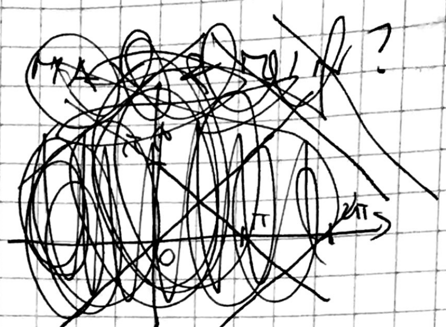
max & min

$$\boxed{f = \cos x + \cos y + \cos(x+y)}$$



$$\begin{cases} D_1 f = -\operatorname{sen} x - \operatorname{sen}(x+y) = 0 \\ D_2 f = -\operatorname{sen} y - \operatorname{sen}(x+y) = 0 \end{cases}$$

$$\begin{cases} \operatorname{sen} x = \operatorname{sen} y \\ \operatorname{sen} x = -\operatorname{sen}(x+y) \end{cases}$$

Basta lavorare con $x, y \in [0, 2\pi[$ (poi periodico)

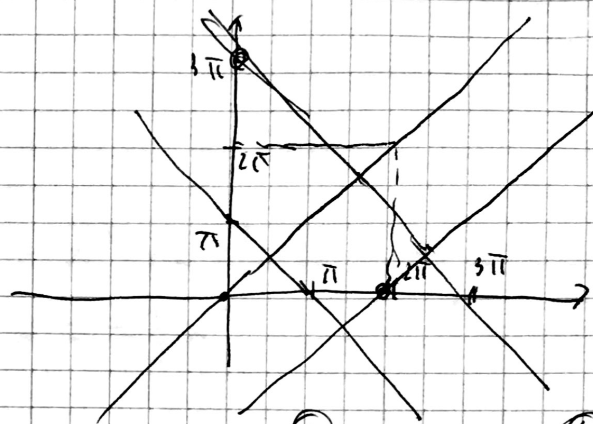
$\operatorname{sen} x = \operatorname{sen} y$

$\Downarrow$

I. $x = y$

II. $x = \pi - y$ cioè $x + y = \pi$

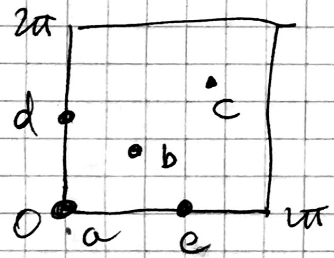
(III. $x = 3\pi - y$ cioè $x + y = 3\pi$)



I. $x = y \Rightarrow \operatorname{sen} x + 2 \operatorname{sen} x \cos x = 0$
$\operatorname{sen} x (1 + 2\cos x) = 0 \Rightarrow (0,0)$ ⓐ $\left(\frac{2}{3}\pi, \frac{2}{3}\pi\right)$ ⓑ $\left(\frac{4}{3}\pi, \frac{4}{3}\pi\right)$ ⓒ

II $x + y = \pi \Rightarrow \operatorname{sen}(x+y) = 0 \Rightarrow (0, \pi)$ ⓓ $(\pi, 0)$ ⓔ

(III $x + y = 3\pi \Rightarrow$ // $\Rightarrow$ no)



$D_{11} f = -\cos x - \cos(x+y)$

$D_{22} f = -\cos y - \cos(x+y)$

$D_{12} f = -\cos(x+y)$

$Hf(0,0) = \begin{pmatrix} -2 & -1 \\ -1 & -2 \end{pmatrix}$ $\det > 0$, $tr < 0$ $\Rightarrow \lambda_j < 0$ ⓐ MAX

$Hf(0,\pi) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $Hf(\pi, 0) =$ // $\det < 0$ d, e SELLA

$Hf\left(\frac{2}{3}\pi, \frac{2}{3}\pi\right) = \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix}$ $\det > 0$, $tr > 0$ $\Rightarrow \lambda_j > 0$ MIN ⓑ

$Hf\left(\frac{4}{3}\pi, \frac{4}{3}\pi\right) = (\ //\ )$ // MIN ⓒ

$$\boxed{f(x,y)=y^2-x^3}$$ MAX & MIN

$$\begin{cases} D_1 f = -3x^2 = 0 \\ D_2 f = 2y = 0 \end{cases} \quad (0,0) \text{ STAZ} \quad Hf = \begin{pmatrix} -6x & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$$

det $= 0$ che si fa?

