in R$, then either $R = D$ or $R = M_2(D)$, the ring of all 2×2 matrices over D, or $R = D \oplus D$, where D is a division ring.

In this paper we will consider the structure of a prime ring R having

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a mapping f such that $f([x,y]) - [x,y]$ is zero or invertible, for any x, y in a non-zero two-sided ideal of R.

More precisely we will prove the following:

THEOREM 1. *Let R be a non-commutative prime ring, I a non-zero two-sided ideal of R, f a non-trivial automorphism or a non-zero derivation of R such that $f([x,y]) - [x,y]$ is zero or invertible, for any $x, y \in I$. Then either R is a division ring, or R is the ring of all 2×2 matrices over a division ring.*

By continuing this line of investigation we will also study the case when R is a semiprime ring and f a non-zero derivation of R, and we will prove the following:

THEOREM 2. *Let R be a non-commutative semiprime ring, f a non-zero derivation of R such that $f([x,y]) - [x,y]$ is zero or invertible, for any $x, y \in R$. Then either R is a division ring, or R is the ring of all 2×2 matrices over a division ring.*

1 – Automorphisms in prime rings

In this section we will prove Theorem 1 in the case when R is a prime ring, I a non-zero two-sided ideal of R and f a non-trivial automorphism on R such that $f([x,y]) - [x,y]$ is zero or invertible, for any $x, y \in I$.

LEMMA 1.1. *Let R be a prime ring and I a non-zero ideal of R. If $f(x) = x$, for any $x \in I$, then $f(r) = r$, for any $r \in R$.*

PROOF. Let $x \in I, r \in R$.

Because $f(xr) = xr$, then $xr = xf(r)$. Therefore $x(r - f(r)) = 0$ and, since R is prime, $r - f(r) = 0$. □

LEMMA 1.2. *Let R be a prime ring and I a non-zero ideal of R such that $f([x,y]) = [x,y]$, for any $x, y \in I$, then $f(r) = r$, for any $r \in R$.*

Proof. Let $x, y \in I$.

$$f([xy, x]) = f(x[y, x]) = f(x)f([y, x]) = f(x)[y, x]\,.$$

Moreover, because $f([xy, x]) = [xy, x] = x[y, x]$, we have

$$(f(x) - x)[y, x] = 0\,.$$

Let $r \in I$.

$$\begin{aligned} 0 = (f(x) - x)[ry, x] &= (f(x) - x)(r[y, x] + [r, x]y) = \\ &= (f(x) - x)r[y, x] \end{aligned}$$

that is $(f(x) - x)I[y, x] = 0$, for any $x, y \in I$.

If $x \in I - Z(R)$ then there exists $y \in I$ such that $[y, x] \neq 0$ and, by primeness of R, $f(x) = x$.

Now let x a non-zero element, with $x \in Z(R) \cap I$ and let $r \in I - Z(R)$. Hence $xr \in I - Z(R)$. So $f(xr) = xr$. On the other hand $f(xr) = f(x)f(r) = f(x)r$.

Therefore $(x - f(x))r = 0$, for any $r \in I$, and the element $x - f(x)$ is zero since it is central and annihilates a non-zero element $r \in I - Z(R)$. However $f(x) = x$, for any $x \in I$ and by previous lemma we are done. □

Remark 1.1. Since if R is commutative then $0 = f([x, y]) = [x, y]$, for all $x, y \in R$, in all that follows we assume R a non-commutative ring.

Lemma 1.3. *If $f \neq 1$ then R is a simple ring.*

Proof. Suppose R is not simple. For any ideal $M \neq 0$ of R, $I \cap M = J \neq 0$. For every $x, y \in J$, $f([x, y]) - [x, y]$ is zero or invertible and

$$f([x, y]) - [x, y] \in f(J) + J$$

which is a two-sided ideal of R.

If $f([x, y]) - [x, y] = 0$, for all $x, y \in J$, then, by Lemma 1.2, $f = 1$ in R, a contradiction.

