ANALISI II MAT – 2017/2018 – Canale M/Z (A.C. Ponsiglione)
II esonero 17 gen 2018

(1) $\omega(x,y) = (\varphi(y) - y^2)\,dx + (\varphi(y)(x+1) - 2xy)\,dy$, $\Omega = \mathbb{R}^2$

$D_1\omega_2 = \varphi(y) - 2y$
$D_2\omega_1 = \varphi'(y) - 2y$ $\Big] \Leftrightarrow \varphi(y) = \varphi'(y)$

A) $\varphi(y) = 1 \Rightarrow \omega$ CHIUSA $[\varphi \neq \varphi']$ $\boxed{F}$

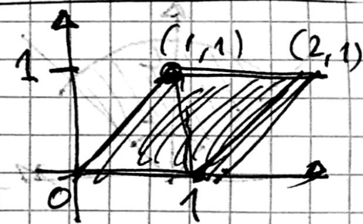
B) $\varphi(y) = e^y \Rightarrow \omega$ ESATTA $[\varphi = \varphi', \Omega$ sempl. conn.$]$ $\boxed{V}$

C) $\varphi = -e^y \Rightarrow \omega$ ESATTA [" "] $\boxed{V}$

D) $\varphi = e^y$, $\gamma = (1+\cos t, \operatorname{sen} t)$, $t \in [0,\pi]$ $\boxed{F}$
$\Rightarrow \int_\gamma \omega = 1$

[ω esatta $= df$: $D_1 f = e^y - y^2 \Rightarrow f = x(e^y - y^2) + C(y)$
$D_2 f = xe^y - 2xy + C'(y) = \omega_2 = e^y(x+1) - 2xy$
$\Rightarrow C' = e^y \Rightarrow C(y) = e^y + C \Rightarrow f = e^y(x+1) - xy^2 + C$.
$\gamma(0) = (2,0)$, $\gamma(\pi) = (0,0)$
$\int_\gamma \omega = f(0,0) - f(2,0) = 1 - 3 = -2$]

(2) $f = xe^y$ $D = \{0 \le y \le 1,\ y \le x,\ y \ge x-1\}$



A) D insieme normale risp. a x $\boxed{V}$

B) f misurabile [è continua] $\boxed{V}$

C) $\int_D f < \infty$ [f limitata su D] $\boxed{V}$

D) $\int_D f = 1/2$ $\boxed{F}$

$\left[\int = \int_0^1 \left(\int_y^{y+1} xe^y\,dx\right)dy = \int_0^1 e^y \frac{x^2}{2}\Big|_{x=y}^{x=y+1} dx = \frac{1}{2}\int_0^1 e^y(2y+1)\,dy\right.$
$= \frac{1}{2}\left(2e^y(y-1) + e^y\right)\Big|_0^1 = \frac{1}{2}(2ye^y - e^y)\Big|_0^1 = \frac{1}{2}(e+1)$]

(3) $\omega = (\alpha y - 2x^2 + 2y^2)\,dx + (\alpha^2 x + 4xy)\,dy$, $\alpha \in \mathbb{R}$, $\Omega = \mathbb{R}^2$

(i) Per quali α ω è CHIUSA?

$D_2\omega_1 = \alpha + 4y$ $D_1\omega_2 = \alpha^2 + 4y$ $\Rightarrow \alpha = \alpha^2 \Rightarrow \alpha = \begin{cases} 0 \\ 1 \end{cases}$

(ii) Per quali α ω è ESATTA?

$\Omega = \mathbb{R}^2$ sempl. connesso quindi ω CHIUSA $\Leftrightarrow$ ESATTA. $\alpha = \begin{cases} 0 \\ 1 \end{cases}$

(iii) Per α tale che ω ESATTA trovare PRIMITIVA

$\boxed{\alpha=0}$: $\omega = 2(y^2-x^2)\,dx + 4xy\,dy = df$

$D_2 f = 4xy \Rightarrow f = 2xy^2 + C(x)$; $D_1 f = \omega_1 \Leftrightarrow$

$\Leftrightarrow 2y^2 + C'(x) = 2y^2 - 2x^2 \Rightarrow C'(x) = -2x^2 \Rightarrow C(x) = -\frac{2x^3}{3} + c$

$\Rightarrow f = 2xy^2 - \frac{2}{3}x^3 + c$.

