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Atomic decomposition for preduals of some Banach spaces

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Dedicated to professor Umberto Mosco on his eightieth birthday

Abstract. *Given a Banach space E with a supremum type norm induced by a sequence $\mathcal{L} = (L_j)$ of linear forms $L_j : X \to \mathbb{R}$ on the Banach space X, we prove that if the unit ball $\mathbb{B}_X$ is $\sigma(X, \mathcal{L})$-compact then E has a predual $E_\star$ with an atomic decomposition. We extend results from [7] where X is assumed a reflexive Banach space.*

1. Introduction

A Banach space X is a *dual space* if there exists a Banach space Y such that $Y^\star \cong X$ (isometric isomorphism) and we say that Y is a *predual* of X.

Among all Banach spaces, *dual* spaces $X^\star$ have some special properties. A test for duality is suggested by the Banach-Alaoglu-Bourbaki Theorem (see for example [3]) which guarantees compactness of the closed unit ball $\mathbb{B}_{X^\star}$ of the dual space $X^\star$ in the weak* topology $\sigma(X^\star, X)$ on $X^\star$. On the other hand, a reflexive Banach space X, which is always dual with unique predual, has the basic property that $\mathbb{B}_X$ is compact in the weak topology $\sigma(X, X^\star)$. A possible approach to see if the non reflexive Banach space E is dual, is to dispose of an auxiliary reflexive Banach space $X \supset E$ with weaker norm then E, "governing" the duality of E [12]. Namely, the Banach space E is supposed to be defined and normed by the fact that $x \in E$ if and only if $x \in X$ and

$$\sup_{L \in \mathcal{L}} |\langle L, x \rangle| < \infty \tag{1.1}$$

where $\mathcal{L}$ is a *collection* of linear forms

$$L : X \to \mathbb{R}$$

belonging to the dual $X^\star$ of the reflexive and separable Banach space X and E is continuously embedded in X.

Following this way , we characterized predual $E_\star$ of E in terms of an *atomic decomposition* of their elements ([7]). Our aim here is to relax the reflexivity assumption on X by assuming that

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(j) *the closed unit ball* $\mathbb{B}_X$ *of the separable Banach space* X *is compact with respect to the weak topology* $\sigma(X, \mathcal{L})$ *on* X *given by the chosen total in* X *and countable set* $\mathcal{L} \subset X^\star$.

A set $\mathcal{L}$ of linear forms $L\colon X \to \mathbb{R}$ is *total* in X if, for $x \in X$, $\langle L, x \rangle = 0$, for any $L \in \mathcal{L}$, implies $x = 0$.

This classically implies that X is dual of the closure in $X^\star$ of span $\mathcal{L}$ (see [5]) and so we reconcile this framework with the classical characterization of dual separable Banach spaces (see [11] and Section 4 with example $X = L \log L$). We obtain in Theorem 2.1 the simple description of $E_\star$ in terms of ℓ^1, in Theorem 3.1 the atomic decomposition of elements of $E_\star$ and in Theorem 3.2 the imbedding $\mathcal{L} \subset E_\star$.

Our approach is natural once we observe that any predual $\mathcal{P}$ of a Banach space X should be viewed as a suitable subspace of the dual space $X^\star$, as from the following (see [14]).

Proposition 1.1. *Let* $X, \mathcal{P}$ *be Banach spaces and* X *separable. If* $\mathcal{P}$ *is a predual of* X *with isometric isomorphism*

$$\Phi\colon X \to \mathcal{P}^\star$$

then, there is a linear isometric injection

$$I\colon \mathcal{P} \to X^\star$$

such that

(i) $I(\mathcal{P})$ *separate points of* X *(is total on* X*);*

(ii) $\mathcal{B}_X$ *is* $\sigma(X, I(\mathcal{P}))$*-compact.*

Proof. Let us consider the adjoint $\Phi^\star$ of Φ, that is for $y \in \mathcal{P}^{\star\star}$ and $x \in X^\star$ we have

$$\langle \Phi^\star(y), x \rangle_{X^\star, X} = \langle \Phi(x), y \rangle_{\mathcal{P}^\star, \mathcal{P}^{\star\star}}$$

with

$$\Phi^\star\colon \mathcal{P}^{\star\star} \to X^\star$$

an isomorphim. Let us denote by

$$J\colon \mathcal{P} \to \mathcal{P}^{\star\star}$$

the canonical embedding of $\mathcal{P}$ into its second dual $\mathcal{P}^{\star\star}$, and $I\colon \mathcal{P} \to X^\star$ the linear isometry given by composition:

