Probability ACSAI 2024-25
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Week 5

Exercise 1. A binary signal is transmitted via a channel. Due to the background noise, it may happen that when 0 is transmitted 1 is received, and similarly it may happen that when 1 is transmitted 0 is received. Assume that:

- the probability that 0 is received correctly is 0.94;
- the probability that 1 is received correctly is 0.91.

A single bit is transmitted, which is 0 with probability 0.45 and 1 with probability 0.55. Compute:

1) the probability of receiving 1;

2) the probability of receiving 0;

3) the probability that 1 was transmitted, given that 1 was received;

4) the probability that 0 was transmitted, given that 0 was received;

5) the probability that the signal is wrongly received (i.e. the probability that the received signal is different from the transmitted signal).

Exercise 2. Alice and Bob each toss a biased coin: Alice's coin gives head with probability 1/3, while Bob's coin gives head with probability 1/4.

1) Compute the probability that both Alice and Bob get head.

2) Compute the probability that they get exactly one head.

3) Knowing that the coin tosses resulted in one head and one tail, compute the probability that Alice got head.

Exercise 3. It is known that twins can be identical, in which case they are necessarily of the same sex, or non-identical, in which case they are of the same sex in 50% of the cases. Let p denote the probability that the twins are identical.

1) Compute, as a function of p, the probability that two twins are identical, knowing that they are of the same sex.

2) Compute, as a function of p, the probability that two twins are not of the same sex.

Exercise 4. In a factory three machines A, B, and C make respectively 40%, 10%, and 50% of the produced items. The respective percentages of faulty items are 2%, 3% and 4%. Pick an item at random.

1) Compute the probability that the item is faulty.

2) Knowing that the item is faulty, compute the probability that it was produced by machine A, B or C.

Exercise 5. An urn contains three coins: the first coin is fair, and has head (H) on one side and tail (T) on the other side, the second coin has H on both sides and the third coind has T on both sides. A coin is chosen at random from the urn, and it is tossed without looking at which one it is.

1) Compute the probability that the coin toss gives H.

2) Given that the coin toss gave H, compute the probability that on the other side of the coin there is T.

3) Knowing that the coin toss gave H, we pick it up and, without looking at the other side, toss it again. Compute the probability that the coin gives H again.

Exercise 6. Let S be a set of cardinality n. Pick two subsets of S at random. Compute the probability that the first set is a subset of the second set.

Exercise 7. Three roads connect the houses A, B and C in such a way that from each house one can get to any other house with a direct path. Due to bad weather, the roads may be (temporarily) closed. Let $p_{AB} \in (0,1)$ (respectively p_{BC}, p_{AC}) denote the probability that the road linking A and B (respectively B and C, A and C) is open. You can assume that each road is open or closed independently of the state of the other roads. You are at house A.

1) Compute the probability that you can get to house C.

2) Someone told you that it is not possible to get to house C due to bad weather. Compute the probability that you can get to house B.

Now suppose that between A and B there are 3 direct paths, each one open with probability q independently of the others.

3) Compute again the above probabilities, without redoing the computations from scratch.