Rendiconti di Matematica, Serie VII
Volume 25, Roma (2005), 175-184

Extension problems in Complex Analysis

GIUSEPPE TOMASSINI

Abstract: *We present some new results on the extension problem of analytic objects. In particular we discuss two classes of them, namely sections of coherent sheaves and Levi flat hypersurfaces.*

1 – Introduction

One of the recurring problems in Complex Analysis is that of extending "analytic objects": Hartogs theorem (holomorphic functions "fill compact holes" in $\mathbb{C}^n$, $n \geq 2$) is the prototype of all extension theorems. Analytic objects are meant to be those geometric objects which can be constructed starting from holomorphic functions (e.g. analytic subsets of a complex space, coherent sheaves and their sections, cohomology classes with value in a coherent sheaf, etc. ...).

The theme is classic and there exists a rather vaste literature on the subject (see for instance [Si] for a general account). In addition, in the first part of this paper, we sketch some new results. In the second part we will discuss the extension problem for a different class of geometric objects, namely the Levi flat hypersurfaces of $\mathbb{C}^n$: a real hypersurface of $\mathbb{C}^n$ is said to be *Levi flat* if it is foliated by complex hypersurfaces. Originally introduced as biholomorphic transforms of real hyperplanes (cfr. [Se]), Levi flat hypersurfaces appear in many global problems in Complex Analysis and during the last twenty years they became a very fruitful area of research.

Key Words and Phrases: *Extension of analytic bjects – Levi flatness.*
A.M.S. Classification: 32D15 – 32F32 – 32T15 – 32V15 – 32V25 – 32W50
Conferenza tenuta nel convegno in onore del Prof. B. Segre

2 – Extending analytic objects

Let X be a (reduced) complex space. For every open subset U of X let $\mathfrak{A}(U)$ a class of analytic objects defined on U. We assume that, if U, V are open subsets of X and $V \subset U$, there is a "restriction map"

$$\varrho_V^U : \mathfrak{A}(U) \to \mathfrak{A}(V)$$

satisfying the usual compatibility condition: $\varrho_W^V \circ \varrho_V^U = \varrho_W^U$ whenever $W \subset V \subset U$.

The extension problem can be formulated as follows: given a family $\mathcal{C}$ of closed subset C of X, decide whether on not the restriction map

$$\varrho_{X \smallsetminus C}^X : \mathfrak{A}(X) \to \mathfrak{A}(X \smallsetminus C)$$

is onto for every $C \in \mathcal{C}$. The most interesting cases for applications are the following

(I) $\mathcal{C}$ is a family of compact sets of X;

(II) $\mathcal{C}$ is the family of all analytic (or, more generally, thin) subsets of X;

(III) $\mathcal{C}$ is a family of closed subsets of X of the form $K \cap \{\mathrm{Re}F \geq 0\}$ where K is a compact subset of X and F if a holomorphic function in X (and in this situation $\{\mathrm{Re}F \geq 0\} \smallsetminus C$ is said to be a *semi-corona*).

2.1 – Classical results

The results relative to cases (I) and (II) are rather classical. Let us recall the main ones

(I) Hartogs type extension theorems

- Let X be a normal complex space, K a compact subset such that $X \smallsetminus K$ is connected. Then the natural homomorphism

$$\mathcal{O}(X) \to \mathcal{O}(X \smallsetminus K)$$

  is an isomorphism.
- Let X be a Stein space, $\varphi : X \to \mathbb{R}$ a strongly plurisubharmonic function and $K = \{\varphi \leq c\} \Subset X$. Let $\mathcal{C}oh(X)$ be the category of all coherent sheaves over X and $\mathcal{F} \in \mathcal{C}oh(X)$. The natural homomorphisms

$$H^j(X, \mathcal{F}) \to H^j(X \smallsetminus K, \mathcal{F})$$

  are isomorphisms for $1 \leq j \leq \mathrm{depth}\,(\mathcal{F}) - 1$.

