





## Foreword

Bruno de Finetti had a very versatile personality and was actively involved in many different fields of Science and Culture. In fact, not only he gave his determinant contributions to Mathematics and its applications, in particular Probability, Statistics, Financial and Actuarial Mathematics, but he was also deeply interested in Epistemology, Philosophy, Economics, Politics. Still in our days, the contributions that he gave to different fields are often revisited and, in many cases, they have been discovered to contain foresight and fertile suggestions for contemporary research.

Bruno de Finetti was born, from Italian parents, in Innsbruck on June 13, 1906. During 2006 a number of different events have been organized to celebrate the centenary of his birth. Several scientific and cultural institutions have organized scientific events with the purpose of highlighting the different facets of his scientific and human personality and to discuss his advanced contributions.

In particular, on November 15-17, 2006, in Rome, the International Symposium “Bruno de Finetti Centenary Conference” was held at the Department of Mathematics of University “La Sapienza” and Accademia dei Lincei, two institutions of which he had been a member for many years.

Already in 1981, the celebration of the 75<sup>th</sup> birthday of Bruno de Finetti offered University La Sapienza and Accademia dei Lincei the opportunity for the organization of an international conference, that was specifically devoted to the theme of Exchangeability (the contributions presented at that conference were collected in the Volume *Exchangeability in Probability and Statistics* (G. Koch and F. Spizzichino, Eds), North Holland, Amsterdam, 1982).

The Symposium held on November 2006 consisted of two different parts, respectively devoted to the themes *de Finetti’s Legacy in Probability Today* and *Bruno de Finetti and Economic Analysis*.

The Organizing Committee was formed by Edoardo Vesentini (Politecnico di Torino and Accademia dei Lincei), Pierluigi Ciocca (Banca d’Italia), Giorgio

Letta (Università di Pisa and Accademia dei Lincei) Giorgio Lunghini (Università di Pavia and Accademia dei Lincei), Carlo Sbordone (Università Federico II, Napoli, and Accademia dei Lincei), and Fabio Spizzichino (Università La Sapienza, Roma). The Local Organizing Committee was formed by Giovanna Nappo, Mauro Piccioni and Fabio Spizzichino at the Department of Mathematics of Università La Sapienza.

The aim of the Symposium was just to discuss some problems, arising in the frame of recent research in the fields of Probability and Economics, respectively, and to trace the connections with ideas and anticipations contained in the work by Bruno de Finetti.

In particular the Sessions on *de Finetti's Legacy in Probability Today* were devoted to rethink some purely mathematical aspects of the contributions that he gave in the Thirties of last Century, and to present related developments that have been obtained in recent times.

These sessions were constituted by general lectures, given by a few selected speakers. Listed in the order they appeared in the Programme, the invited speakers were Eugenio Regazzini (Università di Pavia), Olav H. Kallenberg (Auburn University), Persi Diaconis (Stanford University), Steffen L. Lauritzen (Oxford University), Paul Ressel (Katholische Universität Eichstatt), Yoseph Rinott (Hebrew University, Jerusalem), and Murad Taqqu (Boston University).

The complete scientific programme of the Symposium can be found in the next pages.

More details about the Symposium can be found at the web-site:

[www.mat.uniroma1.it/ricerca/convegni/2006/deFinetti/](http://www.mat.uniroma1.it/ricerca/convegni/2006/deFinetti/)

The programme of the first session, in particular, included also an opening address by Fulvia de Finetti, the daughter of Bruno de Finetti, the presentation of *Opere di Bruno de Finetti*, and the Exhibition of Historical Books *Probability from Cardano to de Finetti*.

With her kind permission, the text of the contribution by Fulvia de Finetti is reported in this Volume.

*Opere di Bruno de Finetti* is a two Volumes edition published by Unione Matematica Italiana (UMI); the first Volume, in particular, is devoted to his contributions to Probability, Statistics and Decision Theory.

The book exhibition was held in the Library of the Department of Mathematics and was organized by Giovanna Nappo with the precious collaboration of the staff of the Library; more details can be found at the web-site address <http://www.mat.uniroma1.it/ricerca/convegni/2006/deFinetti/mostra/historical-books-details.html>

The present Volume, edited by Giovanna Nappo, Mauro Piccioni and Fabio Spizzichino, collects the contributions that the Authors, invited to lecture on the theme *de Finetti's Legacy in Probability Today*, delivered after the Symposium.

