Generalized Hough transform for segmentation of X-ray Computed Tomography images

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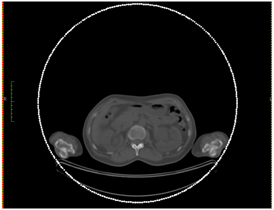
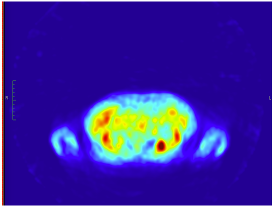
- The problem: segmentation in medical imaging
- The methods: active contour and Hough Transform
- Caveat(s): theoretical and practical aspects

CT and PET

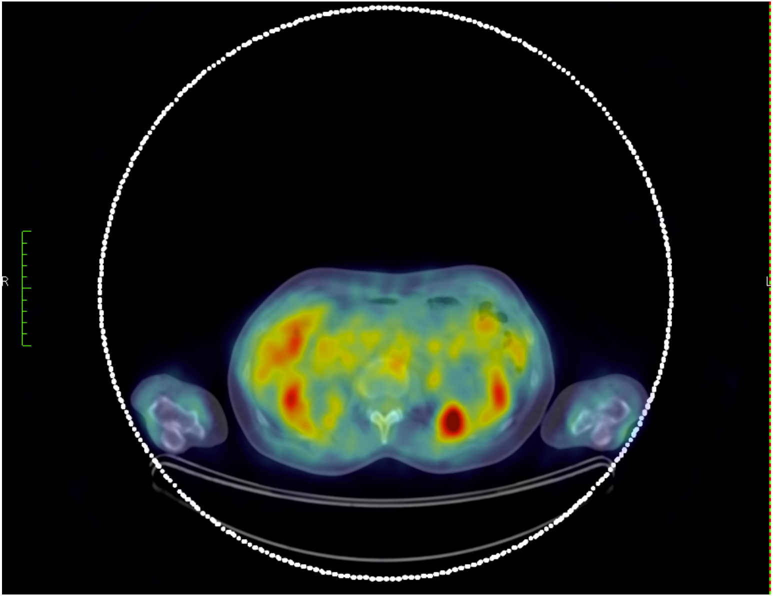
X-ray Computed Tomography (CT):
Anatomical information



Positron Emission Tomography (PET):
Metabolic information



CT/PET



Pattern recognition in medical imaging

Recognition of specific districts on high resolution CT images and coregistration on (low resolution) PET images to extract metabolic activity

1. recognition on CT images;
2. binary masks in which all pixels inside the profiles are set equal to one and all the other pixels are set equal to zero;
3. pixel-by-pixel application to PET images

We perform the recognition task using:

Active contour: detection of the compact bone tissue in the CT axial image by applying an iterative optimization scheme based on level set

or

Extension of Hough transform to algebraic curves

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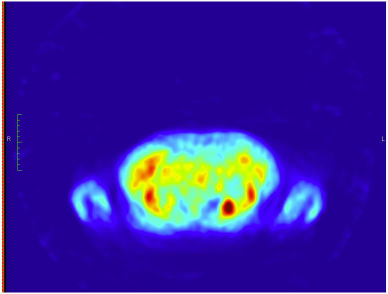
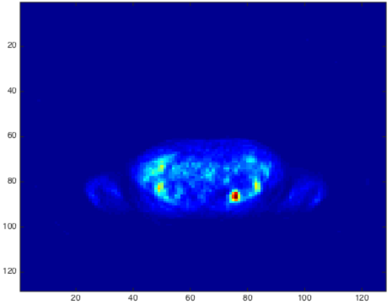
Native format vs graphics file format





Tomato sauce using pictures of tomatoes

The resolution is poor:
PET images 5.33×5.33 mm, CT images 1.33×1.33 mm

Active contour (T.F. Chan and L.A.Vese *IEEE Transactions on Image Processing*, 2001) is represented by the zero level set of a function $\phi : \mathbb{R}^2 \to \mathbb{R}$.
The functional we minimized is given by:

$$F(\phi) = \left(\int_\Omega \delta(\phi)|\nabla\phi|\, d\mu\right)^2 + \\ \int_\Omega (I - m_1(\phi))^2 H(\phi)\, d\mu + \int_\Omega (I - m_2(\phi))^2 (1 - H(\phi))\, d\mu$$