Hence there exist $r, s \in J$ such that $f([r, s]) - [r, s]$ is invertible, so $f(J) + J = R$.

If $J \cap f(J) = 0$ then

$$Jf(J) \subseteq J \cap f(J) = 0\,.$$

Therefore $J = 0$, because R is prime, that is $M = 0$.

Hence $J \cap f(J) \neq 0$.

Let $x, y \in J \cap f(J)$.

There exist $r, s \in J$ such that

$$x = f(r), y = f(s)\,.$$
$$[J \cap f(J), J \cap f(J)] \subseteq [J, J]$$
$$f([x, y]) \in f(J)$$
$$[x, y] = f([r, s]) \in f(J)$$
$$f([x, y]) - [x, y] \in f(J)\,.$$

If $f([x, y]) - [x, y] = 0$, for any $x, y \in J \cap f(J)$, then, again by Lemma 1.2, we obtain the contradiction $f = 1$ in R.

In the other case $R = f(J)$, that is $J = R$, and so $I = M = R$, that is R is a simple ring. ∎

Remark 1.2. In all that follows R will be a simple ring with 1. So it is a dense ring of linear transformations on a vector space V over a division ring D.

We will show that R is a simple artinian ring, that is R contains a non-zero transformation of finite rank (see [7, page 75], [6, Lemma 1.2.2]).

Lemma 1.4. *Suppose R is not artinian. If $vr = wr = 0$, where $v, w \in V$ are linearly independent and $r \in R$, then $vf(r)$ and $wf(r)$ are linearly D-dependent.*

Proof. Suppose $vf(r)$ and $wf(r)$ linearly independent.

There exists $u \in Vf(r)$, $u = v'f(r)$, such that $u, vf(r), wf(r)$ are linearly independent.

Moreover there exist $s, t \in R$ such that:

$$vf(r)f(s) = 0, \quad wf(r)f(s) = w, \quad uf(s) = 0$$
$$vf(r)f(t) = 0, \quad wf(r)f(t) = v', \quad uf(t) = w\,.$$

Therefore $vf([rs,rt]) = 0$ and $wf([rs,rt]) = v'$.

$$v\,(f([rs,rt]) - [rs,rt]) = 0$$
$$w\,(f([rs,rt]) - [rs,rt]) = v'\,.$$

Hence $(f([rs,rt]) - [rs,rt])$ is not zero nor invertible, a contradiction. □

LEMMA 1.5. *Suppose R is not artinian. If $v, w \in V$ are linearly independent and $vr = wr = 0$, for some $r \in R$, then $vf(r) = wf(r) = 0$.*

PROOF. Suppose $vf(r) \neq 0$.

By previous lemma $vf(r), wf(r)$ are linearly dependent, that is there exist $a, b \in D$ such that $(a,b) \neq (0,0)$ and

$$avf(r) + bwf(r) = 0 \text{ that is } (av + bw)f(r) = 0\,.$$

Moreover there exists $u = v'f(r) \in Vr$ such that $u, vf(r)$ are linearly independent.

Hence there exist $s, t \in R$ such that

$$vf(r)f(s) = v', \quad uf(s) = 0$$
$$vf(r)f(t) = 0, \quad uf(t) = v\,.$$

Since $(av + bw)\,(f([rs,rt]) - [rs,rt]) = 0$ then $f([rs,rt]) - [rs,rt]$ is not invertible, so it is zero.

Moreover $0 = v\,(f([rs,rt]) - [rs,rt]) = v$, a contradiction. □

LEMMA 1.6. *Suppose R is not artinian. If $vr = 0$, where $v \in V$, $r \in R$, then $vf(r) = 0$.*

PROOF. Suppose by contradiction $vf(r) \neq 0$.

Since $\dim_D(Vr) = \infty$, then there exist u, $w \in V$ such that ur, wr, $vf(r)$ are linearly independent.