**Scientific Program  
of International Symposium**

*Bruno de Finetti Centenary Conference*

Rome, November 15-17, 2006

Università "La Sapienza"  
Piazzale A. Moro, 5  
Accademia Nazionale dei Lincei  
Via della Lungara, 230

**Wednesday, November 15 : Mathematical Session 1*****De Finetti's Legacy in Probability Today***

Aula Magna  
Università "La Sapienza"  
Piazzale A. Moro, 5

8:30 – 9:15 Registration of Participants

9:15 Opening of Conference

Chairman: Marco Scarsini

9:45 – 10:45 Eugenio Regazzini (Università di Pavia, Italy)

*De Finetti's contribution to the theory of random functions*

10:45 – 11:15 Coffee Break

11:15 – 12:00 Presentation of the Volumes

*Opere di Bruno de Finetti* Edited by Unione Matematica Italiana

Chairman: Giorgio Koch

15:00 – 16:00 Olav H. Kallenberg (Auburn University , USA)

*Some highlights from the theory of multivariate symmetries*

16:00 – 16:30 Coffee Break

16:30 – 17:30 Persi Diaconis (Stanford University, USA)

*Exchangeability in the Twenty First Century*

12:00 – 18:00 Exhibition of Historical Books

*Probability from Cardano to de Finetti*

Library of Department of Mathematics "G. Castelnuovo"

**Thursday, November 16: Mathematical Session 2*****De Finetti's Legacy in Probability Today***

Accademia Nazionale dei Lincei  
Via della Lungara, 230

Ore 9:00 Opening

Chairman: Carlo Sbordone

10:00 – 11:00 Steffen L. Lauritzen (Oxford University, UK)

*Exchangeable Rasch Models*

11:00 – 11:30 Coffee Break

11:30 – 12:30 Paul Ressel (Katholische Universität Eichstatt, Germany)  
*Exchangeability and semigroups*

Chairman: Wolfgang Runggaldier

15:00 – 16:00 Yoseph Rinott (Hebrew University, Israel)  
*Exchangeability, concepts of dependence, and statistical implications*

16:00 – 16:30 Coffee Break

16:30 – 17:30 Murad Taqqu (Boston University, USA)  
*Dependence structures of some infinite variance stochastic processes*

### **Friday, November 17: Economics Session**

#### ***Bruno de Finetti and Economic Analysis***

Accademia Nazionale dei Lincei  
Via della Lungara, 230

Ore 9:00 Opening

Chairman: Siro Lombardini

9:30 – 10:30 Giorgio Lunghini (University of Pavia, Italy)  
*Bruno de Finetti and Economic Theory*

10:30 – 11:00 Coffee Break

11:00 – 12:00 Luca Barone (Goldman-Sachs International, UK)  
*Mean-Variance Portfolio Selection: de Finetti scoops Markowitz*

12:00 – 13:00 Flavio Pressacco (University of Udine, Italy)  
*B. de Finetti hero of the two worlds: (applied) mathematician  
and (quantitative) economist*

13:00 Conclusions and Closure of Conference





## **Bruno de Finetti Centenary Conference**

### **Rome, November 15, 2006**

**FULVIA DE FINETTI**

It is a pleasure for me to be once more in this University where my father spent many years of his academic career. As you all probably know my father left the University of Trieste in 1954 when he conquered a chair in the University of Rome and it was in 1961 that he moved from the Faculty of Economics to the Faculty of Science to teach Probability. This was the happy end of a story that goes back to 1927 when my father, just graduated, reached Rome to work as head of the Mathematical Service of the Istituto Centrale di Statistica. He had already in his mind Probability and after finishing his work at the Institute he used to attend the seminars that took place in Panisperna Street, next door to his office. He was particularly impressed by Enrico Fermi whose rapid career became a target for him. He immediately gained the attention of the great mathematicians that worked in Rome at that time: Guido Castelnuovo, Tullio Levi-Civita, Federigo Enriques.

Guido Castelnuovo at that time taught probability and so he explained why he found probability an interesting topic to teach: “Probability is a science of recent formation; hence in it, better than in other branches of mathematics, one can see the relationship between the empirical contribution and the one given by reasoning, and between the process of inductive and deductive logic used in it. The fact that it is a science in the making explains why it is appropriate to give frequent examples to show the applications of known methods or to introduce new ones.”

If you compare Castelnuovo’s sentence with what Bruno wrote to his mother when student in Milan: . . . Mathematics is not by now a field already explored, just to learn and pass on to posterity as it is. It is always progressing, it is

enriching and lightening itself, it is a lively and vital creature, in full development and just for these reasons I love it, I study it and I wish to devote my life to it ...

It will not surprise you to know that Castelnuovo often invited Bruno at his home to see the progresses in the works of this promising young mathematician. To open the door was a little girl wearing her hair in pigtailed. She was Emma the daughter of Castelnuovo. In a letter dated July 28, 1928 Castelnuovo examines the work of Bruno, recognizes his capabilities as analyst, gives advices on how to present the work and concludes, "I feel sure that you will be able to give important contributions to Probability Calculus and its applications". And so it seems he did if important names in this field have accepted to come from U.S.A. and Europe for this Centenary Conference.

