Thursday, 21 January 2021 18:35

CORREZIONE MOLTO GROSSOLANA DELL'ESAME DEL 25 Gennaio 2021

I a) $f(x,y)=\sqrt{x^2+2y}$, $(x_0,y_0)=(1,4)$ Dominio [ ]

$\sqrt{\quad} = \sqrt{9} = 3$

$f(x_0,y_0)=\sqrt{1+8}$

$D=\{(x,y)$ t.c. $x^2+2y\geq 0\}$

$y=-\frac{x^2}{2}$

parabola inclusa

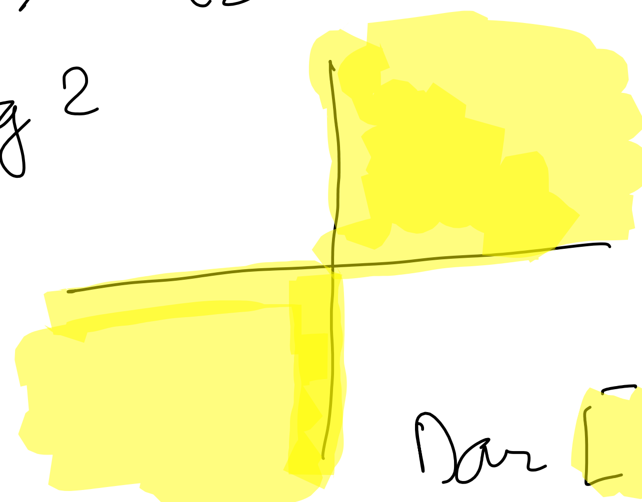
b) $f(x,y)=\log(xy)$, $(x_0,y_0)=\left(\frac{1}{2},4\right)$

$f(x_0,y_0)=\log\left(\frac{1}{2}\cdot 4\right)=\log 2$

$D=\{(x,y)$ t.c. $xy>0\}$

$x>0$ e $x<0$

$y>0$ $\quad$ $y<0$



Dom [ ]

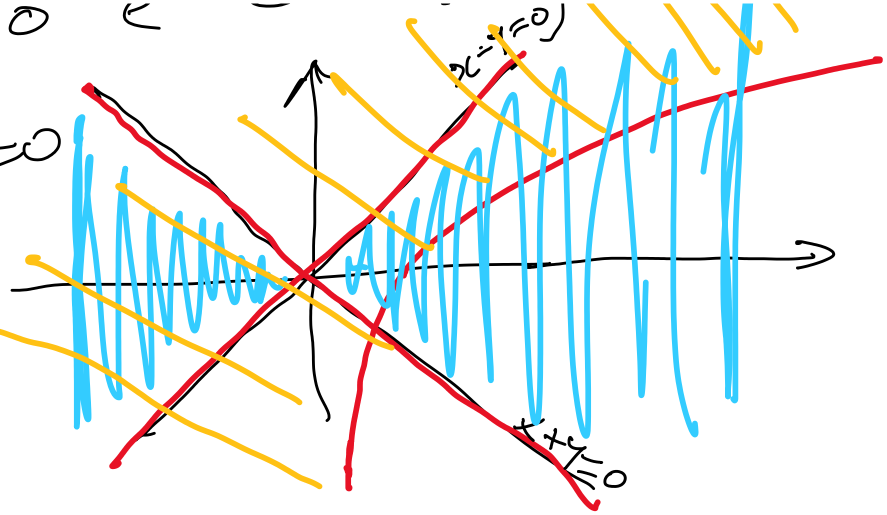
esclusi gli assi

c) $f(x,y)=\log\left(\frac{x^2-y^2}{e^y-x}\right)$ $\qquad (x_0,y_0)=\left(1,\frac{1}{2}\right)$

$f\left(1,\frac{1}{2}\right)=\log\left(\frac{1-\frac{1}{4}}{e^{1/2}-1}\right)=\log\left(\frac{3}{4(e^{1/2}-1)}\right)$