Moreover there exists $s \in R$ such that $(ur)s = 0$, $(wr)s = 0$, $vf(r)s = v$. Since $v(rs) = w(rs) = 0$, by previous lemma $vf(rs) = vf(r)f(s) = 0$. Analogously we also have $urf(s) = wrf(s) = 0$. Let $g = f^{-1}$, the inverse automorphism of f. It satisfies the same properties of f.

Again by previous lemma, since $vf(r)f(s) = urf(s) = 0$ and $vf(r)$, ur are linearly independent, we get

$$0 = vf(r)g\left(f(s)\right) = vf(r)s$$

but $vf(r)s = v \neq 0$, a contradiction. ∎

Lemma 1.7. *Suppose R is not artinian. For any $v \in V, r \in R$, $vf(r) = \lambda vr$, where $\lambda \in D$ is independent on the choise of v.*

Proof. Suppose $vf(r)$ and vr linearly independent.

There exists $s \in R$ such that $vrs = 0$, $vf(r)s = v$. By Lemma 1.6

$$0 = vf(rs) = vf(r)f(s)\,.$$

Since the previous lemma holds obviously for $g = f^{-1}$ too, then $0 = vf(r)g(f(s)) = vf(r)s = v$, a contradiction.

Hence $vr, vf(r)$ are linearly dependent.

Let $w \in V$ such that wr, vr are linearly independent.

$$vf(r) = \lambda_v vr, \quad wf(r) = \lambda_w wr$$

and so $(v + w)f(r) = (\lambda_v + \lambda_w)r$. Moreover $(v + w)f(r) = \lambda_{v+w}(v + w)r$ then $\lambda_v = \lambda_w = \lambda_{v+w}$, $\forall v, w \in V$. ∎

Lemma 1.8. *If R is a simple ring and $f \neq 1$, then R is artinian.*

Proof. Fix $v \in V$, $r \in R$.

Suppose that R is not an artinian ring. By Lemma 1.7

$$vf(r) = \lambda_r vr$$
$$vf(r)r = \lambda_r vr^2\,.$$

Moreover, again by Lemma 1.7,

$$(vr)f(r) = \lambda_r(vr)r = \lambda_r vr^2\,.$$

Therefore, for any $v \in V$, $r \in R$, $v[f(r), r] = 0$, that is $V[f(r), r] = 0$ and so $[f(r), r] = 0$. Because R is not commutative, and by main result in [12], one has $f = 1$, a contradiction. ∎

Now we complete the proof of theorem 1 in the case f is a non-trivial automorphism of R:

Proof of Theorem 1. By previous lemmas R is simple artinian with 1, then $R = M_n(D)$, the ring of all $n \times n$ matrices over a division ring D.

Suppose $n \geq 3$.

Let e_{ij}, the matrix unit with 1 in the (i,j)-entry and zero elsewhere. Since e_{ij} has rank 1, $f(e_{ij}) - e_{ij}$ and $f(ae_{ij}) - ae_{ij}$ have rank ≤ 2 one has $f(e_{ij}) = e_{ij}$, $f(ae_{ij}) = ae_{ij}$, for all i, j and $a \in D$. This one implies $ae_{ij} = f(ae_{ij}) = f(a)f(e_{ij}) = f(a)e_{ij}$, that is $f(a) = a$, for any $a \in D$. Therefore we conclude that if $n \geq 3$ then $f = 1$ in R, which is a contradiction. $\square$

2 – Derivations in prime rings

In this section we will prove Theorem 1 in the case f is a derivation. In all that follows R will be a prime ring, I a non-zero two-sided ideal of R and f a non-zero derivation of R such that $f([x, y]) - [x, y]$ is zero or invertible, for any $x, y \in I$.