- Let X be Stein and $\Omega = \{r < \varphi < s\} \Subset X$ where $\varphi : \Omega \to \mathbb{R}$ is a strongly plurisubharmonic function such that $\{r + \varepsilon < \varphi < s - \varepsilon\} \Subset \Omega$, $\varepsilon > 0$. Let Y be an analytic subset of Ω and $\mathfrak{I}_Y$ the ideal of Y. If depth$(\mathfrak{I}_Y) \geq 3$, Theorem A of Oka-Cartan-Serre holds true for $\mathfrak{I}_Y$. In particular, Y extends to $\{\varphi < s\}$.

(II) Remmert-Stein type theorems

Let Y be an analytic subset of a Stein space X. Then

a) if X is a normal space and $\operatorname{codim} Y_x \geq 2$, $\forall x \in Y$, the natural homomorphism
$$\mathcal{O}(X) \to \mathcal{O}(X \smallsetminus Y)$$
is an isomorphism;

b) if $\dim_{\mathbb{C}} \mathrm{Y} = \mathrm{d}$ every analytic subset $Z \subset X \smallsetminus Y$ such that $\dim Z_x \geq d + 1$, $\forall x \in Z$, extends through Y;

c) let $\mathcal{F} \in \mathcal{C}oh(X)$ be such that depth$(\mathcal{F}) \geq q$; then the natural homomorphisms
$$H^j(X, \mathcal{F}) \to H^j(X \smallsetminus Y, \mathcal{F})$$
are isomorphisms for $j \leq j \leq q - d - 1$.

2.2 – Semi-coronae

The interest for the case (III) is relatively more recent. The starting point is maybe the following extension theorem for CR-functions stated in [LT] (see also [St1]):

Theorem 2.1. *Let Σ be a real connected C^1-hypersurface in $\mathbb{C}^n$, $n \geq 2$, oriented, compact, with boundary $b\Sigma$. Assume that the following conditions are fullfilled:*

(i) *$b\Sigma$ belongs to a C^∞-hypersurface M and there exists a relatively open subset A of M such that $b\Sigma = bA$;*

(ii) $\Sigma \cap M = b\Sigma$;

(iii) *M is the zero set of a pluriharmonic function in a neighbourhood of the open set D bounded by $\Sigma \cup A$.*

Then every Lipschitz CR-function in $\Sigma^0 = \Sigma \smallsetminus b\Sigma$ extends to D by a holomorphic function which is continuous on $D \cup \Sigma^0$.

This result is, in fact, an extension theorem for semi-coronae, in view of the Plemelj formula for Bochner-Martinelli Transforms in $\mathbb{C}^n$ (cfr. [HL]).

For a generalization to CR-forms see [P]. Similar results are proved in [L], in the context of Stein manifold.

Further generalizations to CR-objects can be found in [T].

It is worthwhile observing that Theorem 2.1 motivated the notion of "removable set". Let X be a complex manifold and $\Omega \Subset X$ a bounded domain with regular boundary; a (closed) subset $E \subset b\Omega$ is said to be *removable* if all CR-functions in $b\Omega \smallsetminus E$ extend holomorphically to Ω. In [St2] and [J] can be found a large number of references on the subject.

Very recently the case of semi-coronae was considered again with reference to the extension problem for coherent sheaves.

Let X be a Stein space and $\Omega = \{r < \varphi < s\} \Subset X$, where $\varphi : \Omega \to \mathbb{R}$ is a strongly plurisubharmonic function such that $\{r + \varepsilon < \varphi < s - \varepsilon\} \Subset \Omega$, $\forall \varepsilon > 0$ sufficiently small. Let $B = \{\varphi < s\}$ and F be a holomorphic function in a neighbourhood of $\overline{\Omega}$. Let $\Omega^+ \cup \Omega^-$ and $B^+ \cup B^-$ be the connected decompositions of $\Omega \smallsetminus \{\mathrm{Re} F = 0\}$ and $B \smallsetminus \{\mathrm{Re} F = 0\}$ respectively. Then, using the cohomological thecniques by Andreotti and Grauert (cfr. [AG]) one proves the following result (cfr. [ST]):

