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Some analytical contributions to a mathematical model of resource curse

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Abstract: *In this paper we provide some simple generalizations, that generate all the same results, of a model originally presented, along with its simplified version, in two works of Robinson, Torvik and Verdier [13, 14] about the understanding of the resource curse phenomenon, in particular we study how political objectives influence economical choices. More specifically we show how the extraction rate of various public nonrenewable resources, the rate of employment in the public sector and, last but most important, the overall income of a country depend in some way on the aspiration of the politician in charge to keep his power. In addition we analyze some particular cases, related to the same model, in which extraction or employment rates are fixed to a boundary value. We give also a lot of graphics about the results achieved in this extended work.*

1 – Introduction

Scholars and economic historians traditionally emphasized the great benefits which natural resources give to a nation (see for example [17] on the British case). However in some cases it seems that resources are a sort of curse for currently developing countries. In this regard the expression resource curse thesis was first used in 1993 by Auty in [4] to describe how countries rich in natural resources were unable to use that wealth to boost their economies and how these countries had lower economic growth than countries without an abundance of natural resources.

A fundamental question is to learn the mechanism linking natural resources availability and their prices to development of countries. Empirical literature on the resource curse suggests that many different reasons, for example government mismanagement of resources or weak, ineffectual, unstable or corrupt institutions can lead to an incorrect exploitation of abundant natural resources. The resources

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more often considered are especially those that are publicly owned like coal, natural gas or oil. For such resources the rent accrues directly to the government and it decides how to spend this rent.

The work in this paper draws inspiration by two works of Robinson, Torvik and Verdier, namely [13] and its simplified version [14], in which they model a situation with two politicians, an incumbent and a competitor, two periods with an election in the middle, an amount of natural resource than can be extracted and sold in the two periods at two (different) prices, a population of voters/workers which gives to the incumbent a certain probability of reelection and an equilibrium policy to choose that maximizes a given income.

In this work, like in the original one, we consider an incumbent that distribute its rent as patronage to influence the outcome of the election. Patronage is to be understood in the definition given by Weingrod in [16] that is the way in which party politicians distribute public jobs or special favors in exchange for electoral support. It is widely believed that public employing is, politically speaking, a very profitable way to distribute rents (see for example [5], about the phenomenon of political recommendation in Palermo, or [3]). So we choose to model patronage as offering job in the public sector made by the incumbent to a voter to take the favors. For an alterative form of patronage we can think to an incumbent who try to influence the outcome of elections by investing in white elephants, as in [12], namely valuable but burdensome possessions or investment projects whose costs are out of proportion to its usefulness or worths or, in the definition given by the authors, a project with a negative social surplus[1].

The paper proceeds as follows. In the next section we describe the model under consideration. It is basically the model contained in the simplified version of the original work, but with some important changes. We consider in fact not only a sigle natural resource but d stocks of different nonrenewable resources, each with its own selling prices in the two periods. Moreover, for every resource we set an upper bound which is the total amount of good available. On the function which describes the reserves remaining for the second period we made further technical assumptions to ensure consistency with the situation under study. The last change concerns the function which describes the reelection probability. In fact we consider the case in which the second derivative is not zero but less than or equal to zero to model a sort of saturation phenomenon of the labor market.

Section 3 describes the mathematical tools used to obtain rigorous proofs of the results already achieved by the authors. Section 4 is the main section. We consider

[1]An example of white elephant comes from the activities of the Industrial Development Corporation of Zambia, that was subject to a series of political and most importantly uneconomic directives on specific operational issues, including type and location of investments (see [15] for further details). Other examples can be founded in [8].

the case $d = 1$ and we analyze in the first subsection the situation in which the maximum point of the income function is in the interior of the considered region, as was done by the original authors. In the second subsection there are instead some new results concerning a maximum point localized on the boundary of the same region under consideration. Finally in Section 5 we consider the case of two different natural resources. We show several graphical simulations showing both expected and unexpected phenomena.

We want at the end to point out to readers two other interesting works about resource curse. In [2] Andersen and Aslaksen studied the question about what form of government, presidentialism or parliamentarism, leads to the resource curse phenomenon, while in [11] Mehlum, Moene and Torvik studied what kind of institution, grabber friendly or producer friendly, carries to the same phenomenon.

2 – Model overview

We deal with a model in which there are two politicians, an incumbent politician wishing to be reelected and a competitor. The mass of voters is normalized to 1. There are two periods with an election occurring at the end of the first one in which the incumbent is challenged by the alternative politician. There are also d stocks of different nonrenewable natural resources $E = (E_1, \dots, E_d)$ and all the income from selling E accrues directly to the government. The selling prices of the natural resources in the two periods are $p_1 = (p_1^1, \dots, p_1^d)$ in the first period and $p_2 = (p_2^1, \dots, p_2^d)$ in the second one and we assume are determined on world market[2] and taken as given by the country under consideration.

The incumbent must decide how much of the resource to extract in the first period, denoted $e = (e_1, \dots, e_d)$, and consequently how much to left for the second period. We denote $R(e) = (R_1(e_1), \dots, R_d(e_d))$ the remaining resources available in the second period. We assume that every R_i for $i = 1, \dots, d$ is continuous and $R_i', R_i'' < 0$. These assumptions on the derivatives mean that every R_i is a strictly decreasing and concave function of e_i respectively. This models the fact that, obviously, more resources are extracted less remain and that the total amount of resources that can be extracted depends in turn on the extraction rate in a way such that if too much is taken in the first period the total stock over the two periods falls down. Moreover, the sign of the second derivative implies that for every i there exists a value $\overline{e_i} < E_i$ such that $R_i(\overline{e_i}) = 0$. We make the further assumption that $R_i'(0) \leq -1$, and therefore $R_i'(e_i) < -1$ for $e_i > 0$ since $R_i'' < 0$, to ensure that $R_i(e_i) \leq E_i - e_i$, that is $e_i + R_i(e_i) \leq E_i$ and the equality holds if and only if $e_i = 0$. In conclusion for every different resource i the incumbent can extract a quantity $e_i \in [0, \overline{e_i}]$ and we have $R_i(0) = E_i$ and $R_i(\overline{e_i}) = 0$.

[2]We consider an open market, in which all economic actors have an equal opportunity of entry in that market.

To influence the outcome of the election the incumbent politician engages in clientelism and offers to employ voters in the public sector. We denote the reelection probability $\Pi = \Pi(G)$ where $G \in [0,1]$ is the number of voters employed in the public sector. We assume Π continuous and different from 0 and 1, $\Pi' > 0$ and $\Pi'' < 0$. These assumptions imply that Π is strictly increasing and concave function and model a situation in which the reelection probability increases with respect to the number of voters employed but in a way such that if the number of workers employed is too high the reelection probability increases less if the incumbent assumes other workers. We also assume $\Pi(0) = \frac{1}{2}$ so that if the incumbent does not employ any worker he has a fifty-fifty chance to be reelected.

Private sector individuals have a productivity H while productivity of public sector, which is lower, is set to 0. Private sector workers receive a wage equal to their productivity while public sector workers receive a wage W. We make the assumption that $W > H$ so that for a worker is better if he is offered a job in the public sector. On the other hand employing people in the public sector will be socially and economically inefficient because their productivity is lower than productivity of private sector workers.

Resource income can be spent by the incumbent politician or can be redistributed as patronage to increase reelection probability and to influence the outcome of voting. So the incumbent chooses its economic policy, namely $e \in [0, \overline{e_1}] \times \ldots \times [0, \overline{e_d}]$ and $G \in [0,1]$, in order to maximize his own expected income[3]

$$I(e, G; p_1, p_2) := p_1 \cdot e - WG + \Pi(G)(p_2 \cdot R(e) - WG). \tag{2.1}$$

The first term $p_1 e - WG$ in the expression above is the difference between the income from the resource extraction and the outcome to employ workers while the second term $\Pi(G)(p_2 R(e) - WG)$ is the same for the second period yet discounted by a factor that is the reelection probability.

3 – Mathematical tools

To give a more rigorous mathematical fundament to the main results we describe first of all the tools used later in the paper, the most important of which are the Karush-Kuhn-Tucker conditions [7, 10] that give necessary conditions so that $x \in \mathbb{R}^n$ is a maximum point of $f(x)$ in the region

$$E := \{x \in \mathbb{R}^n \,|\varphi_j(x) \le 0,\ j = 1, \ldots, m\}$$

where $f, \varphi_1, \ldots, \varphi_m \colon \mathbb{R}^n \to \mathbb{R}$ are continuously differentiable functions.

