ancora MOLT. di LAGRANGE

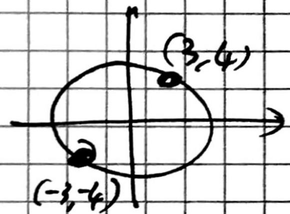
$$\boxed{f = 3x + 4y \quad \text{su} \quad \{x^2+y^2 \le 25\}}$$

$\nabla f = (3,4) \neq (0,0)$ NO PTI STAZ.

Lagrange $\quad G = 3x + 4y - \lambda(x^2+y^2-25)$

$$\begin{cases} 3 = 2\lambda x \\ 4 = 2\lambda y \\ x^2+y^2=25 \end{cases} \qquad \frac{3}{4} = \frac{x}{y} \qquad \left(\frac{16}{9}+1\right)x^2 = 25$$

$\Rightarrow \begin{cases} x = \pm 3 \\ y = \pm 4 \end{cases}$ (in accordo) $\Rightarrow$ 2 PTI $\quad (3,4)$, $(-3,-4)$



$f(3,4) = 25 \quad \Downarrow$ MAX

$f(-3,-4) = -25 \quad \Downarrow$ MIN

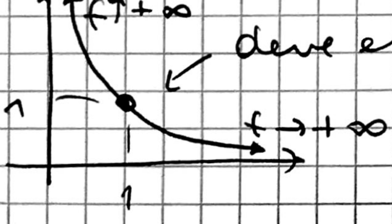
$$\boxed{f = x+y \quad \text{su} \quad S = \{x>0,\ y>0,\ xy=1\}}$$

$G = x+y-\lambda(xy-1)$

$$\begin{cases} 1 = \lambda y \\ 1 = \lambda x \\ xy = 1 \end{cases} \quad \rightarrow \quad x = y = 1/\lambda \quad (\lambda \neq 0\,!)$$

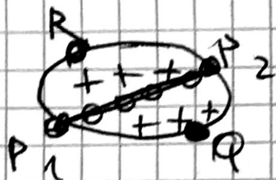
$x^2 = 1 \Rightarrow x = 1 = y$

$(1,1)$ è MINIMO perché il minimo deve esistere



$$\boxed{f = (x-2y)^2 \quad \text{su} \quad \frac{x^2}{4} + \frac{y^2}{3} \le 1}$$

PTI STAZ. $\begin{cases} 2(x-2y) = 0 \\ 2(x-2y)\cdot(-2) = 0 \end{cases} \rightarrow \underline{x = 2y}$



Sulla retta $x=2y$ la f fa 0

Altrove $f > 0$

$\Rightarrow$ i punti della retta $x = 2y$ sono MINIMI (assoluti).

BORDO

$$G = (x-2y)^2 - \lambda\left(x^2/_4 + y^2/_3 - 1\right)$$

$$\begin{cases} 4(x-2y) = \lambda x \\ -6(x-2y) = \lambda y \\ \dfrac{x^2}{4} + \dfrac{y^2}{3} = 1 \end{cases} \qquad \begin{aligned} x - 2y &= \tfrac{1}{4}\lambda x \\ x - 2y &= -\tfrac{1}{6}\lambda y \end{aligned} \Bigg\} \; \lambda y = -\tfrac{3}{2}\lambda x$$

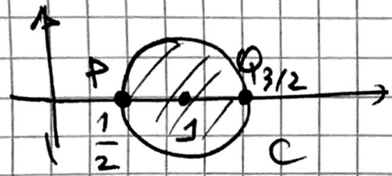
$$\lambda\left(y + \tfrac{3}{2}x\right) = 0$$

$\lambda = 0 \rightarrow x = 2y \rightarrow$ minimi locali P_1, P_2
(già visti).

$$\lambda \neq 0 \Rightarrow \begin{cases} y = -\dfrac{3}{2}x \\ \dfrac{x^2}{4} + \dfrac{3}{4}x^2 = 1 \Rightarrow x = \pm 1 \end{cases} \Rightarrow \begin{aligned} (1, -3/2) &= Q \\ (-1, 3/2) &= R \end{aligned}$$

$f(1, -3/2) = f(-1, 3/2) = 16 \Rightarrow$ MASSIMI

$\boxed{f = x^2+2y^4 \text{ su } C=(x-1)^2+y^2 \le 1/4}$ 3bis



$(0,0)$ è STAZIONARIO ma cade FUORI dal cerchio C.

BORDO $\quad G = x^2+2y^4-\lambda((x-1)^2+y^2-1/4)$

$$\begin{cases} x = \lambda(x-1) \\ 4y^3 = \lambda y \\ (x-1)^2+y^2 = 1/4 \end{cases} \qquad (\lambda = 0 \text{ NO } \to x=y=0)$$

$x(\lambda-1) = \lambda \qquad (\lambda \neq 0 \Rightarrow \lambda \neq 1 \text{ e } x \neq 0)$

$\Rightarrow \boxed{x = \dfrac{\lambda}{\lambda-1}}$

$y(4y^2-\lambda) = 0 \to y = 0 \Rightarrow \begin{cases} x = \frac{3}{2} & f(\frac{3}{2},0) = \frac{9}{4} \quad Q \\ x = \frac{1}{2} & f(\frac{1}{2},0) = \frac{1}{4} \quad P \end{cases}$

$y \neq 0 \Rightarrow y^2 = \dfrac{\lambda}{4}$ QUINDI $\lambda > 0$

$\Rightarrow y = \pm \dfrac{\sqrt{\lambda}}{2}$

$$\Rightarrow \begin{cases} x = \frac{\lambda}{\lambda-1} \\ y = \pm\frac{\sqrt{\lambda}}{2} \\ (x-1)^2+y^2 = 1/4 \end{cases}$$

Sostituisco $\Rightarrow \dfrac{1}{(\lambda-1)^2} + \dfrac{\lambda}{4} = \dfrac{1}{4}$

$\Rightarrow \dfrac{1}{(\lambda-1)^2} = \dfrac{1-\lambda}{4} \Rightarrow (1-\lambda)^3 = 4 \Rightarrow \lambda = 1-\sqrt[3]{4}$ IMPOSSIBILE

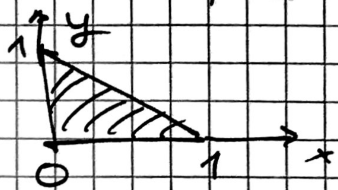
perché $\lambda > 0$

Restano solo i punti $P, Q \Rightarrow \begin{cases} f(\frac{3}{2},0) = \frac{9}{4} & \text{MAX} \\ f(\frac{1}{2},0) = \frac{1}{4} & \text{MIN} \end{cases}$

Altri esercizi

① $f = x^2 + xy + y^2$ su $[-2,2] \times [-2,2]$

② $f = x^2y + xy^2 - xy$ su



③ $f = \dfrac{xy}{1+x^2+y^2}$ su $x^2+y^2 \leq 1$

④ $f = x^2 + xy + y^2$ su $x^2+y^2 \leq 4$