$e^y-x\neq 0\}$

$D = \{(x,y)$ t.c. $\frac{x^2-y^2}{e^y - x} > 0$



$x^2-y^2=(x-y)(x+y)=0$

$x=e^y$

$x^2-y^2 \geq 0$ contiene $y=0 \Rightarrow$

$e^y - x \geq 0 \quad x \leq e^y$

Il dominio è dove ci sono 2 colori o nessuno:

d) $f(x,y) = e^{x^2+y^2-1}$

$$\Big[\qquad \Big]$$

Esercizio II

a) $\iint_D x-y \, dx\,dy$, $D=[-1,3]\times[-2,1]$

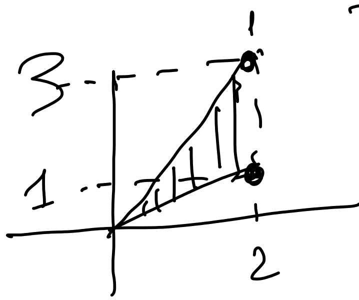
$$\iint_D x-y\,dx\,dy=\int_{-2}^{1}\left[\int_{-1}^{3}x-y\,dx\right]dy=$$

$$=\int_{-2}^{1}\frac{x^2}{2}-yx\Big|_{-1}^{3}dy=\int_{-2}^{1}\left[\frac{9}{2}-3y\right]-\left[\frac{1}{2}+y\right]dy$$

$$=\int_{-2}^{1}(4-4y)\,dy=4y-2y^2\Big|_{-2}^{1}=4-2-(-8-8)$$

$$=2+16=18$$

b) $\iint_T y\,dx\,dy$



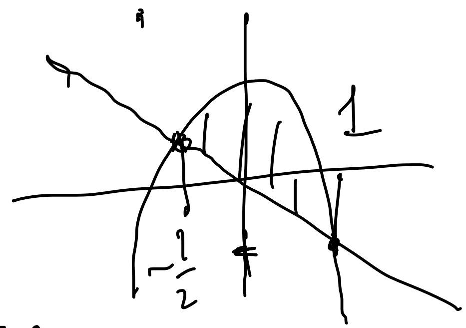
$y=\frac{x}{2}$, $y=\frac{3x}{2}$

$$=\int^{2}\int^{\frac{3x}{2}} y\,dy\,dx=\int^{2}\frac{y^2}{2}\Big|^{\frac{3x}{2}}dx=\frac{1}{2}\int^{2}\frac{9x^2}{4}-\frac{x^2}{4}\,dx$$

$$-\int_0 \int_{\frac{x}{2}} \qquad \int_0 \Big|_{\frac{x}{2}} \qquad \int_0$$

$$= \int_0^2 x^2\,dx = \frac{x^3}{3}\Big|_0^2 = \frac{8}{3}$$

c) $\displaystyle\iint_D xy\,dx\,dy$ con $D = \{-x \le y \le 1-2x^2\}$



$1-2x^2 = -x$

$2x^2 - x - 1 = 0$

$x = 1$

$x = -\frac{1}{2}$

$$\int_0^1 \left(\int_{-x}^{1-2x^2} xy\,dy \right) dx = \int_0^1 x\frac{y^2}{2}\Big|_{-x}^{1-2x^2} dx = \int_0^1 \frac{x}{2}(1-2x^2)^2 - \frac{x^3}{2}\,dx$$

$$= \frac{1}{2}\left[\frac{-1}{4\cdot 3}(1-2x^2)^3 - \frac{x^4}{8} \right]_0^1 = -\frac{1}{24}(-1)^3 - \frac{1}{8} - \left[-\frac{1}{24} \right]$$

$$= -\frac{1}{8}$$

$$= (2\cos t \sin t,\ 3\cos^2 t \sin t)$$

d) $\gamma(t) = (\cos^2 t, \cos^3 t) \to \gamma'(t) = ($

$\gamma'(\frac{\pi}{2}) = (0, 0).$

$$L_\gamma = 2\int_0^{\frac{\pi}{2}} \sqrt{4\cos^2 t \sin^2 t + 9\cos^4 t \sin^2 t}\, dt$$

$$= 2\int_0^{\frac{\pi}{2}} \cos t \sin t \sqrt{4 + 9\cos^2 t}\, dt$$

$$= -\frac{2}{18} (4 + 9\cos^2 t)^{3/2} \cdot \frac{2}{3} \Big|_0^{\frac{\pi}{2}} =$$

$$= -\frac{1}{9} (4)^{3/2} \cdot \frac{2}{3} + (13)^{3/2} \frac{2}{27} = \frac{2}{27}\left[13^{3/2} - 4^{3/2}\right]$$