[3]In the rest of the article we will omit the scalar product symbol for simplicity of notation.

THEOREM 3.1 (Karush-Kuhn-Tucker conditions). *Let $x \in \mathbb{R}^n$ be a maximum point for f in the region E. If the constraints satisfy some regularity conditions in x then there exists $\lambda \in \mathbb{R}^m$ such that (x, λ) is a solution of the system*

$$\begin{aligned} \nabla f(x) - \sum_{j=1}^{m} \lambda_j \nabla \varphi_j(x) = 0 & \\ \varphi_j(x) \leq 0, \qquad & j = 1, \ldots, m \\ \lambda_j \geq 0, \qquad & j = 1, \ldots, m \\ \lambda_j \varphi_j(x) = 0, \qquad & j = 1, \ldots, m. \end{aligned}$$

REMARK 3.2. The well known first order conditions, that say that in an internal maximum (or minimum) point x it results $\nabla f(x) = 0$, are a particular case of the KKT conditions when we consider a solution (x, λ) of the system such that $\varphi_j(x) < 0$, *i.e.* x is an internal point of E, and consequently $\lambda = 0$.

The second important tool is the implicit function theorem that we give in its general formulation.

THEOREM 3.3 (General implicit function theorem). *We suppose that we are given a set of equations*

$$f_i(x_1, \ldots, x_l, y_1, \ldots, y_n) = 0, \qquad i = 1, \ldots, n$$

where all the functions f_i are continuously differentiable. We will assume that $(p, q) = (p_1, \ldots, p_l, q_1, \ldots, q_n)$ is a point such that all the equations hold and at which we have

$$\det\left(\frac{\partial(f_1, \ldots, f_n)}{\partial(y_1, \ldots, y_n)}\right) = \det\begin{pmatrix} \frac{\partial f_1}{\partial y_1} & \cdots & \frac{\partial f_1}{\partial y_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial y_1} & \cdots & \frac{\partial f_n}{\partial y_n} \end{pmatrix} \neq 0.$$

Then there exist a neighborhood $U \subset \mathbb{R}^l$ of p and a continuously differentiable function $\phi\colon U \to \mathbb{R}^n$ such that $\phi(p) = q$ and

$$f_i(x, \phi(x)) = 0,\ i = 1, \ldots, n$$

holds for $x \in U$ and we have

$$\frac{\partial(\phi_1, \ldots, \phi_n)}{\partial(x_1, \ldots, x_l)}(x) = -\left(\frac{\partial(f_1, \ldots, f_n)}{\partial(y_1, \ldots, y_n)}(x, \phi(x))\right)^{-1} \frac{\partial(f_1, \ldots, f_n)}{\partial(x_1, \ldots, x_l)}(x, \phi(x)). \tag{3.1}$$

A proof of this well known result, along with further generalizations and examples, can be found for example in [9] and for the last part in [6].

REMARK 3.4. The case $n = 1$ is of course the classic implicit function theorem for one dependent variable and one equation.

To start the analytical study of the model we recall that the aim of the incumbent is to maximize his own expected income (2.1) in the set Ω that can be described as

$$\Omega = \{(e, G) \in \mathbb{R}^{d+1} \,|\varphi_j(e, G) \leq 0, \qquad j = 1, \dots, 2(d+1)\}$$

where

$$\begin{array}{lll}
\varphi_j(e,G) = -e_j & \nabla\varphi_j = (0,\dots,0,\overbrace{-1}^{j-th},0,\dots,0) & j = 1,\dots,d \\
\varphi_j(e,G) = e_{j-d} - \overline{e_{j-d}} & \nabla\varphi_j = (0,\dots,0,\overbrace{\quad 1 \quad}^{(j-d)-th},0,\dots,0) & j = d+1,\dots,2d \\
\varphi_{2d+1}(e,G) = -G & \nabla\varphi_{2d+1} = (0,\dots,0,-1) & \\
\varphi_{2(d+1)}(e,G) = G - 1 & \nabla\varphi_{2(d+1)} = (0,\dots,0,1) &
\end{array}$$

and we observe that if the constraints are affine, like in this case, then they automatically satisfy in every point the regularity conditions requested in Theorem 3.1 (see for example [1] for further details).

4 − Case $d = 1$: a single natural resource

The case $d = 1$ model the simplest situation in which there is only one natural resource to extract, so we can omit the index associated with the variable e. The goal of the incumbent is simply to maximize his expected income in the set

$$\Omega = \{(e, G) \in \mathbb{R}^2 \,|\varphi_j(e, G) \leq 0, \ j = 1, \dots, 4\}$$

where

$$\begin{array}{ll}
\varphi_1(e,G) = -e & \nabla\varphi_1 = (-1,0) \\
\varphi_2(e,G) = e - \overline{e} & \nabla\varphi_2 = (1,0) \\
\varphi_3(e,G) = -G & \nabla\varphi_3 = (0,-1) \\
\varphi_4(e,G) = G - 1 & \nabla\varphi_4 = (0,1).
\end{array}$$

Theorem 3.1 says that if (e, G) is a maximum point for (2.1) in the set Ω and if the constraints are qualified in this point[4] then there exists $\lambda = (\lambda_1, \dots, \lambda_4) \in \mathbb{R}^4$ such

[4] This is the case.

that (e, G, λ) is a solution of

$$
\begin{aligned}
& p_1 + \Pi(G) p_2 R'(e) + \lambda_1 - \lambda_2 = 0 \\
& -(1 + \Pi(G))W + \Pi'(G)(p_2 R(e) - WG) + \lambda_3 - \lambda_4 = 0 \\
& 0 \le e \le \overline{e},\ 0 \le G \le 1 \qquad (4.1a) \\
& \lambda_j \ge 0,\ j = 1, \dots, 4 \\
& -\lambda_1 e = \lambda_2 (e - \overline{e}) = -\lambda_3 G = \lambda_4 (G - 1) = 0. \qquad (4.1b)
\end{aligned}
$$

To solve this system we can distinguish several cases by studying more deeply the last line and this is the goal of the next two subsections.

4.1 – Maximum point in the interior

If we suppose that the maximum point is internal, this means that inequalities (4.1a) become $0 < e < \overline{e}$ and $0 < G < 1$, then equations (4.1b) become $\lambda_j = 0$ for all j. By virtue of Remark 3.2 the first two equations of system (4.1) are simply the two first order necessary conditions for this maximization problem

$$F^1(e, G; p_1, p_2) := I_e = p_1 + \Pi(G) p_2 R'(e) = 0 \qquad (4.2a)$$

$$F^2(e, G; p_1, p_2) := I_G = -(1 + \Pi(G))W + \Pi'(G)(p_2 R(e) - WG) = 0. \qquad (4.2b)$$

For simplicity of notation we define

$$D_1 := F^1_e F^2_G - F^1_G F^2_e = p_2(-2W\Pi\Pi' R'' + \Pi\Pi'' R''(p_2 R - WG) - p_2 (\Pi')^2 (R')^2)$$

and we suppose that it is strictly positive[5].

Starting from the two first order conditions (4.2) we can prove immediately an important result whose proof is identical to that in [14].

PROPOSITION 4.1. *Let e_0 be the socially optimal extraction rate in the first period, namely*

$$e_0 := \arg\max_{e \in [0, \overline{e}]} \{p_1 e + p_2 R(e)\}.$$

Then $e > e_0$, that is the resources are inefficiently over-extracted.

PROOF. We recall that e is an internal point, and suppose that e_0 is internal too. We observe that e_0 is simply the value that maximize the total income from selling the resource over the two periods and that satisfies

$$p_1 + p_2 R'(e_0) = 0.$$

[5] This hypothesis is related to the second order sufficient condition for this maximization problem.

Now comparing (4.2a) with the last equality one has, since $\Pi < 1$ and $R' < 0$,

$$R'(e_0) = \Pi(G)R'(e) > R'(e)$$

which implies $e > e_0$ because R' is decreasing since $R'' < 0$. □

The main tool to see how prices of the resource influence extraction and public sector employment, in other words how the maximum point (e, G) changes with respect to the parameters of the model p_1 and p_2, is the general implicit functions theorem applied to equations (4.2a) and (4.2b). We state the first result on this.