When in 1961 the Faculty of Science decided to resume for Bruno the chair of Probability that had been of Castelnuovo but extinguished when he left, the main concern of my father was that the same thing might happen when he would leave. In April 1973, Savage had already died, my father received from the University of Michigan an invitation for the year 1973-74. I think it may interest you to read part of the answer of my father to decline the invitation: "...I am very pleased and honoured for such attracting invitation and for the interest in my research ... and in my point of view about subjective probability. I would be surely willing to support it, especially in your University where L.J. Savage spent several years of his admirable activity ... "I am involved in many programs here, highly depending on myself (my collaborators are too young to be fully responsible for the courses)."

I am sure my father would be very happy to know that the Faculty of Science, not only preserved the chair of Probability but even increased to three the number of chairs and that some of those "too young collaborators" are continuing his teaching and research. Among them I want particularly to thank Professor Fabio Spizzichino who promoted and organized this International Centenary Conference.

## Preface

GIOVANNA NAPPO – MAURO PICCIONI

FABIO SPIZZICHINO

De Finetti started his active research in Probability around 1927, at the age of 21. At that time, his interest was mainly addressed to the foundations of Probability. His interest on this topic focused on two different, even though related, aspects: the meaning of the term probability of an event and the axioms that should be imposed in the formalization of the mathematical theory.

In fact, the main motivations for his research can be traced back to his dissent and his objections to ideas and viewpoints about probability that were common among mathematicians and other scientists. His reflections ripped his firm opinion that probability cannot have but a subjective character. This conclusion lead him to rethink the meaning of stochastic independence and to introduce the concept of Exchangeability in the foundations of Probability and Statistics.

On the other hand he also strongly defended the idea that probability must satisfy the simple-additivity property while, in his view, the stronger assumption of countable additivity is not necessary and may give rise indeed to different drawbacks.

Another topic which attracted his interest at that time was the definition and the analysis of Stochastic Processes with Independent Increments. He first analyzed the finite-dimensional distributions of these processes, thus arriving to single out the class of infinitely divisible distributions. This piece of research was carried out, later on, by himself, P. Lévy, A. Y. Khinchin, A. N. Kolmogorov and others, in a series of papers devoted to the characterization of infinitely divisible distributions.

The two basic topics mentioned so far constitute the primary elements for the contributions collected in this Volume. The articles by Eugenio Regazzini

with Federico Bassetti, Olav Kallenberg, Persi Diaconis with Svante Janson, Steffen L. Lauritzen, and Paul Ressel are related to the theme of exchangeability, whereas the article authored by Murad S. Taqqu with Joshua B. Levy, concerns the developments of the theory of stochastic processes with independent increments.

It is well known that exchangeability is nothing but a simple concept of symmetry: a (finite-dimensional) vector of random variables is exchangeable if its joint distribution is invariant under permutation of coordinates. A denumerable sequence is exchangeable if and only if any finite subsequence is exchangeable.

The simplest example of an exchangeable sequence is the case of independent, identically distributed random variables. A natural extension is the case of sequence of conditionally independent, identically distributed random variables. The celebrated de Finetti's theorem about exchangeability guarantees that denumerable sequence of exchangeable variables are necessarily conditionally i.i.d..

Of course the notion of exchangeability can be extended and generalized to cope with many other interesting situations, also in more abstract settings. The interest for this topic dates back at least to late Thirties with de Finetti's work about partial exchangeability ("equivalence partielle").

Among developments related to de Finetti's theorem, a special type of problems stimulated the interest of several probabilists: to characterize the class of exchangeable models manifesting some further conditions of invariance, under different groups of transformations. A related problem, obtained by reversing this point of view, is to find invariance properties which characterize all the joint distributions obtained as mixture of i.i.d. variables with common distribution belonging to a special class (e. g. the cases of conditionally i.i.d. Gaussian variables or of conditionally i.i.d. exponential variables). This class of problems is of basic importance in the construction and the theoretical study of statistical models (especially in a Bayesian context) and is strictly related with the theory of sufficient statistics. From the mathematical point of view this subject translates into the problem of characterizing the extremal points of some convex spaces. Results in this direction have been called de Finetti-type theorems.

The fields of exchangeability and de Finetti-type theorems have been extensively developed in the last few decades and important contributions have been given by Authors of articles that appear in the present collection. The articles presented here contain recent results and also provide a review of relevant issues from the literature.

