Calculus-Unit 1

Applied Computer Science for AI

Blank examination

Voto finale

Postazione:

Cognome:

Nome:

Matricola:

Canale:

Esercizio	Punteggio
1	
2	
3	
4	
Risp. Mult.	
Totale	

Es. 1 [1+2+1 Points] Given the sequence a_n defined in the following way

$$\begin{cases} a_0 = 1 \\ a_{n+1} = \sqrt{a_n + 1} \end{cases}$$

a) Prove by induction that $a_n \leq \frac{1+\sqrt{5}}{2}$

b) Prove that, if the limit exists, it is equal to $\frac{1+\sqrt{5}}{2}$

c) Prove that the sequence is monotone increasing

Es 2 [3 Points] Determine the points of discontinuity and of non differentiability of the function $f(x) = |\sin(2x)|$ (justify your answer).

Es 3 [4 points] Compute the following limit (justify your answer) $\lim_{x\to 0^+} \frac{1-\cos(2\sqrt{x})}{\log(1+\sin(3x))}$

Es 4 [1+2+1+2+1 points] Given the function $f(x) = e^{\frac{x}{x^2-1}}$. Determine:
a) Domain:
b) The limit at the boundary of the domains
c) The asymptotes
d) The derivative
e) The interval of monotonicity

Es 5 [2 o -1 points] The function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = |e^{-x^2} - \frac{1}{2}|$

(A) Has a minimum and a maximum (**B**) Has a maximum but no minimum

(C) Has a minimum but no maximum (D) Its minimum is at infinity

Es 6 [2 o -1 punti] The derivative of $f(x) = \arctan(\frac{2x}{x-2})$ is:

(A) $\frac{1}{1+x^2} \cdot \frac{-4}{(x-2)^2}$ (B) $\frac{1}{1+\left(\frac{2x}{x-2}\right)^2}$ (C) $\frac{-4}{(x-2)^2}$

(**D**) $\frac{-4}{5x^2-4x+4}$ (E) None of the previous answers is correct

Es 7 Let $f : [0, 2] \to \mathbb{R}$ continuous such that the image of f is $[0, 2]$. Then

(A)[1/2] The function $g(x) = f(x) - x$ has at least a zero in $[0, 2]$ **T** **F**
(B)[1/2] The function is tangent to the bisector **T** **F**
(C)[1/2] f has a maximum and a minimum **T** **F**
(D)[1/2] $\exists x_o \in (0, 1)$ and $x_1 \in (1, 2)$ such that $f(x_o) = f(x_1)$ **T** **F**

Es 8 Given the equation $(z + i)^4 = 1$ in $\mathbb{C}$

(A) It has 2 solutions **T** **F**

Es 9 [3 o -1 punti] The $\lim_{n\to+\infty} \dfrac{-e^{2n} + 2n^4 + \ln(n^2 - 1)}{n \sin n + 2e^{2n} + \sqrt{3}}$ equals

(A) 1 (**B**) $\frac{1}{2}$ (C) $+\infty$

(D) $-\infty$ (E) The limit does not exist (F) None of the previous answers is correct

Es 10 The function $f : [a, b] \to \mathbb{R}$ is differentiable. Say which of the following holds true

(A)[1/2] If $f(a) = f(b)$ then the maximum of f is 0 **T** **F**

(B)[1/2] If f is convexe then the deirvative of f is increasing **T** **F**

(C)[1/2] If $f(b) > f(a)$, then f is increasing in (a, b). **T** **F**

(D)[1/2] If $f(x) = 2f(a) + b(x - a)$, then $f(a) = 0$ **T** **F**

(E)[1/2] There exists an $x \in (a, b)$ such that $f'(x) = \frac{f(a)-f(b)}{a-b}$ **T** **F**