Proposition 4.2. *The resource extraction rate is an increasing function with respect to p_1, decreasing with respect to p_2 and decreasing with respect to both p_1 and p_2 also if they vary simultaneously but proportionally.*

Proof. We apply the result in Theorem 3.3 but in a constructive way. To do this we consider

$$\frac{\partial F^2}{\partial G} = -2\Pi' W + \Pi''(p_2 R - WG)$$

which is negative, and in particular nonzero, if we require in addition the quite natural hypothesis that $D_2 := p_2 R - WG > 0$ if $R \neq 0$. Equation (4.2b) implicitly defines a function $G = G(e; p_1, p_2)$. We substitute in (4.2a) and define

$$H(e; p_1, p_2) := F^1(e, G(e; p_1, p_2); p_1, p_2) = 0$$

We now consider

$$\frac{\partial H}{\partial e} = \frac{\partial F^1}{\partial e} + \frac{\partial F^1}{\partial G}\frac{\partial G}{\partial e} = F^1_e + F^1_G\left(-\frac{F^2_e}{F^2_G}\right) = \Pi p_2 R'' + \Pi' p_2 R'\left(-\frac{\Pi' p_2 R'}{-2\Pi' W + \Pi'' D_2}\right).$$

We suppose $H_e \neq 0$ and then $H = 0$ implicitly defines a function $e = e(p_1, p_2)$. Denoting $h(p_1, p_2) := G(e(p_1, p_2); p_1, p_2)$, the starting system is now

$$\begin{aligned} F^1(e(p_1, p_2), h(p_1, p_2); p_1, p_2) &= 0 \\ F^2(e(p_1, p_2), h(p_1, p_2); p_1, p_2) &= 0. \end{aligned}$$

Differentiating both equations by p_1 one has

$$\begin{aligned} F^1_e \frac{\partial e}{\partial p_1} + F^1_G \frac{\partial h}{\partial p_1} &= -F^1_{p_1} \\ F^2_e \frac{\partial e}{\partial p_1} + F^2_G \frac{\partial h}{\partial p_1} &= -F^2_{p_1} \end{aligned}$$

and by Cramer's rule we get

$$\frac{\partial e}{\partial p_1} = \frac{\begin{vmatrix} -F^1_{p_1} & F^1_G \\ -F^2_{p_1} & F^2_G \end{vmatrix}}{\begin{vmatrix} F^1_e & F^1_G \\ F^2_e & F^2_G \end{vmatrix}} = \frac{-F^1_{p_1} F^2_G}{D_1} = \frac{2\Pi' W - \Pi'' D_2}{D_1} > 0.$$

Differentiating now both equations by p_2 one has

$$F^1_e \frac{\partial e}{\partial p_2} + F^1_G \frac{\partial h}{\partial p_2} = -F^1_{p_2}$$
$$F^2_e \frac{\partial e}{\partial p_2} + F^2_G \frac{\partial h}{\partial p_2} = -F^2_{p_2}$$

and by Cramer's rule we get

$$\frac{\partial e}{\partial p_2} = \frac{\begin{vmatrix} -F^1_{p_2} & F^1_G \\ -F^2_{p_2} & F^2_G \end{vmatrix}}{D_1} = \frac{2W\Pi\Pi' R' - \Pi\Pi'' R' D_2 + (\Pi')^2 R R' p_2}{D_1} < 0.$$

To consider a simultaneous but proportional variation of p_1 and p_2 we compute the directional derivative of function $e(p_1, p_2)$ along the direction $\overrightarrow{u} = c(p_1, p_2)$ with c a normalizing constant[6]. One has

$$\frac{de}{d\overrightarrow{u}} = \langle \nabla e, \overrightarrow{u} \rangle = c\left(\frac{\partial e}{\partial p_1} p_1 + \frac{\partial e}{\partial p_2} p_2\right) = c\frac{(\Pi')^2 R R'}{D_1} p_2^2 < 0 \qquad (4.3)$$

after some calculation and using (4.2a) twice. □

We make now some considerations. The extraction rate is an increasing function of p_1 because if, for example, price in the first period increases the resources become more valuable in the present than in the future, so the optimal response is to increase the extraction.

Vice versa the extraction rate is decreasing with respect to p_2. In fact if p_2 increases resources become more valuable in the future than now, so the optimal response is to decrease the extraction to leave more resources available for the second period.

In the last case the situation is a little bit different. If both p_1 and p_2, for example, increase the optimal response is not to leave the extraction rate unaltered (as in Propositions 4.5 and 4.6) but to decrease the extraction (and to increase the

[6]In this case $c = \frac{1}{\sqrt{p_1^2 + p_2^2}} > 0$.

number of voters employed in the public sector, see Proposition 4.3) because this situation makes more valuable to be in power in the future.

In order to present some graphical results, that show the correct behavior but do not respect the constraints which has not been possible to implement in the resolution of the system, we can choose for example the quadratic function $R(e) = -\frac{5}{16}e^2 - e + 1$ that satisfies all the hypothesis and models a situation in which $E = 1$ and $\overline{e} = \frac{4}{5}$, so the incumbent can extract only 80% of the total in the first period leaving nothing for the second one.

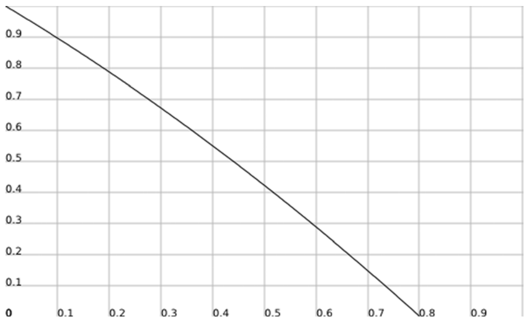

Figure 1 The function $R(e) = -\frac{5}{16}e^2 - e + 1$.

Similarly we can choose for example the function $\Pi(G) = -\frac{3}{10}G^2 + \frac{3}{4}G + \frac{1}{2}$ that satisfies all the hypothesis and at point $G = 1$ it is close to 1.

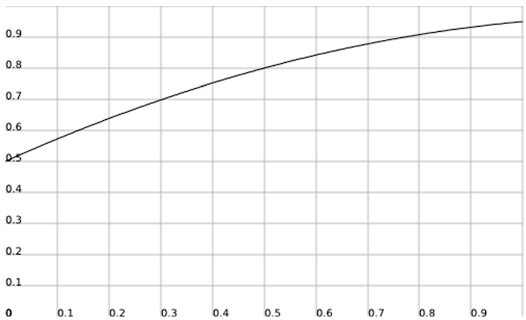

Figure 2 The function $\Pi(G) = -\frac{3}{10}G^2 + \frac{3}{4}G + \frac{1}{2}$.

Now Figure 3 shows an example of function $e(p_1, p_2)$ in the region $1 \leq p_1 \leq 4$ and $6 \leq p_2 \leq 9$ when we choose $W = 1$ and functions R and Π as in Figures 1 and 2.

The second result concerns how the public sector employment is affected by a variation of prices of resource.

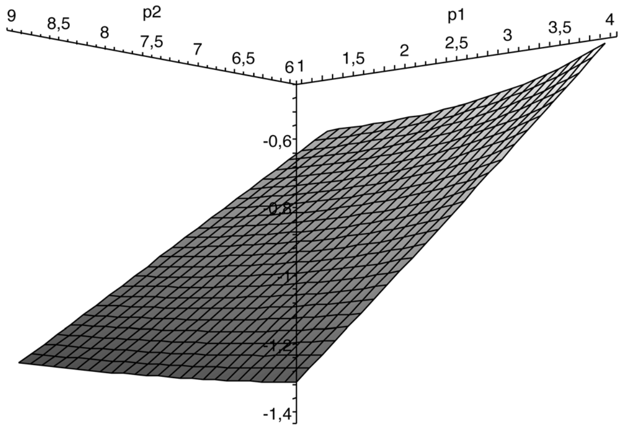

Figure 3 The function $e(p_1, p_2)$ when we choose $W = 1$ and functions $R(e)$ and $\Pi(G)$ as in Figures 1 and 2.