We begin with the following:

Lemma 2.1. *Let f a non-zero derivation of R, $I \neq 0$ a two-sided ideal of R, such that $f([x, y]) = [x, y]$ for any $x, y \in I$. Then R is commutative.*

Proof. By using Kharchenko's theorem in [8], either $f = \mathrm{ad}(A)$ is the inner derivation induced by an element $A \in Q$, the Martindale quotient ring of R, or I satisfies $[t, y] + [x, w] - [x, y] = 0$. In this last case, putting $t = x$, we conclude that I is commutative and so R too. Now let $f = \mathrm{ad}(A)$, and $x, y, r \in I$. The following hold:

$$\begin{aligned}
[x, y]r + x[r, y] = [xr, y] = f([xr, y]) = \\
&= f([x, y]r + x[r, y]) = f([x, y]r) + f(x[r, y]) = \\
&= f([x, y])r + [x, y]f(r) + f(x)[r, y] + xf([r, y]) = \\
&= [x, y]r + [x, y]f(r) + f(x)[r, y] + x[r, y]\,.
\end{aligned}$$

This implies that $[x,y]f(r) + f(x)[r,y] = 0$. Put $r = y$ and obtain $[x,y]f(y) = [x,y][A,y] = 0$, for any $x, y \in I$. Since by [3] I and Q satisfy the same generalized polynomial identities, it follows that $[x,y][A,y] = 0$, for all $x, y \in Q$. For $x = A$ we have $(f(y))^2 = 0$. Because f is an inner derivation of Q, by Theorem 2 in [5], $A \in Z(Q)$ that is $f = 0$ in Q and so in R, which contradicts our assumption. ∎

Remark 2.1. By previous lemma one can say that R is commutative if and only if $f([x,y]) = [x,y]$, for any $x, y \in I$. In all that follows we assume R is not commutative.

Lemma 2.2. *R is a simple ring.*

Proof. Since we assume R is not commutative, there exist $a, b \in I$ such that $f([a,b]) - [a,b]$ is invertible in R. This implies that $I = R$ and $f([x,y]) - [x,y]$ is zero or invertible, for any $x, y \in R$. Suppose that R is not simple. Thus there exists a suitable non-zero two-sided ideal $M \neq 0$ of R such that $M \neq R$. Since M cannot contain any invertible element of R, $f([x,y]) - [x,y] = 0$, for any $x, y \in M$. In this case, by previous lemma, we have the contradiction that R is commutative. ∎

Lemma 2.3. *Let R be a simple artinian ring. Then either R is a division ring or R is the ring of all 2×2 matrices over a division ring.*

Proof. We know that in this case $R = M_k(D)$, the ring of all $k \times k$ matrices over a division ring D.

Suppose $k \geq 3$.

Let e_{ij} the matrix unit with 1 in (i,j) entry and 0 elsewhere.

The matrix $f(e_{ij}) - e_{ij} = f(e_{ii}e_{ij}) - e_{ij} = f(e_{ii})e_{ij} + e_{ii}f(e_{ij}) - e_{ij}$ has rank ≤ 2. Thus it is not invertible in R and so it is zero. Therefore $f(e_{ij}) = e_{ij}$, for all $i \neq j$. Moreover $f(e_{ii}) = f(e_{ij}e_{ji}) = f(e_{ij})e_{ji} + e_{ij}f(e_{ji}) = 2e_{ii}$.

If char $R = 2$ then $f(e_{ii}) = 0$ for all i. Let $u \in [R,R]$, where $u = [e_{11} + e_{12}, e_{21}] = e_{11} - e_{21} - e_{22}$. By calculation we obtain $f(u) = -e_{21}$ and $f(u) - u = -e_{11} + e_{22}$, which is not zero nor invertible in $R = M_k(D)$, when $k \geq 3$. Because this last one gives a contradiction, then char $R \neq 2$ and the following hold, for any $i \neq j$:

$$e_{ij} = f(e_{ij}) = f([e_{ij}, e_{jj}]) = [f(e_{ij}), e_{jj}] + [e_{ij}, f(e_{jj})] = e_{ij} + 2e_{ij} = 3e_{ij}$$

that is $2e_{ij} = 0$, which is impossible because char $R \neq 2$.

