Corso di Laurea in Tecnologie per la Conservazione e il Restauro dei Beni Culturali

Esercizi di Matematica – Foglio n.6

LIMITI (PER $x \to x_0^{\pm}$) E DOMINI DI DEFINIZIONE.

Esercizio. Per ognuna delle seguenti funzioni $f(x)$:
(i) determinare il dominio di definizione D_f;
(ii) calcolare i limiti per x tendente ai bordi (da destra e da sinistra) del dominio di definizione (compresi i casi $x \to \pm\infty$).
Se il risultato del limite è zero, decidere se è 0^+ oppure 0^-.

$(i)\ f(x) = \frac{1}{x-x^3}$	$(ii)\ f(x) = \frac{x^2-x}{x^3-x}$	$(iii)\ f(x) = \frac{x^2+2x+1}{x^3+1}$
$(iv)\ f(x) = \frac{2-x^2}{x^3-3}$	$(v)\ f(x) = \frac{x+1}{x-1}$	$(vi)\ f(x) = \frac{x-1}{x^2+1}$
$(vii)\ f(x) = \frac{x^2-3x+2}{x^2-4x+3}$	$(viii)\ f(x) = \sqrt{\frac{x+1}{x-1}}$	$(ix)\ f(x) = \frac{2^{x-3}}{2^{-x}}$
$(x)\ f(x) = x^{-3} - 2^{-x}$	$(xi)\ f(x) = x2^{-x}$	$(xii)\ f(x) = \frac{x^2}{-\ln x}$
$(xiii)\ f(x) = \frac{2^{x-2}}{x-2}$	$(xiv)\ f(x) = \frac{x^3}{2^{1/x}}$	$(xv)\ f(x) = 2^x \ln\lvert x\rvert$
$(xvi)\ f(x) = x\ln x$	$(xvii)\ f(x) = x^2 \ln x$	$(xviii)\ f(x) = \frac{\ln\lvert x\rvert}{\sqrt{x}}$
$(xix)\ f(x) = (\frac{1}{x} + x)^{(x^3)}$	$(xx)\ f(x) = x^x$	$(xxi)\ f(x) = (\frac{1}{2x})^x$
$(xxii)\ f(x) = (\frac{1}{x^2-1})^{\frac{1}{x}+2}$	$(xxiii)\ f(x) = \frac{x^2}{-\ln x}$	$(xxiv)\ f(x) = (\frac{1}{x^2-1})^x$
$(xxv)\ f(x) = (2-\frac{1}{x})^{\ln x}$	$(xxvi)\ f(x) = (\frac{1}{x^2-2})^{\ln(x)}$	$(xxvii)\ f(x) = \frac{\sin x}{x^2}$
$(xxviii)\ f(x) = \frac{x-\sin x}{x}$	$(xxix)\ f(x) = \frac{1-\cos 3x}{x^2}$	$(xxx)\ f(x) = \frac{1-\cos x}{x}$
$(xxxi)\ f(x) = \frac{1-\cos x}{x^2}$	$(xxxii)\ f(x) = \cos(\frac{1}{x})$	$(xxxiii)\ f(x) = \frac{2^x-1}{x}$
$(xxxiv)\ f(x) = \frac{x^\pi-1}{x-1}$	$(xxxv)\ f(x) = (1+x)^{\frac{1}{x}}$	$(xxxvi)\ f(x) = (\frac{1}{x})^{\frac{1}{3}} - \sqrt{\frac{1}{x}}$

Suggerimento: nel caso di forme indeterminate, utilizzare i criteri di confronto tra infiniti/infinitesimi, i limiti notevoli oppure, come extrema ratio, la regola dell'Hôpital.

Nota: ln indica il logaritmo in base e.

Soluzioni (giuste con buona probabilità)

Le soluzioni sono date in quest'ordine: dominio di definizione; punti nei quali ha senso calcolare il limite (a crescere, da $-\infty$ a $+\infty$), valori dei limiti in quei punti.
Se $l_1 = \lim_{x\to x_0^-} f(x) \neq \lim_{x\to x_0^+} f(x) = l_2$, scriveremo il risultato del limite per $x \to x_0$ come una coppia (l_1, l_2). Quando un limite non esiste, riportiamo il simbolo $\nexists$.