PROPOSITION 4.3. *The rate of voters employed in the public sector is a decreasing function with respect to* p_1*, increasing with respect to* p_2 *and increasing with respect to both* p_1 *and* p_2 *also if they vary simultaneously but proportionally.*

PROOF. We apply again the result in Theorem 3.3 in the same way. To do this we consider

$$\frac{\partial F^1}{\partial e} = \Pi p_2 R''$$

which is negative, and in particular nonzero. Equation (4.2a) implicitly defines a function $e = e(G; p_1, p_2)$. We substitute in (4.2b) and define

$$H(G; p_1, p_2) := F^2(e(G; p_1, p_2), G; p_1, p_2) = 0.$$

We now consider

$$\frac{\partial H}{\partial G} = \frac{\partial F^2}{\partial e}\frac{\partial e}{\partial G} + \frac{\partial F^2}{\partial G}.$$

We suppose $H_G \neq 0$ and then $H = 0$ implicitly defines a function $G = G(p_1, p_2)$. Denoting $h(p_1, p_2) := e(G(p_1, p_2); p_1, p_2)$, the starting system is now

$$F^1(h(p_1, p_2), G(p_1, p_2); p_1, p_2) = 0$$
$$F^2(h(p_1, p_2), G(p_1, p_2); p_1, p_2) = 0.$$

Differentiating both equations by p_1 one has

$$F_e^1 \frac{\partial h}{\partial p_1} + F_G^1 \frac{\partial G}{\partial p_1} = -F_{p_1}^1$$
$$F_e^2 \frac{\partial h}{\partial p_1} + F_G^2 \frac{\partial G}{\partial p_1} = -F_{p_1}^2$$

and by Cramer's rule we get

$$\frac{\partial G}{\partial p_1} = \frac{\begin{vmatrix} F_e^1 & -F_{p_1}^1 \\ F_e^2 & -F_{p_1}^2 \end{vmatrix}}{D_1} = \frac{\Pi' p_2 R'}{D_1} < 0.$$

Differentiating now both equations by p_2 one has

$$F_e^1 \frac{\partial h}{\partial p_2} + F_G^1 \frac{\partial G}{\partial p_2} = -F_{p_2}^1$$
$$F_e^2 \frac{\partial h}{\partial p_2} + F_G^2 \frac{\partial G}{\partial p_2} = -F_{p_2}^2$$

and by Cramer's rule we get

$$\frac{\partial G}{\partial p_2} = \frac{\begin{vmatrix} F_e^1 & -F_{p_2}^1 \\ F_e^2 & -F_{p_2}^2 \end{vmatrix}}{D_1} = \frac{-\Pi\Pi' p_2 R R'' + \Pi\Pi' p_2 (R')^2}{D_1} = \frac{\Pi\Pi' p_2((R')^2 - RR'')}{D_1} > 0.$$

Considering a simultaneous but proportional variation of p_1 and p_2 we have

$$\frac{dG}{d\overrightarrow{u}} = \langle \nabla G, \overrightarrow{u} \rangle = c\left(\frac{\partial G}{\partial p_1} p_1 + \frac{\partial G}{\partial p_2} p_2\right) = c\frac{\Pi\Pi' R R''}{D_1} p_2^2 > 0 \tag{4.4}$$

after some calculation and using (4.2a) again. □

Regarding this case, the situation is exactly the opposite. The rate of voters employed in the public sector in a decreasing function of p_1. In fact if, for example, p_1 increases more resources are extracted in the first period. So the incumbent has less incentive to be in power in the second period and then to influence his reelection probability by employing people in the public sector because there are less resources remaining to exploit.

Vice versa the voters are increasing with respect to p_2 because if, for example, the price in the second period increases then it is more valuable to be in power in this period so the incumbent is forced to increase the number of voters employed to increase his reelection probability.

Even in the latter case the optimal response is, if for example both p_1 and p_2 increase, to increase the number of voters employed in the public sector to increase the reelection probability because it is more valuable (as in Proposition 4.2) to be in power in the period after the elections.

In Figure 4 there is an example of function $G(p_1, p_2)$ in the same region and under the same choices made for the function $e(p_1, p_2)$.

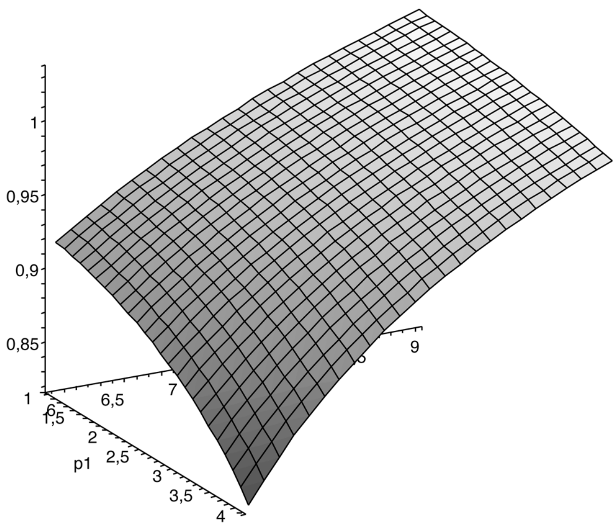

Figure 4 The function $G(p_1, p_2)$ when we choose $W = 1$ and functions $R(e)$ and $\Pi(G)$ as in Figures 1 and 2.

To conclude this subsection we present the most important result of the paper that shows how prices, strength of institutions and total income in the economy of a nation are mutually related.

Proposition 4.4. *The behavior of the total income as function of resource prices is strongly linked to the quality of institutions. In particular it is increasing if the latter are sufficiently strong, conversely it is decreasing if they are not.*

Proof. We consider for simplicity only the dependence with respect to both prices simultaneously, namely we compute the directional derivative with respect to $\overrightarrow{u}$, but the reasoning and the conclusions are the same even if we consider the dependence with respect to a single price. To quantify the total income Y we can use the well known Gross Domestic Product, that measures the market value of all final goods and services produced within a country in a given period of time, that is in this case

$$Y := p_1 e + p_2 R(e) + 2(1 - G)H.$$

We recall that we can see e and G as functions of p_1 and p_2, therefore one can prove that the derivative along the direction $\overrightarrow{u}$ in given by

$$\frac{dY}{d\overrightarrow{u}} = c(p_1 e + p_2 R) + \frac{\partial e}{\partial \overrightarrow{u}}(p_1 + p_2 R') - 2H\frac{\partial G}{\partial \overrightarrow{u}}.$$

Now replacing expressions (4.3) and (4.4) in the previous formula, recalling the definition of D_1 and making a lot of computation one can show that it results

$$\operatorname{sgn}\frac{dY}{d\overrightarrow{u}} = \operatorname{sgn}\left[2R''\left(-eW - \frac{p_2}{p_1}(W-H)R\right) + R''\frac{\Pi''}{\Pi'}(p_2 R - WG)\left(e + \frac{p_2}{p_1}R\right) - \frac{\Pi'}{\Pi}(ep_2(R')^2 - p_2 RR')\right].$$

Here the first and the last addendum, which are respectively positive and negative, are the same contained in the original paper while the second, which is positive, comes from considering a nonzero second derivative of the reelection probability. It is in general not possible to say that this derivative is monotone but if we look at as a function of Π', the argument of sign function is of the form

$$a + b\frac{1}{\Pi'} - c\Pi'$$

with $a, b, c > 0$. The largest zero of this function is

$$\widetilde{\Pi}' = \frac{-a - \sqrt{a^2 + 4bc}}{-2c}$$

which is positive[7]. Moreover it is easy to check that this function is decreasing and continuous in the semiaxis $\Pi' > 0$ (remember that it is an hypothesis of the model). So $\widetilde{\Pi}'$ is a sort of critical value for the derivative of Y because we have $\frac{dY}{d\overrightarrow{u}} > 0$ if $\Pi' < \widetilde{\Pi}'$, that means that if prices increase then the economy of the nation increases and, on the contrary, $\frac{dY}{d\overrightarrow{u}} < 0$ if $\Pi' > \widetilde{\Pi}'$, so to an increase in prices follows a decrease of the total income. The function Π' is in some sense related to the robustness of institutions. A small value means that the incumbent has less chance to influence his reelection probability by employing people in the public sector for different reasons, consequently the institutions are less sensitive to the phenomenon of clientelism. Exactly the contrary happens if Π' is sufficiently big so the statement is proved. □

[7] The other zero is negative.

4.2 – Maximum point on the boundary

We want now to analyze the case in which the maximum point is on the boundary of the set Ω. This set is a rectangle of $\mathbb{R}^2$ and so its boundary Γ is composed by four subsets which are[8] $\Gamma_1 := [0, \overline{e}] \times \{0\}$, $\Gamma_2 := \{\overline{e}\} \times [0, 1]$, $\Gamma_3 := [0, \overline{e}] \times \{1\}$ and $\Gamma_4 := \{0\} \times [0, 1]$. In the next propositions we show how the maximum point on the boundary changes with respect to the parameters and the differences between every piece of the boundary Γ.

In this case we exhibit some graphics that show the evolution of the maximum point in the region $1 \leq p_1, p_2 \leq 2$. We start assuming the maximum point is in the interior of Γ_1.