Esercizio 3: $y'' + 4y' - 5y = f(x)$

a) $y(x) = 2x, \quad y' = 2, \quad y'' = 0$

$0 + 4 \cdot 2 - 5(2x) = 8 - 10x \neq 10x$

y non è soluzione

b) $y'' + 4y' - 5y = 0 \longrightarrow$ Eq. caratteristica

$\lambda^2 + 4\lambda - 5 = 0$

$\lambda = 1, \ \lambda = -5$

$y_0(x) = C_0 e^{x} + C_1 e^{-5x} \quad \forall C_0 \in \mathbb{R}, \forall C_1 \in \mathbb{R}$

c) $y'' + 4y' - 5y = e^{x} \implies y_p(x) = axe^{x}$

$y_p'(x) = (a + ax)e^{x}$

$y_p''(x) = (2a + ax)e^{x}$

$e^{x}[\underbrace{2a + ax}_{y''} + 4\underbrace{(a + ax)}_{y'} - 5\underbrace{ax}_{y}] = 6ae^{x} = e^{x}$

$6a = 1 \implies a = \frac{1}{6}$

$y(x) = \frac{1}{6}xe^{x} + C_0 e^{x} + C_1 e^{-5x} \quad \forall C_0 \in \mathbb{R}, \forall C_1 \in \mathbb{R}$

d) $y(0) = y'(0) = 0$

$y(0) = C_0 + C_1 = 0, \quad y'(x) = \left(\frac{1}{6} + \frac{1}{6}x\right)e^{x} + C_0 e^{x} - 5C_1 e^{-5x}$

$y'(0) = \frac{1}{6} + C_0 - 5C_1 = 0$

$\begin{cases} C_0 + C_1 = 0 \\ C_0 - 5C_1 = \frac{1}{6} \end{cases} \qquad C_0 = -C_1$

$-6C_1 = \frac{1}{6}$

$C_1 = -\frac{1}{36}$

$c_0 = \frac{1}{36}$

$$y(x) = \frac{1}{6} x e^{x} + \frac{1}{36} e^{x} - \frac{1}{36} e^{-5x}$$

e) $y(x) = c e^{-5x}$ è limitata in $(0, +\infty)$ $\forall c \in \mathbb{R}$

Es 4: $F(x,y) = (xy^2 + e^{2x}, x^2 y + \cos(\pi y))$

$\mathrm{Dom}\, F = \mathbb{R}^2$

a) $F(0,0) = (1, 1)$

b) $\frac{\partial F_1}{\partial y} = 2xy$, $\frac{\partial F_2}{\partial x} = 2xy$ $\Rightarrow$ F è irrotazionale

$\mathbb{R}^2$ è semplicemente connesso $\Rightarrow$ F è conservativo.

c) $L(F, \gamma) = 0$ perché $\gamma(0) = (0, 1) = \gamma(2\pi)$

regolare e chiusa.

e F è conservativo.

d) $F(x,y) = \nabla f \Rightarrow$

$\frac{\partial f}{\partial x} = xy^2 + e^{2x} \longrightarrow f(x,y) = \frac{x^2 y^2}{2} + \frac{1}{2} e^{2x} + g(y)$

$$\frac{\partial f}{\partial x} = xy$$

$$\frac{\partial f}{\partial y} = x^2 y + \cos(\pi y)$$

$$\Downarrow$$

$$\frac{\partial f}{\partial y} = x^2 y + g'(y) = x^2 y + \cos \pi y$$

$$\Downarrow$$

$$g'(y) = \cos(\pi y)$$

$$\Downarrow$$

$$g(y) = \frac{1}{\pi} \sin(\pi y).$$

$$f(x,y) = \frac{x^2 y^2}{2} + \frac{1}{2} e^{2x} + \frac{1}{\pi} \sin(\pi y)$$

$$L(F,\gamma) = f(\gamma(1)) - f(\gamma(0)) = f(1,1) - f(0,0) =$$

$$= \frac{1}{2} + \frac{1}{2} e^2 + \frac{1}{\pi} \sin(\pi) - 0 = \frac{1}{2} + \frac{1}{2} e^2$$