- Let $\mathcal{F} \in \mathcal{C}oh(B)$. The natural homomorphism

$$H^0(B^+, \mathcal{F}) \to H^0(\Omega^+, \mathcal{F})$$

is onto in the following two cases:

(i) depth $(\mathcal{O}_{X,x}) \geq 3$, $\forall x \in B$, and $\mathcal{F}$ is locally free;
(ii) $B \cap \mathrm{Sing}\,(X) \cap B = \varnothing$ and depth $(\mathcal{F}_x) \geq 3$ $\forall x \in B$.

In the same context G. Della Sala and A. Saracco have proved that

- if $B \cap \mathrm{Sing}\,(X) \cap B = \varnothing$ every analytic subset Y of Ω such that depth $(\mathfrak{I}_x) \geq 4$, $\forall x$, extends to B^+ (cfr. [DS].

3 – Levi flatness and extension

3.1 – Levi flatness

Let M be a smooth real submanifold of $\mathbb{C}^n$ of dimension m. Let $T_p(M)$ and $HT_p(M) \subset T_p(M)$ be the real and the complex tangent hyperplane to M at p respectively. The distribution $\mathcal{L}$ on M, $p \mapsto HT_p(M)$, $\forall p \in M$, is called the *Levi distribution*. M is said to be *Levi flat* whenever $\mathcal{L}$ is integrable. Levi flatness is characterized by the condition:

$$\mathrm{Levi}\,(\varphi) = \sum_{j,k=1}^{n} \frac{\partial^2 \varphi}{\partial z_j \partial \bar{z}_k} w_j \bar{w}_k = 0$$

when $p \in M$ and $w \in HT_p(M)$, $\{\varphi = 0\}$ being a local equation for M. Let us denote $L(\varphi)(p)$ the hermitian form $\mathrm{Levi}(\varphi)_{|HT_p(M)}$.

In view of Frobenius theorem, it is a easy to show that a Levi flat hypersurface M is foliated by complex hypersurfaces. This geometric property allows to extend the notion of Levi flatness to topological hypersurfaces.

As a consequence of the solution of the Levi problem (cfr. [H]) we have that a Levi flat hypersurface locally divides $\mathbb{C}^n$ in domains of holomorphy.

A smooth hypersurface M is said to be *strongly Levi convex* if $L(\varphi)(p) \neq 0$ for all $p \in M$. A domain D of $\mathbb{C}^n$, $n \geq 2$, with a smooth boundary bD, is said to be *strongly pseudoconvex* if $L(\varphi)(p) > 0$ for every $p \in b\Omega$ and every local equation φ for bD such that $\varphi < 0$ on D.

3.2 – Hulls and Levi flat extension

Let K be a compact subset of $\mathbb{C}^n$. We recall that the *hull of holomorphy* of K is the spectrum $\mathfrak{S}(K)$ of the closure $\mathbf{H}(K)$ of $\mathcal{O}(K)$ in $C^0(K)$. From the Gelfand Theory, we know that $\mathfrak{S}(K)$ is a compact Hausdorff space, ramified over $\mathbb{C}^n$ ([GRS]).

In general the structure of $\mathfrak{S}(K)$ is very involved and describing it is one of the most challenging global problems in Complex Analysis.

We have the following elementary but useful fact: if we know that every $f \in \mathcal{O}(K)$ extends holomorphically to a neighbourhood of a compact subset $K' \supset K$ of $\mathbb{C}^n$ with $K \subset K'$, then $K' \subset \mathfrak{S}(K)$. This raises the following problem:

> given a compact subset K of $\mathbb{C}^n$, can we construct a (possibly maximal) compact subset $\widehat{K} \subset \mathbb{C}^n$ such that $K \subset \widehat{K}$ and all holomorphic functions in $\mathcal{O}(K)$ extend holomorphically on a neighbourhood of $\widehat{K}$?