Proposition 4.5. *If we assume that the maximum point is in the interior of* Γ_1 *so* $G = 0$ *and* $0 < e < \overline{e}$, *then the resource extraction rate is an increasing function with respect to* p_1, *decreasing with respect to* p_2 *and constant with respect to both* p_1 *and* p_2 *if they vary simultaneously but proportionally.*

Proof. By looking at system (4.1), since $G = 0$ and $0 < e < \overline{e}$, equations (4.1b) become $\lambda_1 = \lambda_2 = \lambda_4 = 0$ so the system reduces to

$$\begin{aligned} &F^1(e, \lambda_3; p_1, p_2) := p_1 + \Pi(0) p_2 R'(e) = 0 \\ &F^2(e, \lambda_3; p_1, p_2) := -(1 - \Pi(0))W + \Pi'(0) p_2 R(e) + \lambda_3 = 0. \end{aligned} \tag{4.5}$$

We apply again Theorem 3.3 and consider

$$\frac{\partial F^2}{\partial \lambda_3}$$

which is identically 1, and in particular nonzero. The second equation implicitly defines a function $\lambda_3 = \lambda_3(e; p_1, p_2)$. We substitute in the first one and define

$$H(e; p_1, p_2) := F^1(e, \lambda_3(e; p_1, p_2); p_1, p_2) = 0.$$

We consider now

$$\frac{\partial H}{\partial e} = \frac{\partial F^1}{\partial e} + \frac{\partial F^1}{\partial \lambda_3} \frac{\partial \lambda_3}{\partial e}$$

and suppose $H_e \neq 0$. So $H = 0$ implicitly defines a function $e = e(p_1, p_2)$. Denoting $h(p_1, p_2) := \lambda_3(e(p_1, p_2); p_1, p_2)$ the starting system is now

$$\begin{aligned} &F^1(e(p_1, p_2), h(p_1, p_2); p_1, p_2) = 0 \\ &F^2(e(p_1, p_2), h(p_1, p_2); p_1, p_2) = 0. \end{aligned}$$

[8]We chose to enumerate the subset anticlockwise starting from the bottom rather then in analogy with the φ's functions.

Differentiating both equations by p_1 one has

$$F_e^1 \frac{\partial e}{\partial p_1} + F_{\lambda_3}^1 \frac{\partial h}{\partial p_1} = -F_{p_1}^1$$
$$F_e^2 \frac{\partial e}{\partial p_1} + F_{\lambda_3}^2 \frac{\partial h}{\partial p_1} = -F_{p_1}^2$$

and by Cramer's rule we get

$$\frac{\partial e}{\partial p_1} = \frac{\begin{vmatrix} -F_{p_1}^1 & F_{\lambda_3}^1 \\ -F_{p_1}^2 & F_{\lambda_3}^2 \end{vmatrix}}{\begin{vmatrix} F_e^1 & F_{\lambda_3}^1 \\ F_e^2 & F_{\lambda_3}^2 \end{vmatrix}} = \frac{\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix}}{\begin{vmatrix} \Pi(0)p_2R'' & 0 \\ F_e^2 & 1 \end{vmatrix}} = -\frac{1}{\Pi(0)p_2R''} > 0.$$

Differentiating now both equations by p_2 one has

$$F_e^1 \frac{\partial e}{\partial p_2} + F_{\lambda_3}^1 \frac{\partial h}{\partial p_2} = -F_{p_2}^1$$
$$F_e^2 \frac{\partial e}{\partial p_2} + F_{\lambda_3}^2 \frac{\partial h}{\partial p_2} = -F_{p_2}^2$$

and by Cramer's rule we get

$$\frac{\partial e}{\partial p_2} = \frac{\begin{vmatrix} -F_{p_2}^1 & F_{\lambda_3}^1 \\ -F_{p_2}^2 & F_{\lambda_3}^2 \end{vmatrix}}{\begin{vmatrix} F_e^1 & F_{\lambda_3}^1 \\ F_e^2 & F_{\lambda_3}^2 \end{vmatrix}} = \frac{\begin{vmatrix} -\Pi(0)R' & 0 \\ -F_{p_2}^2 & 1 \end{vmatrix}}{\begin{vmatrix} \Pi(0)p_2R'' & 0 \\ F_e^2 & 1 \end{vmatrix}} = -\frac{R'}{p_2R''} < 0.$$

Lastly considering a simultaneous but proportional variation of p_1 and p_2 we have

$$\frac{de}{d\vec{u}} = \langle \nabla e, \vec{u} \rangle = -c\frac{p_1 + \Pi(0)p_2R'}{\Pi(0)p_2R''} \equiv 0$$

when the last equality turns out from (4.5). □

We observe that since Ω is a rectangle, its boundary is easily parameterizable so we can provide a more direct proof of this and the next results by reducing to the case of maximization of function of only one variable.

PROOF. [Alternative proof of Proposition 4.5] The boundary $\Gamma_1 = [0, \overline{e}] \times \{0\}$ is parameterizable by $e(s) = s$ with $s \in [0, \overline{e}]$ and $G(s) \equiv 0$ so the income to maximize is, with an abuse of notation,

$$I^1(e; p_1, p_2) := I(e, 0; p_1, p_2) = p_1 e + \Pi(0)p_2R(e).$$

If we look for an interior maximum point the first order condition says that

$$\frac{\partial I^1}{\partial e} = p_1 + \Pi(0)p_2 R'(e) = 0. \tag{4.6}$$

We have

$$I^1_{ee} = \Pi(0)p_2 R''(e) < 0$$

which is in particular nonzero[9] then by the implicit function theorem equation (4.6) implicitly defines a function $e = e(p_1, p_2)$ which derivatives are

$$\begin{aligned}\frac{\partial e}{\partial p_1} &= -\frac{I^1_{ep_1}}{I^1_{ee}} = -\frac{1}{\Pi(0)p_2 R''} > 0\\ \frac{\partial e}{\partial p_2} &= -\frac{I^1_{ep_2}}{I^1_{ee}} = -\frac{R'}{p_2 R''} < 0.\end{aligned}$$

□

The next case deals with the maximum point in the interior of Γ_3.

Proposition 4.6. *If we assume that the maximum point is in the interior of* Γ_3 *so* $G = 1$ *and* $0 < e < \overline{e}$*, then the resource extraction rate is an increasing function with respect to* p_1*, decreasing with respect to* p_2 *and constant with respect to both* p_1 *and* p_2 *if they vary simultaneously but proportionally.*

Proof. The proof in essentially the same of the last proposition. The boundary $\Gamma_3 = [0, \overline{e}] \times \{1\}$ is parameterizable by $e(s) = s$ with $s \in [0, \overline{e}]$ and $G(s) \equiv 1$ so the income to maximize is

$$I^3(e; p_1, p_2) := I(e, 1; p_1, p_2) = p_1 e - W + \Pi(1)(p_2 R(e) - W).$$

If we look for an interior maximum point the first order condition says that

$$\frac{\partial I^3}{\partial e} = p_1 + \Pi(1)p_2 R'(e) = 0. \tag{4.7}$$

We have

$$I^3_{ee} = \Pi(1)p_2 R''(e) < 0$$

[9] It is negative so the second order condition for functions of only one variable is fulfilled. The same for all the next two cases.

which is in particular nonzero then by the implicit function theorem equation (4.7) implicitly defines a function $e = e(p_1, p_2)$ whose derivatives are

$$\frac{\partial e}{\partial p_1} = -\frac{I^3_{ep_1}}{I^3_{ee}} = -\frac{1}{\Pi(1)p_2R''} > 0$$
$$\frac{\partial e}{\partial p_2} = -\frac{I^3_{ep_2}}{I^3_{ee}} = -\frac{R'}{p_2R''} < 0$$
$$\frac{de}{d\overrightarrow{u}} = \langle \nabla e, \overrightarrow{u} \rangle = -c\frac{p_1 + \Pi(1)p_2R'}{\Pi(1)p_2R''} \equiv 0$$

when the last equality turns out from (4.7). □

The last two propositions show the same results. If the employment rate is fixed the incumbent can not influence his reelection probability by employing voters in the public sector. This fact is crucial to explain in particular the third result. In fact while the first two results, and related consideration, are identical to those of Proposition 4.2, the latter is different and says that the optimal response to a simultaneous changing of prices is to leave unaltered the extraction rate. This because since the incumbent can not influence his reelection probability the policy is, in some sense, cut off. Consequently, since a proportional increase in both prices keeps unchanged the ratio $\frac{p_1}{p_2}$, the optimal response is what one would obtain by reasoning from a merely economic perspective.