Therefore we conclude that $k \leq 2$ and we are done. ∎

Proof of Theorem 1. By Lemma 3.2 we know that R is a simple ring with 1, otherwise it is commutative. Remark that in this case it easy to see that the Martindale quotient ring Q coincides with R.

We divide the proof in two cases:

Case 1. Suppose there exists a non-zero right ideal ϱ of R such that $f([x,y]) - [x,y] = 0$, for any $x, y \in \varrho$. Let $a \in \varrho, x_1, x_2 \in R$. The following hold:

$$f([ax_1, ax_2]) - [ax_1, ax_2] = 0$$
$$[f(a)x_1 + af(x_1), ax_2] + [ax_1, f(a)x_2 + af(x_2)] - [ax_1, ax_2] = 0\,.$$

Again by Kharchenko's theorem [8], if f is not an inner derivation, then

$$[f(a)x_1 + ay_1, ax_2] + [ax_1, f(a)x_2 + ay_2] - [ax_1, ax_2] = 0$$

for any $x_1, x_2, y_1, y_2 \in R$. Pick $x_1 = 0$. For all $x_2, y_1 \in R$, $[ay_1, ax_2] = 0$, that is R is a GPI ring. Thus, by Martindale's theorem [11] and since R is simple with 1, R is a finite dimensional central simple algebra, that is $R = M_k(D)$, the ring of all $k \times k$ matrices over a division ring D, and we conclude by previous lemma.

Suppose now that f in an inner derivation induced by an element $A \in Q = R$, that is $f(x) = [x, A]$, for any $x \in R$.

Fix $u \in \varrho - \{0\}$. Let $\alpha \in Z(R)$ such that $(A - \alpha)u = 0$. For any $x, y \in R$ we obtain $(A - \alpha)[ux, uy] = 0$ and also

$$[A - \alpha, [ux, uy]] = [ux, uy]$$

that is $-[ux, uy](A - \alpha) = [ux, uy]$ and so $[ux, uy]u = 0$. Again R is a GPI ring and, by above argument, we are done.

Now we can assume that $(A - \alpha)u \neq 0$, for any $\alpha \in Z(R)$. Hence $Au \neq \alpha u$ and so Au, u are linearly independent over $Z(R)$. So using $[A, [ux, uy]] - [ux, uy] = 0$ and right multiplying by u yields

$$(Auxuy - Auyux - uxuyA + uyuxA - uxuy + uyux)u = 0$$
$$Auxuyu - Auyuxu - uxuyAu + uyuxAu - uxuyu + uyuxu = 0$$

which is a non-trivial generalized polynomial identity for R (see [3]). Once again we conclude by previous lemma.

Case 2. Suppose that, for any ϱ right ideal of R, $f([x,y]) - [x,y]$ is not an identity in ϱ.

Of course, for any non-zero right ideal ϱ of R and for any $x, y \in \varrho$, $f([x,y]) - [x,y] \subseteq f(\varrho) + \varrho$.

Because $f([x,y]) - [x,y]$ is not an identity in ϱ, then $f(\varrho)+\varrho$ contains invertible values. This implies $f(\varrho) + \varrho = R$. Let ϱ_1, ϱ_2 be non-zero right ideals of R, such that $\varrho_1 \subseteq \varrho_2$. We know that $f(\varrho_1)+\varrho_1 = R = f(\varrho_2)+\varrho_2$. Let $c \in \varrho_2 - \varrho_1$, so $c = a + f(b)$, where $a, b \in \varrho_1$. Moreover $f(b) \neq 0$ because $c \notin \varrho_1$, and $f(b) \in \varrho_2$.