A good candidate for $\widehat{K}$ is the union of all *analytic discs* with boundaries in K i.e. the images of continuous maps $h : \overline{D}(0,1) \to \mathbb{C}^n$ from the unit disc $\overline{D(0,1)} \subset \mathbb{C}$, which are holomorphic in $D(0,1)$ and such that $h\big(bD(0,1)\big) \subset K$. The reason in doing this is the following result:

> every $f \in \mathcal{O}(K)$ extends holomorphically to a neighborhood of
> $$\bigcup_h h(\overline{D}(0,1))$$
> where h is varying in the family of all analytic discs with boundary in K.

Proving the existence of "sufficiently rich" families of analytic disc is, in general, very difficult. The situation is more clear for $n = 2$, at least when K is a compact subset of a strongly Levi convex hypersurface M. Let us suppose that U is an open bounded domain of M, with smooth boundary $S = bU$. Assume that $S = \widetilde{S}$ where $\widetilde{S} \smallsetminus S$ is Levi flat. Then every function $f \in \mathcal{O}(\overline{U})$ extends holomorphically to the domain Ω between $\overline{U}$ and $\widetilde{S}$. In particular, from what is preceding, we have $\overline{\Omega} \subset \mathfrak{S}(K)$ and thus we are led to the following problem:

> given a smooth compact surface $S \subset \mathbb{C}^2$, find a Levi flat hypersurface $\widetilde{S}$ with boundary S. $\widetilde{S}$ is then said to be a *Levi flat extension* of S.

In '83, using Bishop's theorem on the existence of families of analytic discs (cfr. [B]), Bedford and Gaveau proved the first fundamental result: a generic graph of a smooth function g on a topological 2-sphere $S \subset \mathbb{C}_z \times \mathbb{R}_u$ such that $S \times \mathbb{R}_v$ is a strongly Levi convex hypersurface in $\mathbb{C}^2_{z,w}$, $z = x + iy$, $w = u + iv$, is extendable by a Levi flat graph $\widetilde{S}$ (i.e. bounds a Levi flat graph).

The problem of finding a bounded Levi flat hypersurface $\widetilde{S}$ in $\mathbb{C}^2$ with a prescribed boundary S has been extensively studied in the last twenty years by many authors (cfr. [BG], [BKl], [G], [E], [Sh], [Kr], [CS], [ShT1], [SlT]).

The analytic counterpart of Bedford-Gaveau's statement is an existence theorem for the Dirichlet problem for a quasilinear second order degenerate elliptic equation, the so called *Levi equation* (cfr. 3.1). Levi flat extendability of a surface in $\mathbb{C}^2$ by PDE was brought forward in [SlT] and in a more substantial way in [CM].

In higher dimensions, the situation is completely different from what it is in $\mathbb{C}^2$. The generic (2n-2)-manifold S is not even locally extendable by a Levi flat hypersurface $\widetilde{M}$. Indeed, locally, S is the graph of a smooth function g, so the existence of a local Levi flat graph extending M amounts to solve a boundary problem for a system of (non-linear) differential operators and this requires compatibility conditions for g. Some existence results have been recently obtained for $\mathbb{C}^3$ (cfr. [DTZ]).

4 – Semi-local Levi flat extensions

In this last section we discuss a *semi local version* of Levi flat extendability in $\mathbb{C}^2$, i.e. the Levi flat extension from a part of the boundary.