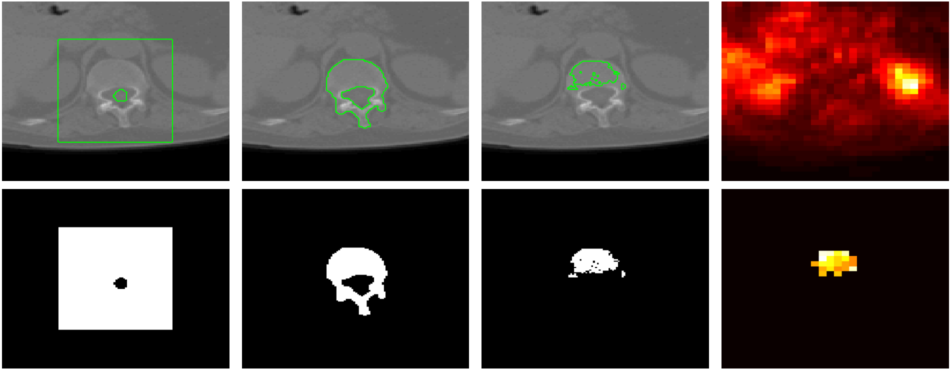
where I is the image, H is the Haviside function, δ is the Dirac delta function and

$$m_1(\phi) = \begin{cases} \frac{\int_\Omega I H(\phi)\, d\mu}{\int_\Omega H(\phi)\, d\mu} & if \int_\Omega H(\phi)\, d\mu \neq 0 \\ 0 & elsewhere \end{cases},$$

$$m_2(\phi) = \begin{cases} \frac{\int_\Omega I(1-H(\phi))\, d\mu}{\int_\Omega 1-H(\phi)\, d\mu} & if \int_\Omega 1 - H(\phi)\, d\mu \neq 0 \\ 0 & elsewhere. \end{cases}$$

Active contours for Advanced Chronic Lymphocytic Leukemia

F. Fiz, C. Marini, C. Campi, et al., *Blood*, 2015



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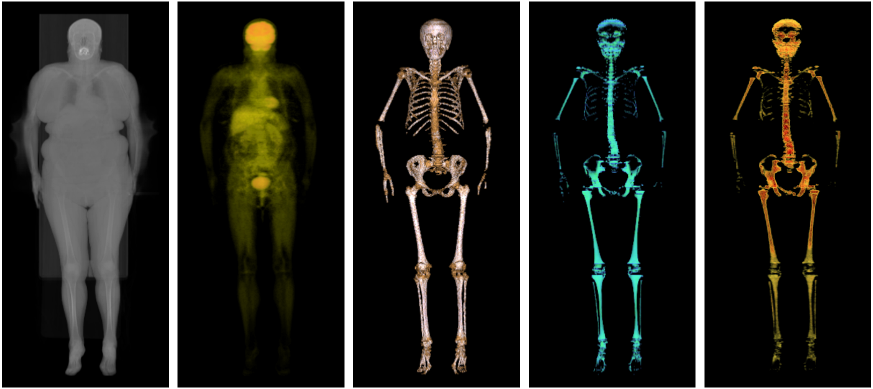
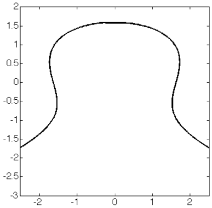


(a) (b) (c) (d) (e)

CT images (a), PET images (b), CT after segmentation (c) , processed CT after thresholding (d), and PET images after coregistration with segmented CT images (e)

Bone profile

$$\mathcal{C}_{a,b} : Y^2 - X^3 - aX - b = 0$$



The roadmap

From contour to equation

Most skeleton districts are characterized by peculiar geometrical properties that can be described by means of specific algebraic families of curves

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Hough Transform (HT)