The difference with respect to the case in which the incumbent maximize also over G variable is that an increase in p_2 makes more valuable to be in power after the elections so the politician is forced to employ voters to increase his chances of success.

Figures 5 and 6 show the function $e(p_1, p_2)$ when G is fixed and takes value on the boundaries.

Now in the next two propositions we consider a maximum point on the sets where e is fixed and G is variable, starting from the interior of Γ_4.

Proposition 4.7. *If we assume that the maximum point is in the interior of Γ_4 so $e = 0$, and consequently $R(e) = E$, and $0 < G < 1$, then the rate of voters employed in the public sector is a constant function with respect to p_1, increasing with respect to p_2 and increasing with respect to both p_1 and p_2 also if they vary simultaneously but proportionally.*

Proof. In this case the boundary $\Gamma_4 = \{0\} \times [0, 1]$ is parameterizable by $e(s) \equiv 0$ and $G(s) = s$ with $s \in [0, 1]$ so the income to maximize is

$$I^4(G; p_1, p_2) := I(0, G; p_1, p_2) = -WG + \Pi(G)(p_2E - WG).$$

If we look for an interior maximum point the first order condition says that

$$\frac{\partial I^4}{\partial G} = -(1 + \Pi)W + \Pi'(p_2E - WG) = 0. \quad (4.8)$$

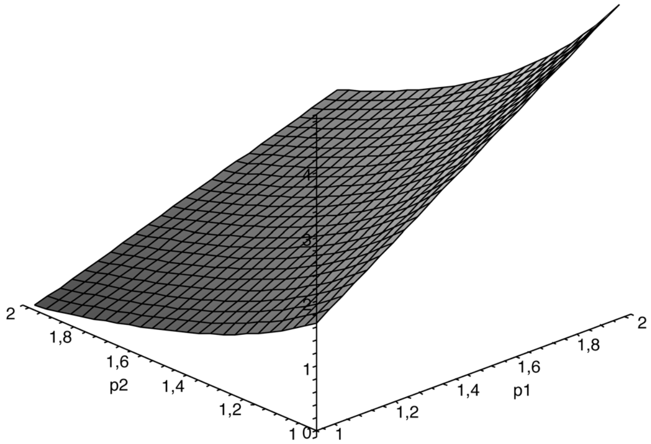

Figure 5 The function $e(p_1, p_2)$ in the case $G = 0$ when we choose $W = 1$ and functions $R(e)$ and $\Pi(G)$ as in Figures 1 and 2.

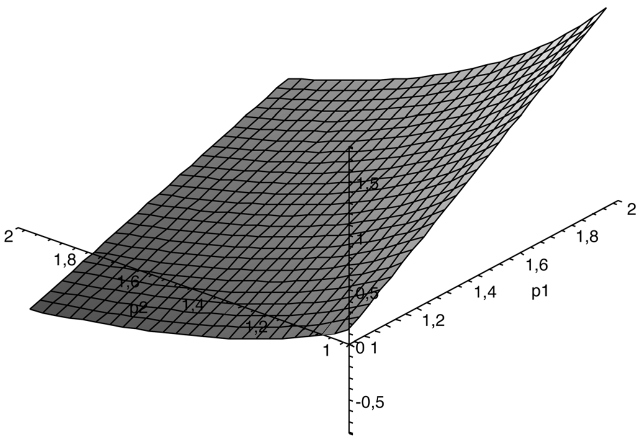

Figure 6 The function $e(p_1, p_2)$ in the case $G = 1$ when we choose $W = 1$ and functions $R(e)$ and $\Pi(G)$ as in Figures 1 and 2.

We have

$$I^4_{GG} = -2\Pi' W + \Pi''(p_2 E - WG) < 0$$

which is in particular nonzero then by the implicit function theorem equation (4.8) implicitly defines a function $G = G(p_1, p_2)$ which derivatives are

$$\begin{aligned} \frac{\partial G}{\partial p_1} &= -\frac{I^4_{Gp_1}}{I^4_{GG}} \equiv 0 \\ \frac{\partial G}{\partial p_2} &= -\frac{I^4_{Gp_2}}{I^4_{GG}} = -\frac{\Pi' E}{-2\Pi' W + \Pi''(p_2 E - WG)} > 0 \\ \frac{dG}{d\overrightarrow{u}} &= \langle \nabla G, \overrightarrow{u} \rangle = c\frac{\partial G}{\partial p_2} p_2 > 0. \end{aligned}$$

□

If we prescribe that in the first period we have no resource extraction, a variation in price p_1 is obviously meaningless. Conversely the number of voters employed by the incumbent is increasing in p_2 and in p_1 and p_2 simultaneously because if, for example, p_2 increases the politician is forced to employ voter in the public sector to guaranteed his victory.

Figure 7 shows the function $G(p_1, p_2)$ when there is no extraction in the first period.

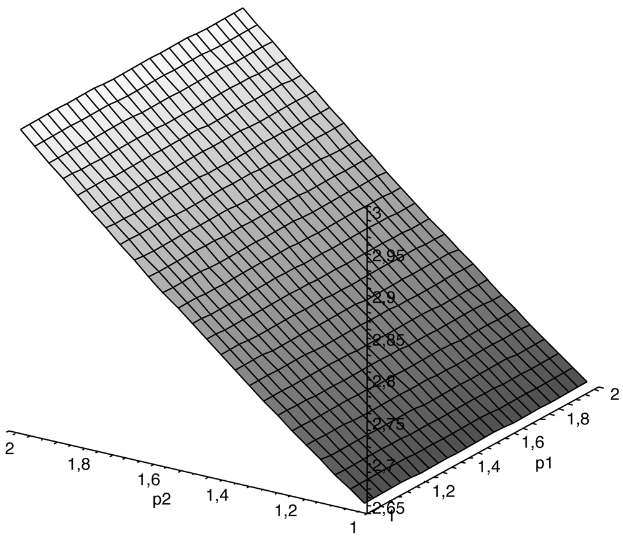

Figure 7 The function $G(p_1, p_2)$ in the case $e = 0$ when we choose $W = 1$ and functions $R(e)$ and $\Pi(G)$ as in Figures 1 and 2.

The last case deals with the maximum point in the interior of Γ_2.

PROPOSITION 4.8. *If we assume that the maximum point is on* Γ_2 *so* $e = \overline{e}$ *and consequently* $R(e) = 0$*, and* $0 < G < 1$*, then the rate of voters employed in the* *public sector is a constant function with respect to* p_1*, constant with respect to* p_2 *and constant with respect to both* p_1 *and* p_2 *also if they vary simultaneously but proportionally.*

PROOF. In this case the boundary $\Gamma_2 = \{\overline{e}\} \times [0,1]$ is parameterizable by $e(s) \equiv \overline{e}$ and $G(s) = s$ with $s \in [0,1]$ so the income to maximize is

$$I^2(G;p_1,p_2) := I(\overline{e}, G; p_1, p_2) = p_1\overline{e} - (1 + \Pi(G))WG.$$

It is easy to see that the maximum with respect to G of I^2 is obtained at $G = 0$ independently of p_1 and p_2, then the optimal rate of voters employed in the public sector does not change. □

We can explain the last result considering that if all the resources are extracted in the first period the incumbent has obviously no interest to be reelected. Therefore the optimal response is to employ nobody in order to cancel hiring costs and this choice does not change if prices p_1 and p_2 vary.

Figure 8 shows the function $G(p_1, p_2)$ when the incumbent exploits all the extractable resource.

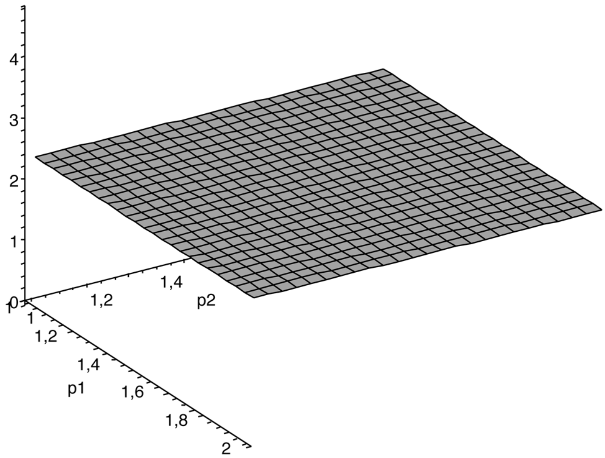

Figure 8 The function $G(p_1, p_2)$ in the case $e = \frac{4}{5}$ when we choose $W = 1$ and functions $R(e)$ and $\Pi(G)$ as in Figures 1 and 2.