Let Ω be a domain in $\mathbb{C}_z \times \mathbb{R}_u$ with a smooth boundary such that $\Omega \times \mathbb{R}_v$ is a strongly pseudoconvex domain in $\mathbb{C}^2_{z,w}$. Let U be an open subset of $b\Omega$. In [ShT3] (see also [ShT2]) we study the following problem: find an open subset Ω^U of $\overline{\Omega}$ with the properties

(i) $\Omega^U \smallsetminus b\Omega = U$;
(ii) every graph $\Gamma(g)$ of a continuous function $g : U \to \mathbb{R}_v$, extends to $\Omega^U \smallsetminus b\Omega$ by a continuous graph $\Gamma(F)$, Levi flat over $\Omega^U \smallsetminus b\Omega$;
(iii) Ω^U is maximal with the properties (i), (ii).

The problem itself interesting can be seen as the first step for a general theory of the domains of existence for Levi flat hypersurfaces.

Construction of Ω^U

Denote $\mathcal{A}(\Omega \times \mathbb{R}_v)$ the Fréchet algebra $\mathcal{O}(\Omega \times \mathbb{R}_v) \cap C^0(\overline{\Omega} \times \mathbb{R}_v)$. Let $E_1 \Subset E_2 \Subset \dots$ be an exhaustion of U by compact subsets such that $E_n \Subset \overset{\circ}{E}_{n+1}$ for every $n \in \mathbb{N}$. For each $n \in \mathbb{N}$ consider the set $K_n = E_n \times [-n, n]$, and the $\mathbb{A}(\Omega \times \mathbb{R}_v)$-hull

$$\widehat{K}_n = \{\zeta \in \overline{\Omega} \times \mathbb{R}_v : |f(\zeta)| \leq \|f\|_{K_n} \forall f \in \mathcal{A}(\Omega \times \mathbb{R}_v)\}$$

of K_n. Define $W = \cup_n \widehat{K}_n \subset \overline{\Omega} \times \mathbb{R}_v$.

Theorem 4.1. *The set W has the following properties*:

(i) *W is open in $\overline{\Omega} \times \mathbb{R}_v$ and $W \cap (\Omega \times \mathbb{R}_v)$ is pseudoconvex.*
(ii) *W is invariant under translation in v-direction. In particular, there is an open subset Ω^U of $\overline{\Omega}$ such that $W = \Omega^U \times \mathbb{R}_v$.*
(iii) *Moreover, if U is the union of simply-connected subdomains of bG, then W is the CR-hull of $U \times \mathbb{R}_v$ (i.e. every CR-function in U extends holomorphically to W).*

In the case when Ω is a topological 3-ball or U has only simply-connected components, the domain Ω^U has the following property:

Theorem 4.2. *Let $\Omega \subset \mathbb{C}_z \times \mathbb{R}_u$ be a bounded domain with smooth boundary such that $\Omega \times \mathbb{R}_v \subset \mathbb{C}^2_{z,w}$ is strongly pseudoconvex. Let U be an open subset of $b\Omega$ and let $U_1, U_2, U_3, \dots$ be the connected components of U. Assume that at least one of the following conditions is satisfied*:

(i) *D is diffeomorphic to a 3-ball.*
(ii) *All connected components U_n of U are simply-connected.*

Then the domains Ω^{U_m} are connected, disjoint and $\Omega^U = \cup_m \Omega^{U_m}$.

The conclusion of the above theorem is false in general as it is shown by the following

Example 4.1. Let Ω be the solid torus in $\mathbb{C}_z \times \mathbb{R}_u$ defined by the inequality $(|z| - 2)^2 + u^2 < 1$. By an easy direct computation one shows that the domain $\Omega \times \mathbb{R}_v \subset \mathbb{C}^2_{z,w}$ is strictly pseudoconvex. Consider the open subset $U = U_1 \cup U_2$, where

$$U_1 = \left\{(z, u) \in b\Omega : |z| < \frac{3}{2}\right\}, \quad U_2 = \left\{(z, u) \in b\Omega : |z| > \frac{5}{2}\right\}.$$

Then the connected components U_1 and U_2 of U are obviously disjoint, but the set $\Omega^U = \{(z,u) \in \Omega : |u| < \sqrt{3}/2\}$ is connected (the set $\Omega^U \times \mathbb{R}_v$ is foliated by annuli $A_c = \{(z,w) \in \Omega \times \mathbb{R}_v : u = \operatorname{Re} C, v = \operatorname{Im} C\}$ where C satisfies the inequality $|\operatorname{Re} C| < \sqrt{3}/2\}$.