OVVIAMENTE nè max nè min. → esempio: $f(x,0) = -x^3$ ha FLESSO

Ad esempio: $\varphi = f(x^3, cx^2) = x^6 - c^3x^6 = (1-c^3)x^6$

Se $c > 1$ φ ha max in 0

Se $c < 1$ φ ha min in 0

$$\boxed{f(x,y) = x^4 + y^2 + x^2y}$$ MAX & MIN

$$\begin{cases} D_1 f = 2xy + 4x^3 = 0 \\ D_2 f = 2y + x^2 = 0 \end{cases} \quad \begin{cases} -x^3 + 4x^3 = 0 \\ y = -\frac{1}{2}x^2 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$$

$(0,0)$ STAZ.

$$Hf = \begin{pmatrix} 2y + 12x^2 & 2x \\ 2x & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}, \ \det = 0$$

Che si fa? ~~[illegible]~~ in effetti MINIMO

$$f = (x^2)^2 + x^2 \cdot y + y^2 = \cancel{[illegible]}$$

$$= \frac{(x^2+y)^2}{2} + \frac{x^4}{2} + \frac{y^2}{2} \geq \frac{x^4+y^2}{2} > 0 \quad \text{se } (x,y) \neq 0$$

$$\boxed{f(x,y) = x^2 + xy^3 + 1}$$ MAX&MIN

$$\begin{cases} D_1 f = 2x + y^3 = 0 \\ D_2 f = 3xy^2 = 0 \end{cases} \quad \begin{cases} x = 0 \\ y = 0 \end{cases} \quad (0,0) \text{ staz.}$$

$$Hf = \begin{pmatrix} 2 & 3y^2 \\ 3y^2 & 6xy \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \text{ indet.}$$

ma NÈ MAX NÈ MIN

$$\left.\begin{aligned} f(2y^3, y) &= 3y^6 + 1 \quad \text{MIN} \\ f(-2y^3, y) &= -y^6 + 1 \quad \text{MAX} \end{aligned}\right\} \text{ Sella}$$

$$\boxed{f(x,y) = 4xy + \frac{1}{x} + \frac{1}{y}}$$ $(x, y \neq 0)$ MAX e MIN

$$\begin{cases} D_1 f = 4y - 1/x^2 = 0 \\ D_2 f = 4x - 1/y^2 = 0 \end{cases} \quad \begin{cases} 4y = 1/x^2 \\ 4x = 1/y^2 \end{cases} \quad \begin{cases} y = \frac{1}{4x^2} \\ x(1-4x^3) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x = 0 \ \text{(NO)} \\ x = \frac{1}{\sqrt[3]{4}} \end{cases} \Rightarrow y = \frac{1}{\sqrt[3]{4}}$$

PTO STAZ. $\left(\frac{1}{\sqrt[3]{4}}, \frac{1}{\sqrt[3]{4}}\right)$

$$Hf = \begin{pmatrix} \frac{2}{x^3} & 4 \\ 4 & 2/y^3 \end{pmatrix} \qquad Hf\left(\frac{1}{\sqrt[3]{4}}, \frac{1}{\sqrt[3]{4}}\right) = \begin{pmatrix} 8 & 4 \\ 4 & 8 \end{pmatrix}$$

$\det H > 0$
$\operatorname{tr} H > 0$ $\Rightarrow$ MINIMO

$$\boxed{f(x,y) = (6-x-y)\, x^2 y^3}$$ MAX & MIN

$$\begin{cases} D_1 f = \left(6 - \frac{3}{2}x - y\right) 2xy^3 = 0 \\ D_2 f = \left(6 - x - \frac{4}{3}y\right) 3x^2y^2 = 0 \end{cases}$$

PTI STAZIONARI:

TUTTI i PUNTI del tipo $(x,0)$ e $(0,y)$ (sugli assi)

OPPURE, se $x \neq 0$ e $y \neq 0$ $\begin{cases} 6 - 3/2\, x - y = 0 \\ 6 - x - \frac{4}{3} y = 0 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = 3 \end{cases}$

$(2,3)$ pto stazionario

$$\begin{cases} D_1^2 f = (6 - 3x - y)2y^3 \\ D_{12}^2 f = (6 - 3/2\, x - 4/3\, y) 6xy^2 \\ D_2^2 f = (6 - x - 2y)\, 6x^2 y \end{cases}$$

$$Hf(2,3) = \begin{pmatrix} -586 & -108 \\ -108 & -144 \end{pmatrix} \Leftarrow$$

$\left.\begin{array}{l} \det H > 0 \\ \operatorname{tr} H < 0 \end{array}\right\} \Rightarrow$ MASSIMO ok

e i punti degli assi? SI PUÒ PROSEGUIRE...

(asse y, punti $(0,y)$: per $y < 6$ MINIMI per $y > 6$ MASSIMI
per $y = 6$ né max né min. asse x, punti $(x,0)$: NO