REMARK 4.9. *In the proof of the last result we can not follow the previous strategy because the term*

$$I^2_{GG} = -2\Pi' W - \Pi'' WG$$

is not surely different from 0*. Nevertheless if we suppose* $I^2_{GG} \neq 0$ *we can calculate explicitly that all the derivatives are identically* 0*.*

5 − Case $d = 2$: two different natural resources

In this section we analyze the case in which the incumbent has two different resources to exploit and the maximum point is located in the interior of the region under consideration. We will just present the situation from a merely graphical point of view because the explicit expression of the derivatives of functions e_1, e_2 and G with respect to the prices can be obtained directly from (3.1).

We decided to take the same choices made for the case of one single resource, so we set $W = 1$ and for the first resource we choose the same function, obviously now indexed with index 1, $R_1(e_1) = -\frac{5}{16}e_1^2 - e_1 + 1$ which describes the remaining. For the second resource we choose the very similar function $R_2(e_2) = -\frac{10}{9}e_2^2 - e_2 + 1$ that models a situation in which again $E = 1$ and $\overline{e} = \frac{3}{5}$, so the incumbent can extract only 60% of the total in the first period leaving nothing for the second one (see Figure 9).

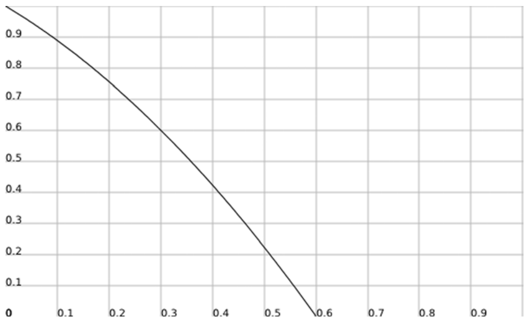

Figure 9 The function $R_2(e_2) = -\frac{10}{9}e_2^2 - e_2 + 1$.

In this situation, with two different natural resources, the are two rates of extraction and four selling prices, two for each period. We are going now to show six graphics that illustrate the trend of the rates of extraction as a function of selling prices. Obviously every rate will be plot as function of only two prices, ranging from 20 to 24 except in a case that will be indicated, so we set the other two prices to an arbitrary value, in this case 28.

Figure 10 shows that the rate of extraction of the first resource e_1 is increasing with respect to its selling price in the first period and decreasing with respect to its price in the second one. The same occurs to the rate of extraction of the second resource e_2 with respect to its selling prices, as shown in Figure 11. The explanation of this behavior is the same provided for Proposition 4.2. We show now the trend of extraction of a resource with respect to the prices of the other one.

Figure 12 shows that the rate of extraction of the first resource as function of prices of the other has the opposite behavior compared to Figure 11. A possible explanation is that if, for example, the price p_1^2 of the second resource in the first

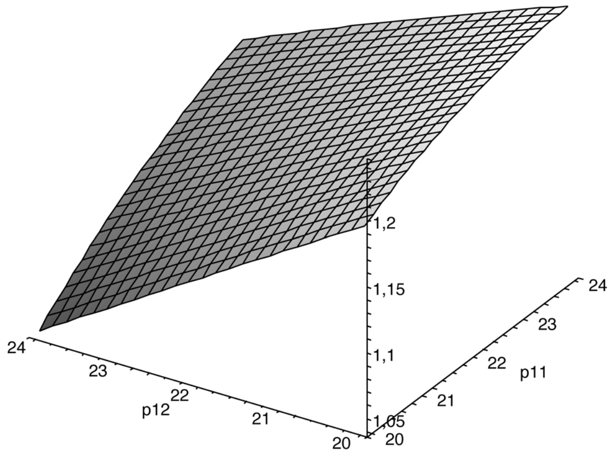

Figure 10 The function $e_1(p_1^1, p_2^1)$ when we choose $W = 1$ and functions $R_1(e_1)$, $R_2(e_2)$ and $\Pi(G)$ as in Figures 1, 9 and 2.

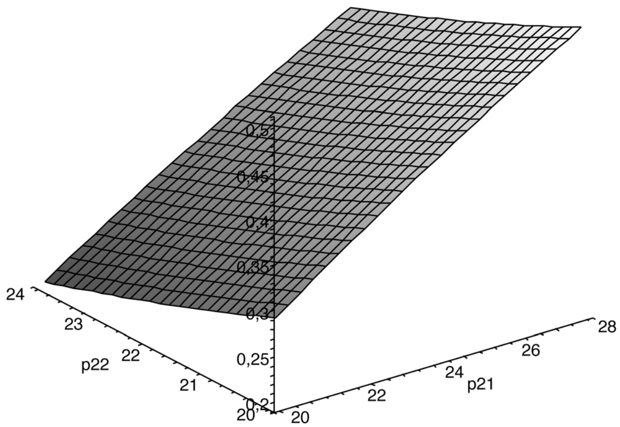

Figure 11 The function $e_2(p_1^2, p_2^2)$ when we choose $W = 1$ and functions $R_1(e_1)$, $R_2(e_2)$ and $\Pi(G)$ as in Figures 1, 9 and 2. In this case we have $20 \leq p_1^2 \leq 28$.

period increases then the incumbent can afford to extract less resource of the first type without compromising the overall gain. Vice versa if price p_2^2 increases then the incumbent has to increase the extraction of the first resource to balance the lower extraction of the second one. The extraction rate of the second resource with respect to the selling prices of the first one has the same behavior, as shown in Figure 13.

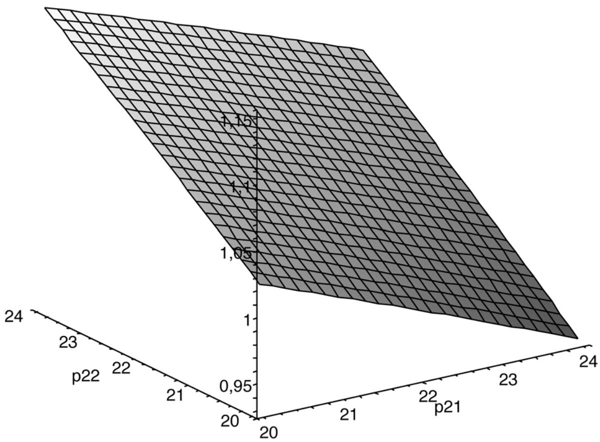

Figure 12 The function $e_1(p_1^2, p_2^2)$ when we choose $W = 1$ and functions $R_1(e_1)$, $R_2(e_2)$ and $\Pi(G)$ as in Figures 1, 9 and 2.

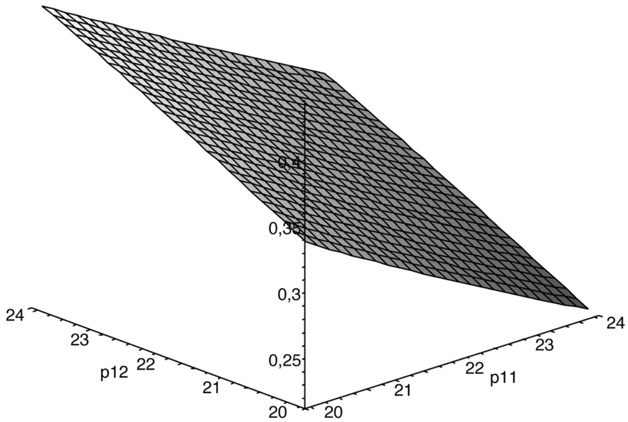

Figure 13 The function $e_2(p_1^1, p_2^1)$ when we choose $W = 1$ and functions $R_1(e_1)$, $R_2(e_2)$ and $\Pi(G)$ as in Figures 1, 9 and 2.

We focus at last on the rate of voters employed in the public sector as function of the selling prices. In this case there is an unexpected phenomenon.

Figure 14 shows that the rate of voters G is increasing with respect to p_1^1 and decreasing with respect to p_2^1, and the same happens if we consider the prices of the second resource, as shown in Figure 15. A possible explanation is that since in any case there is a resource whose extraction rate decreases, the incumbent has interest to be reelected to extract the remaining in the second period, so the optimal response is always to increase the rate of voters employed to guarantee his success.