Now we formulate the central result stated in [ShT3]:

THEOREM 4.3. *Let Ω be a bounded domain in $\mathbb{C}_z \times \mathbb{R}_u$ such that the domain $\Omega \times \mathbb{R}_v \subset \mathbb{C}^2_{z,w}$ is strongly pseudoconvex. Let U be an open subset of $b\Omega$ and Ω^U the defined above subdomain of $\overline{\Omega}$. Then for every $\varphi \in C(U)$ there exist two continuous extensions Φ^+, Φ^- in Ω^U with the properties: the functions $\Phi^\pm$ are continuous on Ω^U, their graphs $\Gamma(\Phi^\pm)$ are Levi flat over $\Omega^U \cap \Omega$ and $\Phi^\pm|_U = \varphi$.*

Moreover, for any function $\Phi \in C(\Omega^U)$ such that $\Gamma(\Phi)$ is Levi flat over $\Omega^U \cap \Omega$ and $\Phi|_U = \varphi$ one has

$$\Phi^-(z,u) \leq \Phi(z,u) \leq \Phi^+(z,u)$$

for each point $(z,u) \in \Omega^U$.

As for the maximality of Ω^U, we have the following

THEOREM 4.4. *Let $\Omega \subset \mathbb{C}_z \times \mathbb{R}_u$ be a bounded domain diffeomorphic to a 3-ball such that the domain $\Omega \times \mathbb{R}_v$ is strictly pseudoconvex. Let U be an open subset of $b\Omega$ constituted by the disjoint union of simply-connected domains each of which is contained either in the "upper" or in the "lower" part of $b\Omega$ (with respect to the u-direction). Then there is a function $\varphi \in C(U)$ such that Ω^U is the maximal domain where the Levi-flat extension of the graph of φ can be defined.*

REFERENCES

[AG] A. ANDREOTTI – H. GRAUERT: *Théorèmes de finitude pour la cohomologie des espaces complexes*, Bull. Soc. math. France, **90** (1961), 193-259.

[BG] E. BEDFORD – B. GAVEAU: *Envelopes of holomorphy of certain 2-spheres in* $\mathbb{C}^2$, Amer. J. Math., **105** (1983), 975-1009.

[BK] E. BEDFORD – W. KLINGENBERG: *On the envelopes of holomorphy of a 2-spheres in* $\mathbb{C}^2$, J. Amer. Math. Soc., **4** (1991), 623-646.

[B] E. BISHOP: *Differentiable manifolds in complex Euclidean space*, Duke Math. J., **32** (1966), 1-22.

[C] H. CARTAN: *Faisceaux analytiques cohérents*, Corso C. I. M. E "Funzioni e varietà complesse", Varenna, 1963.

[CS] E. M. Chirka – N. V. Shcherbina: *Pseudoconvexity of rigid domains and foliations of hulls of graphs*, Ann. Scuola Norm. Sup. Pisa Cl. Sci., **21** (4) (1995), 707-735.

[CM] G. Citti – A. Montanari: *Analytic estimates for solutions of the Levi equation*, J. Differential Equations, **173** (2001), 356-389.

[CLM] G.Citti – E. Lanconelli – A. Montanari: *Smoothness of Lipchitz-continuos graphs with nonvanishing Levi curvature*, Acta Math., **188** (2002), 87-128

[DTZ] P. Dolbeault – G. Tomassini – D. Zaitsev: *On boundaries of Levi flat hypersurfaces in* $\mathbb{C}^3$, preprint.

[E] Y. Eliashberg: *Filling by holomorphic discs and its applications*, London Math. Soc. Lecture Note Ser., **151** (1991), 45-67.