Since bR is a non-zero right ideal of R, $bR + f(bR) = R$ and also $f(bR) = f(b)R + bf(R) \subseteq \varrho_2$. Therefore $R = bR + f(bR) \subseteq \varrho_2$, that is $\varrho_2 = R$. It follows that either R is a division ring or R is the ring of all 2×2 matrices over a division ring. ∎

The results in Sections 1 and 2 give the complete proof of Theorem 1 and we state it again for sake of clearness and completeness:

Theorem 1. *Let R be a non-commutative ring, I a non-zero two-sided ideal of R, f a mapping on R such that $f([x,y]) - [x,y]$ is zero or invertible, for any $x, y \in I$. If R is a prime ring and f a non-trivial automorphism or a non-zero derivation of R, then either R is a division ring, or R is the ring of all 2×2 matrices over a division ring.*

3 – Derivations in semiprime rings

We conclude this note with the proof of Theorem 2. Now we are in the case R is a semiprime ring and f a non-zero derivation of R.

We will make use of the left Utumi quotient ring of R. So we need to mention that the definition, the axiomatic formulation and the properties of this quotient ring can be found in [1], [4], [10].

We begin with:

Lemma 3.1. *Let R be a semiprime ring, f a non-zero derivation of R such that $f([x,y]) - [x,y] = 0$, for any $x, y \in R$. Then R is commutative.*

Proof. Let C be the extended centroid of R, then $Z(U) = C$. It is known that the derivation f can be uniquely extended in U and moreover R and U satisfy the same differential identities (see [9]). Therefore $f([x,y]) - [x,y] = 0$, for any $x, y \in U$. Let M be any maximal ideal of the complete Boolean algebra of idempotents of C, denoted by B. We know that MU is a prime ideal of U. Let $\overline{f}$ the derivation induced by f in $\overline{U} = U/MU$. Therefore $\overline{f}$ satisfies in $\overline{U}$ the same property of f on U. By Lemma 3.1, for all M maximal ideal of B, either $f(U) \subseteq MU$ or $[U,U] \subseteq MU$. In any case $f(U)[U,U] \subseteq \bigcap_M MU = 0$. Without loss of generality we have $f(R)[R,R] = 0$. In particular $f(R)[R^2,R] = 0$, that is

$$0 = f(R)R[R,R] + f(R)[R,R]R = f(R)R[R,R]\,.$$

Therefore $[R, f(R)]R[R, f(R)] = 0$ and, by semiprimeness of R, $[R, f(R)] = 0$, that is $f(R) \subseteq Z(R)$. Because $f([x,y]) = [x,y]$, for all $x, y \in R$, then $[x,y] \in Z(R)$.

Since R is a semiprime ring for which $[[x,y],z]$ is a polynomial identity, then R is a subdirect product of prime rings R_α, each of which still satisfies the identity $[[x,y],z] = 0$. In this case it is easy to prove that any R_α is commutative. Thus we conclude that R must be commutative. ∎

Now we are ready to prove the following:

Theorem 2. *Let R be a non-commutative semiprime ring, f a non-zero derivation of R such that $f([x,y]) - [x,y]$ is zero or invertible, for all $x, y \in R$. Then either R is a division ring or R is the ring of all 2×2 matrices over a division ring.*

Proof. Since R is not commutative, by Lemma 3.1, we can assume that $f([x,y]) - [x,y]$ is not an identity in R, i.e. there exist $a, b \in R$ such that $f([a,b]) - [a,b]$ is invertible. Suppose there exists a non-zero two-sided ideal of R such that $I \neq R$. Since I cannot contain any invertible element of R, $f([x,y]) - [x,y] = 0$, for all $x, y \in I$. By main result in [9] we have $(f([x,y]) - [x,y])i = 0$, for all $x, y \in R, i \in I$. In particular $(f([a,b]) - [a,b])i = 0$. Because $(f([a,b]) - [a,b])$ is invertible in R then $i = 0$, which gives the contradiction $I = 0$. This says that R must be a simple ring. In this case we may conclude by the arguments in Section 2. ∎

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