- The Hough transform (P.V.C. Hough, 1962) is a classical pattern recognition algorithm used to detect straight lines, circles and ellipses in images
- algebraic geometry arguments generalize this approach to special classes of curves
- an effective computational tool for applications in hematology and oncology

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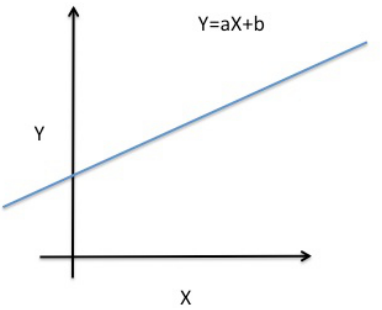
Hough Transform of a straight line



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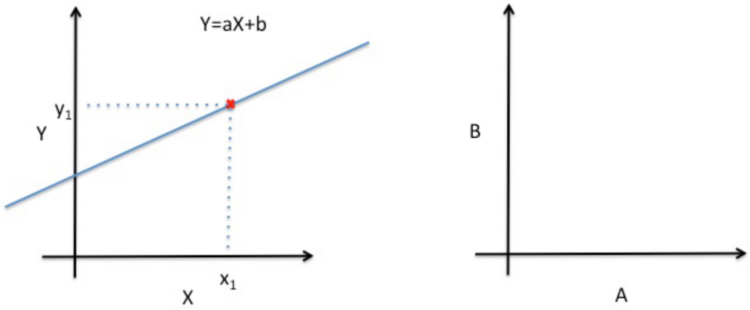

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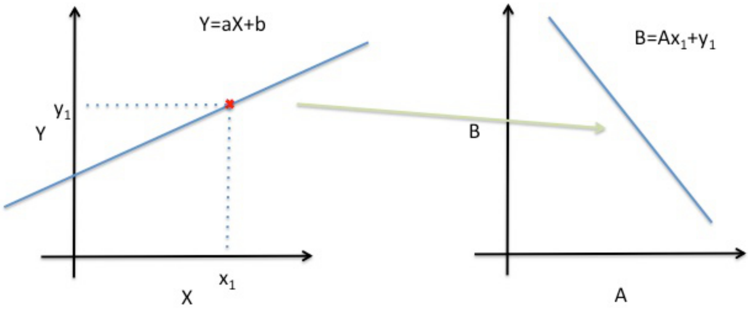

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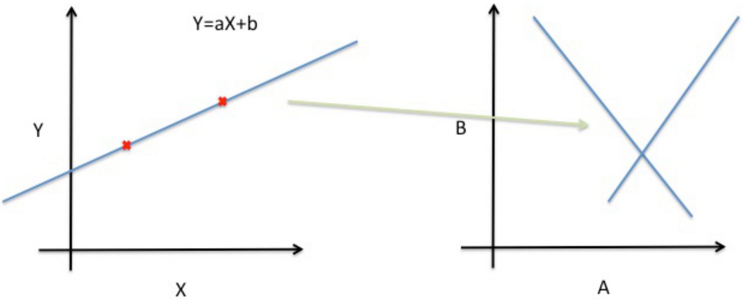

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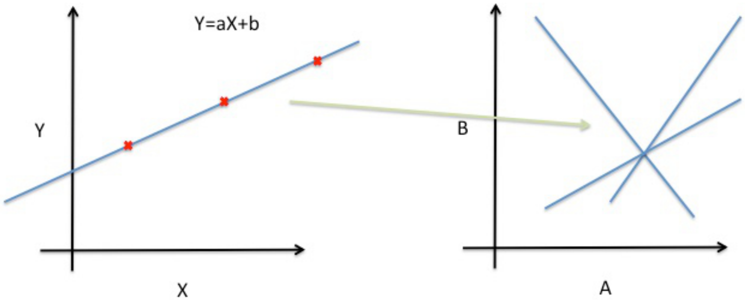

