

General exercises on functions

1. Consider the following functions, and determine whether the image of the function is bounded above or below:

$$\begin{cases} f : \mathbb{R} \rightarrow \mathbb{R} \\ f(x) = \frac{x^2}{x^2+1} \end{cases} \quad \begin{cases} f : \mathbb{R} \rightarrow \mathbb{R} \\ f(x) = \cos x + \sin x \end{cases}$$
$$\begin{cases} f : [-2, 2] \rightarrow \mathbb{R} \\ f(x) = \sqrt{x^2 - 4} \end{cases} \quad \begin{cases} f : \mathbb{R} \rightarrow \mathbb{R} \\ f(x) = \sqrt{x^2 + 4} \end{cases}$$

2. Let  $f$  be a function with the following characteristics:  $f : A \rightarrow \mathbb{R}$ , with  $-1 \in A$  and  $0 \in A$ ,

- it is even ( $f(-x) = f(x)$ ),
- it is bounded (its image set is bounded),
- $f(0) = 1$  and  $f(1) = -1$ .

Answer to the following requests, without determine explicitly

- Determine a possible set  $A$  and draw two possible graphs of  $f$
  - Draw a possible graph of  $f$  which is never zero
  - Considering that the first graph of  $f$  that you have drawn is the correct one, draw the graph of  $y = |f(x)|$ , the graph of the functions  $g(x) = f(x + 1)$ ,  $h(x) = f(x) + 1$
3. Let  $f(x) = \sqrt{x}$  and  $h(x) : A \rightarrow \mathbb{R}$  defined by  $h(x) = \tan(x)$ . Determine a set  $A$  such that it is possible to define  $f \circ h$ .
4. For the following functions, determine the largest domain of definition:

$$f(x) = \sqrt{x^2 + 3x - 1}, \quad g(x) = \frac{x^2 - 3}{\log(x^2 - 1)}, \quad h(x) = \sqrt{\frac{x + 2}{3 - x}}$$