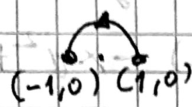
$\boxed{\alpha=1}$: $\omega = (y-2x^2+2y^2)\,dx + (x+4xy)\,dy$

$D_2 f = x + 4xy \Rightarrow f = xy + 2xy^2 + C(x)$

$\Rightarrow y + 2y^2 + C'(x) = y - 2x^2 + 2y^2 \Rightarrow C'(x) = -2x^2$

$\Rightarrow C(x) = -\frac{2}{3}x^3 \Rightarrow f = xy + 2xy^2 - \frac{2}{3}x^3 + c$

(iv) Per $\alpha = 1$, calcolare $\int_\gamma \omega$ dove $\gamma = (\cos t, \operatorname{sen} t)$ e $t \in [0, \pi]$: $\int_\gamma \omega = f(-1,0) - f(1,0) = 4/3$.



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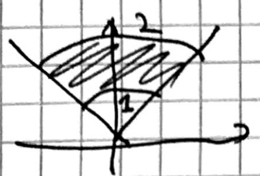
(14) $\iint_D \left(1 + \frac{x}{y^2} + \frac{1}{y}\right) dx\,dy$, $D = \{1 \le x^2+y^2 \le 4,\ y \ge |x|\}$



POLARI $D = \{1 \le r \le 2,\ \frac{\pi}{4} \le \theta \le \frac{3\pi}{4}\}$

$$\iint = \int_{\pi/4}^{3\pi/4} \int_1^2 \left(1 + \frac{r\cos\theta}{r^2 \operatorname{sen}^2\theta} + \frac{1}{r\operatorname{sen}\theta}\right) r\,dr\,d\theta$$

$$= \int_{\pi/4}^{3\pi/4} \left[\frac{r^2}{2}\Big|_1^2 + \frac{\cos\theta}{\operatorname{sen}^2\theta}\cdot(2-1) + \frac{1}{\operatorname{sen}\theta}\cdot(2-1) \right] d\theta$$

$$= \frac{3\pi}{4} + \int_{\pi/4}^{3\pi/4} \left[\frac{\cos\theta}{\operatorname{sen}^2\theta} + \frac{1}{\operatorname{sen}\theta}\right] d\theta$$

$$\int \frac{\cos\theta}{\operatorname{sen}^2\theta}\,d\theta = \int \frac{d(\operatorname{sen}\theta)}{\operatorname{sen}^2\theta} = -\frac{1}{\operatorname{sen}\theta} \Rightarrow \int_{\pi/4}^{3\pi/4} \frac{\cos\theta}{\operatorname{sen}^2\theta}\,d\theta = 0$$

$$\int \frac{d\theta}{\operatorname{sen}\theta} = \int \frac{\operatorname{sen}\theta}{\operatorname{sen}^2\theta}\,d\theta = \int \frac{d(\cos\theta)}{\cos^2\theta - 1} = \int \frac{dt}{t^2-1} =$$

$$= \frac{1}{2}\int \left(\frac{1}{t-1} - \frac{1}{t+1}\right) dt = \frac{1}{2}\log\frac{|t-1|}{|t+1|}$$

$$= \frac{1}{2}\log\left|\frac{\cos\theta - 1}{\cos\theta + 1}\right|$$

$$\Rightarrow \iint = \frac{3\pi}{4} + 0 + \frac{1}{2}\log\left|\frac{\cos\theta-1}{\cos\theta+1}\right| \Bigg|_{\theta=\pi/4}^{\theta=3\pi/4} = \frac{3\pi}{4} + \log\left(\frac{2+\sqrt{2}}{2-\sqrt{2}}\right).$$