$$I = \Phi^\star \circ J\colon \mathcal{P} \to X^\star$$

Our aim is to prove that the family $I(\mathcal{P})$ is total, that is , for $x \in X \setminus \{0\}$ there exists $\varphi \in I(\mathcal{P})$ such that $\langle \varphi, x \rangle \neq 0$. So let us fix $x \in X \setminus \{0\}$ and notice that

$$\begin{aligned} 0 < ||x||_X &= ||\Phi(x)||_{\mathcal{P}^\star} \\ &= \sup_{y \in \mathbb{B}_\mathcal{P}} |\langle \Phi(x), y \rangle_{\mathcal{P}^\star, \mathcal{P}}| \\ &= \sup_{z \in \mathbb{B}_\mathcal{P}} |\langle \Phi(x), J(z) \rangle_{\mathcal{P}^\star, \mathcal{P}^{\star\star}}| \end{aligned}$$

where in the last equality we use the Goldstine Theorem (see [3]). So we get

$$0 < ||x||_X = \sup_{z\in\mathbb{B}_\mathcal{P}} |\langle x, \Phi^\star \circ J(z)\rangle_{\mathcal{P}^\star,\mathcal{P}^{\star\star}}| = \sup_{z\in\mathbb{B}_\mathcal{P}} |\langle x, I(z)\rangle_{\mathcal{P}^\star,\mathcal{P}^{\star\star}}|\,.$$

Hence there exists $z \in \mathbb{B}_\mathcal{P}$ such that $\varphi = I(z) \in I(\mathcal{P})$ and $\langle x, \varphi\rangle \neq 0$.

Let us now show that $\mathbb{B}_X$ is $\sigma(X, I(\mathcal{P}))$-compact. If we know $X \equiv \mathcal{P}^\star$ is separable then also $\mathcal{P}$ is separable (see [3]). Moreover if $\varphi_j \in X$ is a bounded sequence, say $||\varphi_j|| \leq 1$, then there exists a subsequence (φ_{j_k}) that converges in the weak* topology $\sigma(X,\mathcal{P})$ on X. We complete the proof observing that I is an isometry. □

2. The space E generated by a general Banach space X

Suppose that a Banach space E is defined and *normed* by the condition

$$x \in E \text{ if and only if } \sup_{L\in\mathcal{L}} |\langle L, x\rangle| < \infty \tag{2.1}$$

where $\mathcal{L}$ is a family of linear forms $L \in X^*$, the dual of a given Banach space X, and $E \hookrightarrow X$, i.e E is continuously embedded in X, with the norm

$$\|x\|_E = \sup_{L\in\mathcal{L}} |\langle L, x\rangle|. \tag{2.2}$$

If X is reflexive, then in [7] it was shown that

$$E \textit{ has a predual } E_* \tag{2.3}$$

and

$$\textit{the elements of } E_* \textit{ enjoy an atomic decomposition}, \tag{2.4}$$

(see Section 3).

Our aim here is to replace the assumption that X is reflexive with the more general condition

$$\mathbb{B}_X \text{ is a } \sigma(X,\mathcal{L}) - \text{compact} \tag{2.5}$$

with respect to weak topology in X generated by $\mathcal{L}$, where $\mathbb{B}_X$ denotes the closed unit ball of X.

As an example, we show (see Section 4) that the Zygmund space

$$X = L\log L(Q_0)$$

of $x \in L^1(Q_0)$, Q_0 the unit cube of $\mathbb{R}^n$, such that

$$\|x\|_{L\log L} = \int_{Q_0} |x(t)| \log\left(e + \frac{|x(t)|}{\int_{Q_0}|x|}\right) dt < \infty \tag{2.6}$$

is a non reflexive Banach space that satisfies (2.5) for suitable $\mathcal{L}$ (see [9]).

Our first result is the following

Theorem 2.1. *Let X be a Banach space and $\mathcal{L} \subset X^*$ a countable collection of linear functionals $L_j : X \to \mathbb{R}$ verifying* (2.5)*.*

Let $E \subset X$ be a Banach space defined by

$$E = \{x \in X \; : \; \sup_j |\langle L_j, x\rangle| < \infty\} \tag{2.7}$$

and normed by (2.2) *so that there exists $c > 0$:*

$$||x||_X \leq c||x||_E \quad \forall x \in E \tag{2.8}$$

and E is dense in X. Then E has an isometric predual

$$E_\star = \frac{\ell^1}{V(E)^\perp \cap \ell^1} \tag{2.9}$$

where

$$V : E \to \ell^\infty \qquad Vx(j) = L_j x, \ \textit{for} \ \ x \in E. \tag{2.10}$$

Remark 2.2. Notice that the family $\mathcal{L}$ separates points of E. Actually if $x, y \in E$ and

$$L_j x = L_j y \quad \text{for any } j$$

then $\|x - y\|_E = 0$ and so $x = y$.

