1) Consider the sequence

$$\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \dots$$

a) Prove that the sequence is bounded

b) Determine if there is a subsequence converging to 1

c) Determine for which $L$, there exist a subsequence converging for $L$

2) Let $\{a_n\}$ defined in the following way

$$\begin{cases} a_1 = 2 \\ a_{n+1} = \frac{1}{4} a_n + \frac{3}{4} \end{cases}$$

a) Determine if $a_n$ is monotone decreasing

b) Prove that it converges and find the limit

c) Using the fact that $\sum_{k=0}^{n} a^k = \frac{a^{N+1} - 1}{a - 1}$

prove that $a_n = \left(\frac{1}{4}\right)^{n-1} 2 + \frac{1}{(4)} - \left(\frac{1}{4}\right)^{n-1}$

3) Let us consider the Fibonacci sequence

$$\begin{cases} a_1 = 1, \ a_2 = 1 \\ a_{n+1} = a_n + a_{n-1} \end{cases}$$

3.1) Compute $a_3, a_4, a_5$

Let $b_n = \frac{a_{n+1}}{a_n}$.

3.a) Prove that $b_n$ is bounded

3.b) Suppose that the sequence is convergent, determine the limit