[Er] O. G. Eroshkin: *On a topological property of the boundary of an analytic subset of a strictly pseudoconvex domain in* $\mathbb{C}^2$, Mat. Zametki, **49** (1991), 149-151.

[GRS] I. Gelfand – D. Raikov – G. Shilov: *Commutative Normed Rings*, Chelsea Publishing Company, 1964.

[G] M. Gromov:*Pseudo-holomorphic curves in symplectic manifolds*,Invent. Math., **82** (1985), 307-347.

[H] L. Hörmander: *An Introduction to Complex Analysis in Several Complex Variables*, North Holland, 1973.

[HL] F. R. Harvey – H. B. Lawson jr: *On boundaries of complex analytic varieties. I*, Ann. of Math., **102** (2) no. 2 (1975), 223-290.

[K] N. G. Kružilin: *Two-dimensional spheres in the boundary of strictly pseudoconvex domains in* $\mathbb{C}^2$, Izv. Akad. Nauk SSSR Ser. Mat., **55** (1991), 1194-1237.

[J] B. Jöricke: *Boundaries of singular sets, removable singularities, and CR-invariant subsets of CR-manifolds*, J. Geom. Anal., **9** no. 2 (1999), 257-300.

[L] C. Laurent-Thiébaut: *Sur l'extension des fonctions CR dans une variété de Stein*, Ann. Mat. pura e appl., (1984), 257-263.

[LT] G. Lupacciolu – G. Tomassini: *Un teorema di estensione per le CR-funzioni*, Ann. Mat. pura e appl., **137** (1984), 257-263.

[P] A. Perotti: *Extension of CR-forms and related problems*, Rend. Sem. Mat. Univ. Padova, **77** (1987), 37-55.

[S] F. Severi: *Lezioni sulle funzioni analitiche di più variabili complesse*, CEDAM, Padova, 1958.

[Sh] N. Shcherbina: *On the polynomial hull of a graph*, Indiana Univ. Math J., **42** (1993), 477-503.

[ST] A. Saracco – G. Tomassini: *Extension problem for q-coronae*, preprint.

[ShT1] N. Shcherbina – G. Tomassini: *The Dirichlet problem for Levi flat graphs over unbounded domains*, Internat. Math. Res.Notices, (1999), 111-151.

[ShT2] N. Shcherbina – G. Tomassini: *Levi-flat extension from a part of the boundary*, C. R. Acad. Sci. Paris, Ser. I, **337** (2003), 699-703.

[ShT3] N. Shcherbina – G. Tomassini: *Semi local Levi-flat extensions*, (Russian) Izv. Ross. Akad. Nauk Ser. Mat., **68** (2004), 195-218; translation in Izv. Math., **68** (2004), 619-641.

[Si] Y. T. Siu: *Extension Problem in Several Complex Variables*, Proc. Int. Congr. IMU, Helsinki, 1978, 669-673.

[SlT] Z. Slodkowski – G. Tomassini: *Weak solutions for the Levi equation and envelope of holomorphy*, J. Funct. Anal., **101** (1991), 392-407.

[St1] E. L. Stout: *Analytic continuation and boundary continuity of functions of several comlex variables*, Proc. Sopc. Edinburgh, **89 A** (1981), 63-74.

[St2] E. L. Stout: *Removable singularities for the boundary values of holomorphic functions of several complex variables*, Proc. Mittag-Leffler special year in complex variables, Princeton Univ. Press, Princeton, 1993.

[T] G. Tomassini: *Extension d'objets CR*, Math Z., **194** (1987), 471-486.

Lavoro pervenuto alla redazione il 14 ottobre 2004
ed accettato per la pubblicazione il 29 novembre 2004.
Bozze licenziate il 1 settembre 2005

Indirizzo dell'Autore:

Giuseppe Tomassini – Scuola Normale Superiore – Piazza dei Cavalieri 7 – 56126 Pisa, Italy
E-mail: tomassini@sns.it