If W is a subset of a Banach space Z, then its annihilator is

$$W^\perp = \{z^\star \in Z^\star : \langle w, z^\star\rangle = 0 \text{ for all } w \in W\}$$

If U is a subset of $Z^\star$, then

$${}^\perp U = \{z \in Z \; : \; \langle z, u\rangle = 0 \text{ for all } u \in U\}$$

Proof of Theorem 2.1. To prove that E is a dual space, it is sufficient to verify that $V(E)$ is weak* closed in ℓ^∞ and so, by Krein-Smulian Theorem, it is enough to check that

$$V(E) \cap \mathbb{B}_{\ell^\infty} \ \text{ is weak-* closed} \tag{2.11}$$

where $\mathbb{B}_{\ell^\infty}$ is the closed unit ball in ℓ^∞ centered at zero:

$$\mathbb{B}_{\ell^\infty} = \{\underline{y} = (y_j)_{j\in\mathbb{N}} \in \ell^\infty \; : \; \sup_{j\in\mathbb{N}} |y_j| \leq 1\}.$$

Notice that, for $x_0 \in E$ and $L_j \in X^\star$, we have

$$(L_j x_0)_{j\in\mathbb{N}} \in V(E) \cap \mathbb{B}_{\ell^\infty}$$

if and only if $x_0 \in \mathbb{B}_E$, because:

$$\|x_0\|_E = \sup_{j\in\mathbb{N}} |\langle L_j, x_0\rangle| \leq 1.$$

Suppose $(x_\alpha)_\alpha$ is a net in $\mathbb{B}_E$ such that

$$(L_j x_\alpha)_{j\in\mathbb{N}} \to (y_j^{\star\star})_{j\in\mathbb{N}} \in \ell^\infty \tag{2.12}$$

with respect to the $\sigma(\ell^\infty, \ell^1)$ topology. We have to prove that there exists $x \in E$ such that

$$(y_j^{\star\star})_{j\in\mathbb{N}} = (\langle L_j, x\rangle)_{j\in\mathbb{N}} \tag{2.13}$$

By (2.8), $\|\frac{x_\alpha}{c}\|_X \leq 1$ and $\mathbb{B}_X$ is supposed $\sigma(X, \mathcal{L})$-compact, then there exists a subnet $(x_{\alpha'})$ of $(x_\alpha)_\alpha$ such that

$$x_{\alpha'} \rightharpoonup x \in X \ \text{ in } \sigma(X, \mathcal{L})$$

for some $x \in X$; that is

$$(\langle L_j, x_{\alpha'}\rangle)_{j\in\mathbb{N}} \to (\langle L_j, x\rangle)_{j\in\mathbb{N}}$$

By (2.12) for any $j \in \mathbb{N}$ we deduce the convergence of the real net

$$\langle L_j, x_{\alpha'}\rangle \to \langle L_j, x\rangle = y_j^{\star\star} \qquad \forall j \in \mathbb{N}$$

hence (2.13) holds true. Using the condition

$$\sup_j |\langle L_j, x_\alpha\rangle| = \|x_\alpha\|_E \leq 1$$

we obtain

$$\|x\|_E = \sup_j |\langle L_j, x\rangle| \leq 1$$

hence $V(x)$ belongs to the unit ball $\mathbb{B}_{\ell^\infty}$. □

Remark 2.3. For $x \in E$ the norm of x as a functional on $E_\star$ is equal to its norm as an element of E, that is

$$||x||_E = \sup_{||L||\leq 1,\, L\in E_\star} |\langle L, x\rangle|.$$

3. Atomic decomposition of predual

In this section we will complement Theorem 2.1 with description of an atomic decomposition for elements of the predual $E_\star$ of E.