	D_f	x_0	$\lim_{x\to x_0^\mp} f(x)$
(i)	$\mathbf{R}\setminus\{\mathbf{-1,0,1}\}$	$-\infty,-1,0,1,+\infty$	$0^+,(-\infty,+\infty),(-\infty,+\infty),(+\infty,-\infty),0^-$
(ii)	$\mathbf{R}\setminus\{\mathbf{-1,0,1}\}$	$-\infty,-1,0,1,+\infty$	$0^-,(-\infty,+\infty),1,\frac{1}{2},0^+$
(iii)	$\mathbf{R}\setminus\{\mathbf{-1}\}$	$-\infty,-1,+\infty$	$0^-,0^\mp,0^+$
(iv)	$\mathbf{R}\setminus\{\sqrt[3]{3}\}$	$-\infty,\sqrt[3]{3},+\infty$	$0^+,(+\infty,-\infty),0^-$
(v)	$\mathbf{R}\setminus\{\mathbf{1}\}$	$-\infty,1,+\infty$	$1,(-\infty,+\infty),1$
(vi)	$\mathbf{R}$	$-\infty,+\infty$	$0^-,0^+$
(vii)	$\mathbf{R}\setminus\{\mathbf{1,3}\}$	$-\infty,1,3,+\infty$	$1,\frac{1}{2},(-\infty,+\infty),1$
(viii)	$(-\infty,-1)\cup(1,+\infty)$	$-\infty,-1^-,1^+,+\infty$	$1,0^+,+\infty,1$
(ix)	$\mathbf{R}$	$-\infty,+\infty$	$0^+,+\infty$
(x)	$\mathbf{R}\setminus\{\mathbf{0}\}$	$-\infty,0,+\infty$	$-\infty,(-\infty,+\infty),0^+$
(xi)	$\mathbf{R}$	$-\infty,+\infty$	$-\infty,0^+$
(xii)	$(0,+\infty)$	$0^+,+\infty$	$0^+,-\infty$
(xiii)	$\mathbf{R}\setminus\{\mathbf{2}\}$	$-\infty,2,+\infty$	$0^-,(-\infty,+\infty),+\infty$
(xiv)	$\mathbf{R}\setminus\{\mathbf{0}\}$	$-\infty,0,+\infty$	$-\infty,(-\infty,0^+),+\infty$
(xv)	$\mathbf{R}$	$-\infty,+\infty$	$+\infty,0^-$
(xvi)	$(0,+\infty)$	$0^+,+\infty$	$0^-,+\infty$
(xvii)	$(0,+\infty)$	$0^+,+\infty$	$+\infty,+\infty$
(xviii)	$(0,+\infty)$	$0^+,+\infty$	$+\infty,0^+$
(xix)	$(0,+\infty)$	$0^+,+\infty$	$1,+\infty$
(xx)	$(0,+\infty)$	$0^+,+\infty$	$1,+\infty$
(xxi)	$(0,+\infty)$	$0^+,+\infty$	$1,0^+$
(xxii)	$(-\infty,-1)\cup(1,+\infty)$	$-\infty,-1^-,1^+,+\infty$	$0^+,+\infty,+\infty,0^+$
(xxiii)	$(0,+\infty)$	$0^+,+\infty$	$0^-,-\infty$
(xxiv)	$(-\infty,-1)\cup(1,+\infty)$	$-\infty,-1^-,1^+,+\infty$	$+\infty,0^+,+\infty,0^+$
(xxv)	$(\frac{1}{2},+\infty)$	$\frac{1}{2},+\infty$	$-\infty,+\infty$
(xxvi)	$(\sqrt{2},+\infty)$	$\sqrt{2},+\infty$	$0^+,0^+$
(xxvii)	$\mathbf{R}\setminus\{\mathbf{0}\}$	$-\infty,0,+\infty$	$0,(-\infty,+\infty),0$
(xxviii)	$\mathbf{R}\setminus\{\mathbf{0}\}$	$-\infty,0,+\infty$	$1,0^+,1$
(xxix)	$\mathbf{R}\setminus\{\mathbf{0}\}$	$-\infty,0,+\infty$	$0^+,\frac{9}{2},0^+$
(xxx)	$\mathbf{R}\setminus\{\mathbf{0}\}$	$-\infty,0,+\infty$	$0^-,0,0^+$
(xxxi)	$\mathbf{R}\setminus\{\mathbf{0}\}$	$-\infty,0,+\infty$	$0^+,\frac{1}{2},0^+$
(xxxii)	$\mathbf{R}\setminus\{\mathbf{0}\}$	$-\infty,0,+\infty$	$1,(\nexists,\nexists),1$
(xxxiii)	$\mathbf{R}\setminus\{\mathbf{0}\}$	$-\infty,0,+\infty$	$0^+,\ln 2,+\infty$
(xxxiv)	$(0,1)\cup(1,+\infty)$	$0^+,1,+\infty$	$1,\pi,+\infty$
(xxxv)	$(-\frac{1}{2},0)\cup(0,+\infty)$	$-\frac{1}{2},0,+\infty$	$+\infty,(\frac{1}{e},e),1$
(xx)	$(0,+\infty)$	$0^+,+\infty$	$-\infty,0$