Exercises on functions

1. Given two sequences a_n, b_n, consider the following function $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x) = \begin{cases} e^{\frac{-x}{n}} & \text{if } x \in [2n, 2n+1)\, \forall n \in \mathbb{N}* \\ a_n x + b_n & \text{if } x \in [2n+1, 2n+2)\, \forall n \in \mathbb{N} \\ 0 & \text{if } x < 1 \end{cases}$$

   (a) For $a_n = 1$ and $b_n = 0$, for any $n \in \mathbb{N}$, draw the graph of f for $x \in [1, 5]$.

   (b) Find two sequences a_n and b_n such that f is continuous in $\mathbb{R}$.

   (c) For the sequences found in (b), is f differentiable in $\mathbb{R}$.

   (d) Find the limit at infinity of the sequences in (b)

2. Given three sequences a_n, b_n, c_n consider the following function $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x) = \begin{cases} e^{\frac{-x}{n}} & \text{if } x \in [2n, 2n+1)\, \forall n \in \mathbb{N}* \\ c_n x^2 + a_n x + b_n & \text{if } x \in [2n+1, 2n+2)\, \forall n \in \mathbb{N} \\ 0 & \text{if } x < 1 \end{cases}$$

   (a) For $a_n = 1$ and $b_n = 0$, $c_n = 1$ for any $n \in \mathbb{N}$, draw the graph of f for $x \in [1, 5]$.

   (b) Find three sequences a_n, b_n and c_n such that f is continuous in $\mathbb{R}$.

   (c) Is it possible to find three sequences a_n, b_n and c_n such that is f differentiable in $\mathbb{R}$?

3. Define a function, different from the one in exercise 1, which is continuous in $\mathbb{R}$ but has and infinite number of points where it is not differentiable.

4. Let $f(x) = |\sin x|$. Determine the points of *discontinuities* and the points of *non differentiability* of f.

5. Given $f(x) = x^3$, find the equations of the tangent lines to f that are parallel to the bisector $y = x$.

6. Using Newton's method, find a sequence of rational numbers converging to $\sqrt{3}$.