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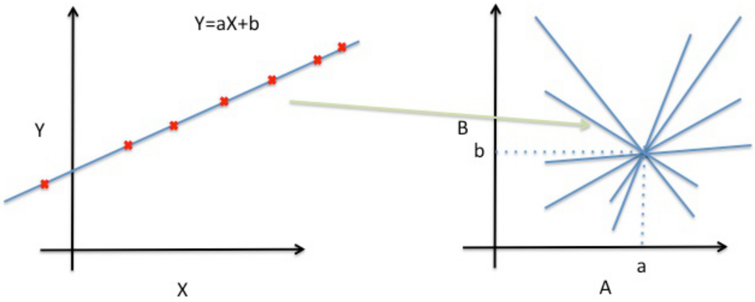

Algorithm

1. select some points on the line;
2. compute the HT of all points;
3. discretize the parameter space by means of an appropriate number of cells of appropriate dimensions;
4. apply an accumulator function to count how many times each cell is voted by the Hough transforms of the points of interest in the image space;
5. compute the local maxima of the accumulator function.

Some questions arise

- can we define the HT for general curves?
- does exist a duality between the image and parameter spaces like in the case of straight lines?
- can we write an efficient algorithm in the case of a general curve?

Beltrametti et al., *SIAM Journal on Imaging Sciences*, 2013.

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Definition

Let $\mathcal{C}_{a,b}$ be a curve such that $\mathcal{C}_{a,b}: \quad F(X, Y; a, b) = 0$, where $F(X, Y; a, b)$ is an irreducible polynomial in the indeterminates X, Y of a given degree, whose coefficients are polynomials in the real parameters a, b.

If $P = (x, y)$ is a point of the image space $\mathbb{A}^2_{(X,Y)}(\mathbb{R})$, then the HT $\Gamma_P(A, B)$ of the pair $(\mathcal{C}_{a,b}, P)$ is the curve defined in the parameter plane $\langle A, B \rangle$ by the equation $F(x, y; A, B) = 0$.

The theorem

Let $\mathcal{F}$ be a family of irreducible curves, and let $\mathcal{C}_{a,b}$ be a curve of the family. Then the following hold:

- The Hough transforms $\Gamma_P(A, B)$ of the pair $(\mathcal{C}_{a,b}, P)$, when P varies on $\mathcal{C}_{a,b}$, all pass through the point (a, b).
- Assume that the Hough transforms $\Gamma_P(A, B)$, when P varies on $\mathcal{C}_{a,b}$ have a point in common other than (a, b), say (a', b'). Thus the two curves $\mathcal{C}_{a,b}$, $\mathcal{C}_{a',b'}$ coincide.

Lemma

Hough regularity: for every family $\mathcal{F}$ of irreducible curves the following conditions are equivalent:

- for any curves $\mathcal{C}_{a,b}$, $\mathcal{C}_{a',b'}$ in $\mathcal{F}$ the equality $\mathcal{C}_{a,b} = \mathcal{C}_{a',b'}$ implies $(a,b) = (a',b')$;
- for each curve $\mathcal{C}_{a,b}$ in $\mathcal{F}$, one has $\bigcap_{P \in \mathcal{C}_{a,b}} \Gamma_P(A,B) = (a,b)$.

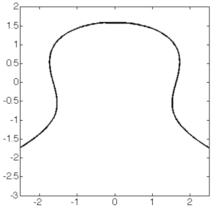
Key Lemma (suggests): maxima in parameter space = curves best approximating the profiles of interest

Elliptic curve

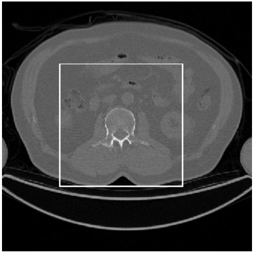
A.M. Massone, A. Perasso, C. Campi, M. Piana *Journal of Mathematical Imaging and Vision* 2015

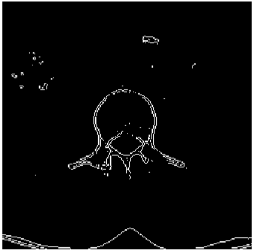
$$\mathcal{C}_{a,b} : Y^2 - X^3 - aX - b = 0$$