The collection opens with the article by Eugenio Regazzini and Federico Bassetti whose main purpose is to present a detailed review of the first paper published by de Finetti on the theme of Exchangeability, which is not very well known. In this paper de Finetti performed the analysis of exchangeable events by means of the method of the characteristic functions. Since this was also the main mathematical tool used in de Finetti's studies about processes with independent increments, this paper can be considered as a bridge between exchangeability

and processes with independent increments. However, the interest of the article by Regazzini and Bassetti goes beyond strictly historical aspects. In particular, following a suggestion contained in the de Finetti's paper, they obtain necessary and sufficient conditions of an algebraic type for the extendibility of a finite sequence of exchangeable events.

The article by Olav Kallenberg provides a useful review concerning the area of probabilistic symmetries, that can be seen as a natural development of the studies about exchangeability and partial exchangeability. An important part of this theory has been systematically developed by Kallenberg himself in a recent monograph (2005). In the present article representation theorems for multi-dimensional arrays of random variables, which are invariant under suitable groups of transformations, are discussed. More precisely, for two-dimensional arrays, the following results are presented: the theorems by Aldous and Hoover on the separate and joint exchangeability, the results by Kallenberg himself on the equivalence between *contractibility* (the operation of taking minors) and exchangeability, and those about invariance under rotations (*rotatability*) obtained by Aldous (in the separate case) and Kallenberg (in the joint case). In order to discuss rotatability in dimension greater than 2, the framework of continuous linear random functionals (CLRF) is introduced, which allows to obtain representations involving multiple Wiener-Ito integrals.

The article by Persi Diaconis and Svante Janson gives a somewhat different perspective on the concept of partial exchangeability through random graphs. The paper develops a theoretical framework with the purpose of giving a probabilistic interpretation to the notions of convergence of graphs and of infinite graph limits which have been recently investigated by Lovász and coauthors. Within this framework, the Lovász-Szegedy characterization of infinite graph limits is translated, in terms of adjacency matrix of the graph, into the Aldous-Hoover representation for two dimensional jointly exchangeable arrays. A similar analysis is performed for bipartite graphs (leading to separately exchangeable arrays) and directed graphs, showing how the results discussed by Kallenberg in the previous paper can be used in this graph theoretical setting.

Some general aspects of the field of de Finetti-type theorems are analyzed in Paul Ressel's article. This article discusses the central role of semigroups in the description of the general mathematical structure that is at the basis of the theory. Actually, the author proves a general theorem on the representation, as a mixture of *characters*, of positive definite functions defined on a semigroup. By using this result a method is provided to give alternative proofs to known theorems on exchangeability, such as the Hewitt-Savage theorem, and characterizations of mixtures of i.i.d. samples from specific parametric families. The paper presents also an application to the exchangeable partitions introduced by Kingman.

The contribution by Steffen L. Lauritzen deals with random infinite matrices which are not only row-column exchangeable (called *separately exchangeable*

by Kallenberg) but also *row-column summarized* (RCES). This means that their finite-dimensional distribution is a function of the row and the column sums (these sums are actually semigroups statistics, hence they fall within the theory developed in the paper by Ressel). It is shown that, within the class of “regular” RCES random matrices, the random Rasch matrices (that were introduced in mathematical psychology as models for intelligence tests) play a special role: they constitute the extreme points. A similar analysis is performed for weakly exchangeable matrices (called *jointly exchangeable* by Kallenberg). The relations with random graphs are also addressed in the article; in particular weakly exchangeable Markov graphs, used in social network analysis, are discussed.

As mentioned, the contribution delivered by Murad S. Taqqu and Joshua B. Levy is related with developments of the original de Finetti’s work on processes with independent increments. Their contribution deals with the phenomenon of long-range dependence of symmetric  $\alpha$ -stable (S- $\alpha$ -S) log-fractional motions with index  $\alpha \in (1, 2)$ . Any such process is self-similar with index  $H = \frac{1}{\alpha}$  and it has infinite variance; therefore the structure of dependence between the values of the process at two different time-instants cannot be described in terms of the covariance, but rather through the use of the so-called codifferences and covariations, whose behavior is described in detail.

Bruno de Finetti gave relevant contributions in many different areas and for his contributions he has been often celebrated. But, first of all, he was a mathematician. We are particularly glad to have the possibility, with the publication of the present Volume, to honour the memory of him as a mathematician. For this reason, we would like to thank all the authors for their enthusiastic and high level collaboration.

We also thank Fulvia de Finetti for contributing this Volume with her opening address and for her stimulating and friendly suggestions during the preparation of the Symposium.

We gratefully acknowledge the kind attitude of the Scientific Board of *Rendiconti di Matematica* in dedicating an issue of the Journal to these Proceedings. In particular we thank our colleague Alessandro Silva, who promoted this editorial initiative and gave us a friendly support along the whole publication process.

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