Theorem 3.1. *Under the same assumptions of Theorem 2.1, every $\varphi \in E_\star$ is of the form*

$$\varphi = \sum_{j=1}^{\infty} \lambda_j g_j \tag{3.1}$$

with $(\lambda_j) \in \ell^1$ and g_j are elements of $E_\star$ with $\|g_j\|_{E_\star} = 1$. Moreover

$$||\varphi||_{E_\star} \cong \inf \sum |\lambda_j| \tag{3.2}$$

where inf *is taken over all representation of φ.*

Proof. By Theorem 2.1 for $\varphi \in E_\star$ and $\delta > 0$, there exists $(y_j^\star) \in \ell^1$ such that

$$\sum_{j=1}^{\infty} |y_j^\star| < ||\varphi||_{E_\star} + \delta$$

$$\varphi = \sum_{j=1}^{\infty} L_j^\star y_j^\star \tag{3.3}$$

where $L_j^\star \in \mathcal{L}(\mathbb{R}, E_\star)$ is the adjoint of $L_j \in E^\star = \mathcal{L}(E, \mathbb{R})$. Recall that finite sums $e_\star = \sum_{j=1}^{k} L_j^\star y_j^\star$ belong to $E_\star$ and the action of elements of $x \in E$ on $e_\star$ is identical

$$\langle x, (y_j^\star)\rangle = e_\star(x)$$

(see [7]). Recall also that

$$||L_j||_{E^\star} = ||L_j||_{\mathcal{L}(E,\mathbb{R})} = ||L_j^\star||_{\mathcal{L}(\mathbb{R},E_\star)}\,.$$

Since

$$||x||_E = \sup_j |\langle L_j, x\rangle| < \infty$$

by Banach-Steinhaus Theorem there exists a constant c_0 such that for any $j \in \mathbb{N}$.

$$||L_j^\star||_{\mathcal{L}(\mathbb{R},E_\star)} = ||L_j||_{E^\star} \le c_0\,.$$

Set for $j \in \mathbb{N}$

$$\lambda_j = ||L_j^\star y_j^\star||_{E_\star}$$

and

$$g_j = \frac{L_j^\star y_j^\star}{||L_j^\star y_j^\star||_{E_\star}}$$

to obtain

$$\varphi = \sum_{j=1}^{\infty} \lambda_j g_j$$

Since $||g_j||_{E_\star} = 1$ we deduce

$$||\varphi||_{E_\star} \le \sum_j \lambda_j$$

Moreover

$$\lambda_j = ||L_j^\star y_j^\star||_{E_\star} \le ||L_j^\star||_{\mathcal{L}(\mathbb{R},E_\star)}||y_j^\star|| \le c_0||y_j^\star||$$

□

Under the same assumptions of Theorem 2.1, we have the following

Theorem 3.2. *$\mathcal{L}$ is continuously contained in $E_\star$*

$$\mathcal{L} \hookrightarrow E_\star \tag{3.4}$$

i.e. for any $x^\star \in \mathcal{L}$ there is a sequence $(y_j^\star) \in \ell^1$ such that for a $C > 0$

$$\|(y_j^\star)\|_{\ell^1} \leq C\|x^\star\|_{X^\star} \tag{3.5}$$

and

$$\langle x^\star, x\rangle = \sum_{j=1}^{\infty} \langle L_j^\star y_j^\star, x\rangle, \qquad \forall x \in E. \tag{3.6}$$

Proof. First of all we have the continuous injection:

$$X^\star \hookrightarrow E^\star$$

due to the fact that $E \hookrightarrow X$.

To see that, actually, for $x^\star \in \mathcal{L}$ we have $x^\star \in E_\star$, it is sufficient to verify that any $x^\star \in \mathcal{L}$ is continuous on E with respect to $\sigma(E, \mathcal{L})$ topology.

To this aim it is sufficient to prove that

$$E \cap \ker x^\star$$

is $\sigma(E, \mathcal{L})$-closed in E or, by virtue of Krein-Smulian theorem that if we fix $x^\star \in \mathcal{L}$ then

$$\mathbb{B}_E \cap \ker x^\star \text{ is } \sigma(E, \mathcal{L}) - \text{closed in } E. \tag{3.7}$$

Let (x_α) be a net in $\mathbb{B}_E \cap \ker x^\star$, so

$$\langle x^\star, x_\alpha\rangle = 0 \qquad \forall \alpha \tag{3.8}$$

and assume

$$x_\alpha \to x \in E \qquad \text{in } \sigma(E, \mathcal{L}).$$

Since $E \hookrightarrow X$ and $\|x_\alpha\|_X \leq c\|x_\alpha\|_E \leq c$, using $\sigma(X, \mathcal{L})$–compactness of $\{x \in X : \|x\|_X \leq c\}$, there exists a subnet $(x_{\alpha'})$ such that

$$x_{\alpha'} \to x_0 \in X \quad \text{in } \sigma(X, \mathcal{L})$$

that is $\forall j \in \mathbb{N}$

$$\langle L_j, x_{\alpha'}\rangle \to \langle L_j, x_0\rangle \qquad \text{in } \mathbb{R}.$$

Since by definition of the norm in E

$$\sup_j |\langle L_j, x_\alpha\rangle| = \|x_\alpha\|_E \leq 1$$

we have also

$$\sup_j |\langle L_j, x_0\rangle| = \|x_0\|_E \leq 1.$$

Then $x_0 \in \mathbb{B}_E \cap \ker x^\star$.