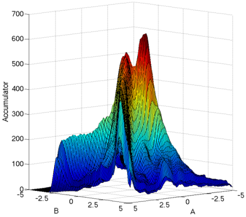
$$\Gamma_P(A, B) : y^2 - x^3 - Ax = B$$



Application to the vertebra







A whole body CT acquisition is made of around 400 images

Physicians time is precious

Graphical user interface (with minimum user involvement)

C. Campi, et al. *Proc. of SPIE* 2016

Technical questions

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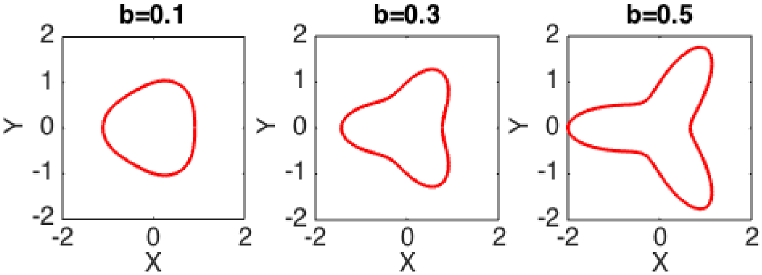
Three convexities curve

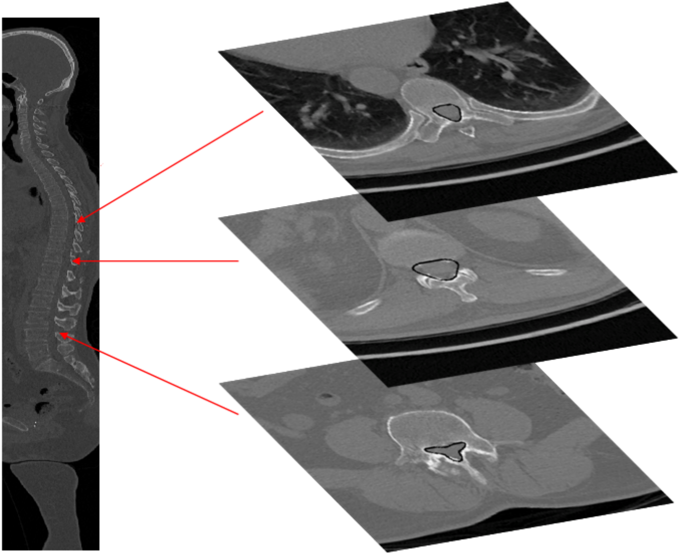
$$\mathcal{C}_{a,b} : (X^2+Y^2)^3 = \left[a(X^2+Y^2) - b(X^3 - 3XY^2)\right]^2$$

$$\Gamma_P(A,B) : (sA - tB + s^{3/2})(sA - tB - s^{3/2}) = 0$$

with $s = x^2 + y^2$, $t = x^2 - 3xy^2$



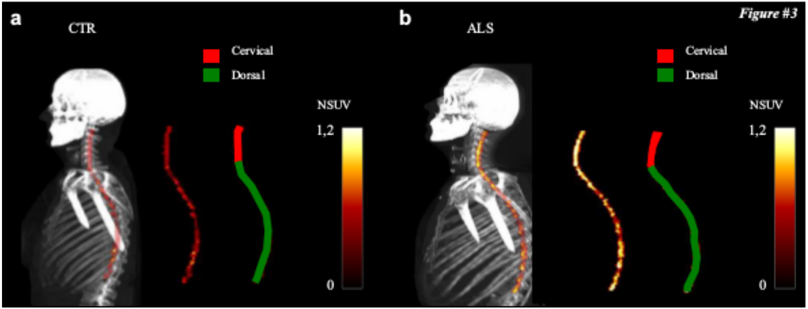
Application to ALS



Application to ALS

C. Marini, A. Cistaro, C. Campi, et al. *European Journal of Nuclear Medicine and Molecular Imaging* 2016

Amyotrophic Lateral Sclerosis (ALS) induces functional alterations in primary motor neuron and progressively in lower motor neuron Evaluation of spinal cord metabolism as a tool to monitor disease mechanisms.



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