IV) Given $f(x) = x^2 + 3x - 1$.

- Find the points of the graph of f for wich the tangent is parallel to "$y = x$".
- Determine if there exists a tangent to f that passes through the origin.
- Determine the sets of points for which the graph of f is below the tangent at zero

VI) Determine the intervals of monotonicity of f in its domain of definition.

a) $f(x) = x^4 - 3x^2$

b) $f(x) = 2x e^{x^2 - 3x}$

c) $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$

d) $f(x) = x - e^x$

e) $f(x) = \log(x^2 + 3x)$

VII) Find the minimum and the maximum of $f(x) = x^3 - 4x^2 + 3x$ in $[0, 3]$.

VIII) Suppose that f and g are both differentiable functions. Let f be monotone increasing and g be monotone decreasing. Prove that $f \circ g$ and $g \circ f$ are monotone decreasing.

Is this true also for f and g not differentiable?

I) Using the formula of derivation, determine the derivative and the domain of definition of the derivative function for the following functions:

a) $f(x) = |2x-1|$, b) $g(x) = \sqrt{4-x^2}$ c) $h(x) = e^{4x}$

d) $k(x) = x\,e^{x^2}$ e) $f(x) = \frac{x^2+3x}{x-2}$, f) $f(x) = \log\left(\frac{x^2+2}{x^2+3}\right)$

g) $f(x) = \begin{cases} x^2+3x-2 & x>0 \\ 3x-2 & x \leq 0 \end{cases}$

II) Let $f(x) = \begin{cases} e^{2x+1} & x > 1 \\ ax+b & x \leq 1 \end{cases}$

Determine a and b such that f is continuous and differentiable in $\mathbb{R}$

III) Let $f(x) = \cot g(2x)$.

Determine the equation of the tangent of f at $x_0 = \frac{\pi}{8}$.

IV) Let $f_\alpha(x) = x^\alpha$. Consider the equation

$$t^2 + f_\alpha'(1)t + 1 = 0.$$

Determine for which α, the equation has two solutions.