Furthermore for $j \in \mathbb{N}$ and $y^\star \in \mathbb{R}$

$$y^\star L_j(x_0) = \lim_{\alpha'} y^\star L_j(x_{\alpha'}) = y^\star L_j(x)$$

hence $L_j(x_0) = L_j(x)$ $\forall j$ and then $x_0 = x$ by the fact that $\| \, \|_E$ is a norm. □

4. An example

The dual space of $X = L\log L$ (see (2.6)) is the space $EXP(Q_0)$ of exponentially integrable functions, that is of functions $\varphi \in L^1(Q_0)$ defined by the condition (see [1])

$$\exists\lambda > 0 \text{ such that } \int_{Q_0} e^{\frac{|\varphi(x)|}{\lambda}}\,dx < \infty \tag{4.1}$$

(see [13, Remark 4.13.8]). Let us indicate by $\exp(Q_0)$ the closure of $L^\infty(Q_0)$ in EXP with respect to its norm

$$\|\varphi\|_{\mathrm{EXP}} = \inf\left\{\lambda > 0 : \int_{Q_0} e^{\frac{|\varphi(x)|}{\lambda}}\,dx \le 2\right\} \tag{4.2}$$

and recall that $L\log L$ is the dual of the so called little- EXP space *exp* (see [13, Remark 4.13.8]), a separable space ([13, Theorem 4.12.11]) which can be identified as the set of $\varphi \in$ EXP such that

$$\int_{Q_0} e^{\frac{|\varphi(x)|}{\mu}}\,dx < \infty \qquad \forall\mu > 0 \tag{4.3}$$

(see Section 2). Of course $L\log L$ is separable ([13, Theorem 4.11.1]) and not reflexive ([13, Theorem 4.13.9])and we will choose

$$\mathcal{L} \subset \exp(Q_0).$$

Actually, the unit ball $\mathbb{B}_{L\log L}$ is not weakly compact, but it is weakly $\star$ compact:

Lemma 4.1. *The unit ball $\mathbb{B}_{L\log L}$ is $\sigma(L\log L, exp)$ compact.*

Proof. By de la Vallee Poussin criterion the assumption $u_j \in \mathbb{B}_{L\log L}$, that is

$$\|u_j\|_{L\log L} \le 1 \qquad \forall j \tag{4.4}$$

implies that the sequence (u_j) has a subsequence u_{j_k} weakly converging to $u \in L\log L$ in $\sigma(L^1, L^\infty)$.

Let us verify that

$$\lim_k \int_{Q_0} (u_{j_k} - u)\varphi\,dx = 0 \qquad \forall\varphi \in \exp. \tag{4.5}$$

Fix $\varphi \in \exp$ and denote by $\varphi_h \in L^\infty$ a sequence such that

$$\lim_h \|\varphi_h - \varphi\|_{\mathrm{EXP}} = 0. \tag{4.6}$$

Then for $h, j \in \mathbb{N}$ we have

$$\begin{aligned}\left|\int_{Q_0} \varphi u_j\,dx - \int_{Q_0} \varphi u\,dx\right| &= \left|\int_{Q_0} (\varphi - \varphi_h)u_j\,dx + \int_{Q_0} \varphi_h u_j\,dx - \int_{Q_0} \varphi_h u\,dx\right.\\ &\qquad \left. + \int_{Q_0} \varphi_h u\,dx - \int_{Q_0} \varphi u\,dx\right|\\ &\le \|u_j\|_{L\log L}\|\varphi - \varphi_h\|_{\mathrm{EXP}} + \|u\|_{L\log L}\|\varphi_h - \varphi\|_{\mathrm{EXP}}\\ &\qquad + \left|\int_{Q_0} (\varphi_h u_j - \varphi_h u)\,dx\right|\end{aligned}$$

Fix h and let $j \to \infty$, then for a $M > 0$ we have

$$\Big| \int_{Q_0} \varphi u_j \, dx - \int_{Q_0} \varphi u \, dx \Big| \leq M \|\varphi_h - \varphi\|_{\mathrm{EXP}}$$

to the limit as $h \to 0$, by (4.6), we obtain (4.5). $\square$

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