## Esercizio 1

Dire se esistono i limiti seguenti e in caso affermativo calcolarli

1. $\lim_{x \to +\infty} \log(x) - \sqrt{x}$
2. $\lim_{x \to 0} \frac{x^5 + x^7 - x^2}{x^5 + x^3 - x}$
3. $\lim_{x \to -\infty} \frac{x^5 + x^7 - x^2}{x^5 + x^3 - x}$
4. $\lim_{x \to 0} \frac{x + 2x^2}{x^5 + x^4 - x^3}$
5. $\lim_{x \to +\infty} \frac{\log(x^3 + 1)}{x}$
6. $\lim_{x \to +\infty} \frac{\log \sqrt{x+1}}{x}$
7. $\lim_{x \to 0} \frac{\log(x^3 + 1)}{x}$
8. $\lim_{x \to 0} \frac{\log \sqrt{x+1}}{x}$
9. $\lim_{x \to 0} \frac{x}{1 - e^{2x}}$
10. $\lim_{x \to +\infty} \left(1 - \frac{1}{x}\right)^{2x}$
11. $\lim_{x \to -\infty} \left(\frac{2x+3}{2x}\right)^{1-x}$
12. $\lim_{x \to +\infty} \left(\frac{x+2}{x+1}\right)^{x}$
13. $\lim_{x \to 1} x^{\frac{2}{x-1}}$
14. $\lim_{x \to 0+} x \log(x)$
15. $\lim_{x \to 0+} x^{x}$
16. $\lim_{x \to 1+} \frac{\log(1 + \sqrt{x-1})}{\sqrt{x^2 - 1}}$

## Esercizio 2

1. $\lim_{x \to 0} \frac{\sin(5x)}{\sin(2x)}$

2. $\lim_{x \to +\infty} x \sin\left(\frac{1}{x}\right)$

3. $\lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$

4. $\lim_{x \to 0} \frac{1 - \cos(x) + \sin(x)}{1 - \cos(x) - \sin(x)}$

5. $\lim_{x \to 0} \frac{\sin^4(x)}{(1 - \cos x)^2}$

6. $\lim_{x \to 0^+} \frac{\sqrt{1 - \cos(x)}}{x}$

7. $\lim_{x \to 0} \frac{\sin 4x}{\operatorname{tg} x}$

8. $\lim_{x \to 0} \frac{\operatorname{tg}^2(x)}{1 - \cos(x)}$

9. $\lim_{x \to 0} \frac{x^3}{\operatorname{tg}(x) - \sin(x)}$

10. $\lim_{x \to 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{\sin^2(x)}$

11. $\lim_{x \to \pi} \frac{1 + \cos(x)}{\sin^2(x)}$

12. $\lim_{x \to 0} \frac{e^{\sin x} - 1}{x}$

13. $\lim_{x \to +\infty} \left(\frac{x}{1 + x}\right)^{2x}$

14. $\lim_{x \to 0} \frac{e^x - e^{-x}}{e^{2x} - e^{-2x}}$

## Esercizio 3

1. $\lim_{x \to 0} \frac{\sin(2x)}{x}$

2. $\lim_{x \to 0} \frac{1 - \cos(x)}{\sin(4x)}$

3. $\lim_{x \to 0} \frac{1 - \cos^3 x}{3x^2}$

4. $\lim_{x \to 0} \frac{\arctg(x)}{x}$

5. $\lim_{x \to 0} \frac{\arcsin(x)}{x}$

6. $\lim_{x \to 0} \frac{\sin^2(2x)}{1 - \cos(3x)}$

7. $\lim_{x \to 0} \frac{\sin 4x}{\tg x}$

8. $\lim_{x \to 0} \frac{\tg^2(x)}{1 - \cos(x)}$

9. $\lim_{x \to +\infty} (1 + e^{-x})^x$

10. $\lim_{x \to 0} \frac{x^2 \log(1 + 2x)}{(2\cos(3x) - 2)\sin(x)}$

11. $\lim_{x \to 0} \frac{\log(x + x^2)}{\log(x)}$

12. $\lim_{x \to 0} \frac{\log(1 - x + x^2)}{x}$

13. $\lim_{x \to 0} \frac{\log(\cos x)}{x^2}$

14. $\lim_{x \to +\infty} \left(1 + \frac{1}{\log(x)}\right)^x$

~~15~~ $\lim_{x \to 0} \frac{\log(1+x) + \log(1-x)}{x^2}$

Esercizio 4*

Verificare che $\lim_{x \to 0} \left(\cos(x)\right)^{1/x^2} = 1/\sqrt{e}$