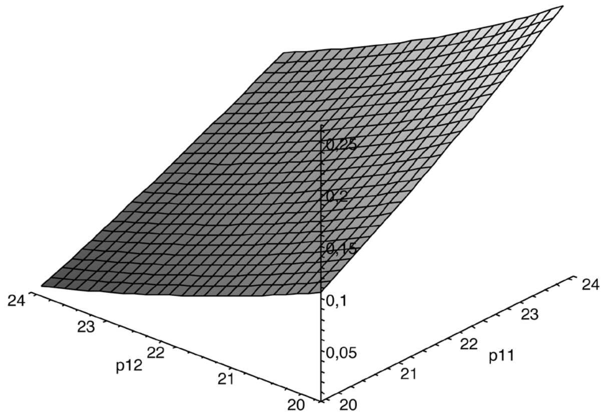

Figure 14 The function $G(p_1^1, p_2^1)$ when we choose $W = 1$ and functions $R_1(e_1)$, $R_2(e_2)$ and $\Pi(G)$ as in Figures 1, 9 and 2.

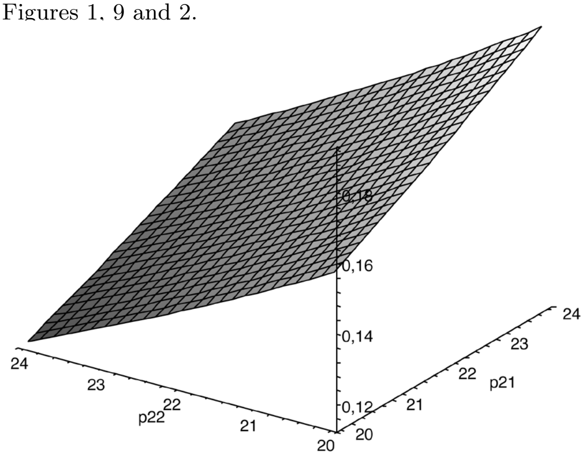

Figure 15 The function $G(p_1^2, p_2^2)$ when we choose $W = 1$ and functions $R_1(e_1)$, $R_2(e_2)$ and $\Pi(G)$ as in Figures 1, 9 and 2.

– Appendix: Some discontinuous behaviors

In Subsection 4.1 we studied, in the case $d = 1$, how the interior maximum point (e, G) changes with respect to the prices p_1 and p_2, and Figures 3 and 4 shows the graphs of e and G as function of prices in the region $1 \leq p_1 \leq 4$ and $6 \leq p_2 \leq 9$. But if we consider a bigger region, for example $1 \leq p_1, p_2 \leq 25$ we can see a quite strange behavior of the two functions.

Figure 16 shows in fact that there are two sorts of shock curves in the plane $p_1 p_2$. The function $e(p_1, p_2)$ remains increasing in p_1 and decreasing in p_2 but at the two lines of shock there are two discontinuities.

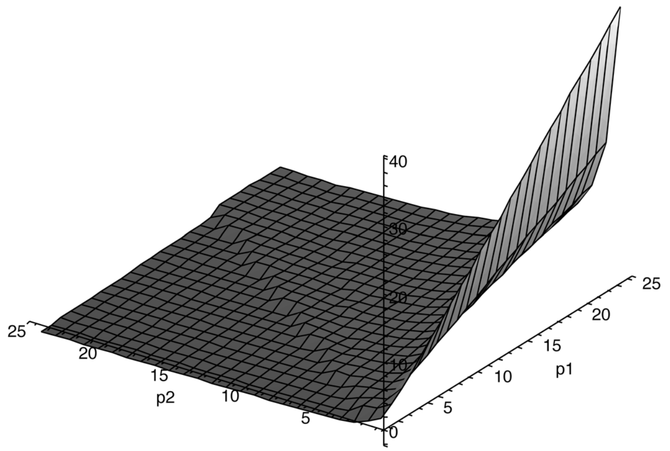

Figure 16 The function $e(p_1, p_2)$ in the region $1 \leq p_1, p_2 \leq 25$ when we choose $W = 1$ and functions $R(e)$ and $\Pi(G)$ as in Figures 1 and 2.

The behavior of $G(p_1, p_2)$ is totally different. In fact if we plot its graph in the same region $1 \leq p_1, p_2 \leq 25$ (see Figure 17) we can see that at the two lines of shock there are again two discontinuities but in this case there is also a change of monotonicity. There is a mathematical explanation of this phenomenon. In fact, as one can see from Figures 18 and 19, exactly in correspondence of the two shock curve there is a change of sign of D_1 and D_2 (again in a discontinuous way) contrary to what is assumed in the model about their positivity.

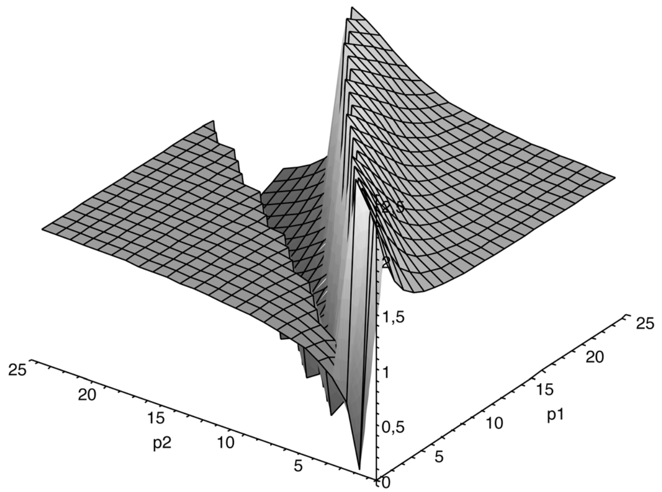

Figure 17 The function $G(p_1, p_2)$ in the region $1 \leq p_1, p_2 \leq 25$ when we choose $W = 1$ and functions $R(e)$ and $\Pi(G)$ as in Figures 1 and 2.

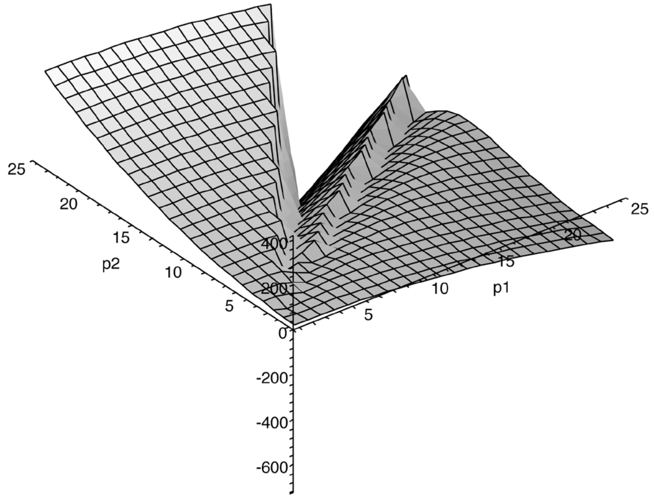

Figure 18 The function $D_1(p_1, p_2)$ in the region $1 \leq p_1, p_2 \leq 25$ evaluated at the maximum point given from functions $e(p_1, p_2)$ in Figure 3 and $G(p_1, p_2)$ in Figure 4 when we choose $W = 1$ and functions $R(e)$ and $\Pi(G)$ as in Figures 1 and 2.

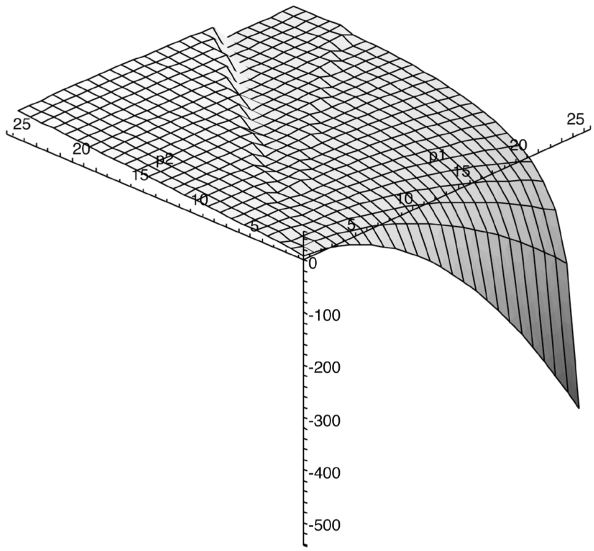

Figure 19 The function $D_2(p_1, p_2)$ in the region $1 \leq p_1, p_2 \leq 25$ evaluated at the maximum point given from functions $e(p_1, p_2)$ in Figure 3 and $G(p_1, p_2)$ in Figure 4 when we choose $W = 1$ and functions $R(e)$ and $\Pi(G)$ as in Figures 1 and 2.

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