Esercizi di Istituzioni di Matematica

Esercizio Studiare le seguenti funzioni $f(x)$. Quindi, per ognuna di esse scrivere l'equazione cartesiana della retta r_1 tangente al grafico in $x = 0$ e disegnarla.

$$(i) \qquad f(x) = \sqrt[3]{1-x}$$

$$(ii) \qquad f(x) = \sqrt{\frac{x^2-1}{x^2-4}}$$

$$(iii) \qquad f(x) = x^2 e^{-x}$$

$$(iv) \qquad f(x) = x - \log(1+x)$$

$$(v) \qquad f(x) = x - \arctan(x)$$

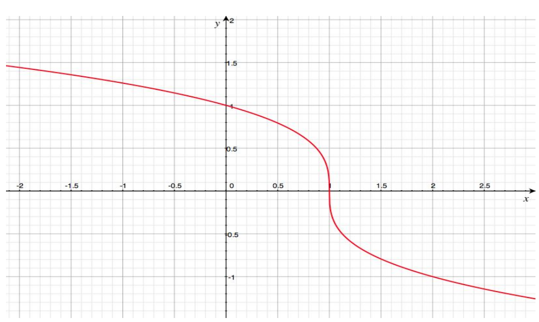
$$(vi) \qquad f(x) = \sin^2(x)$$

Nota: fare sempre una tabella dei segni per f, f', f'' (con gli intervalli di crescita e di convessità di f).

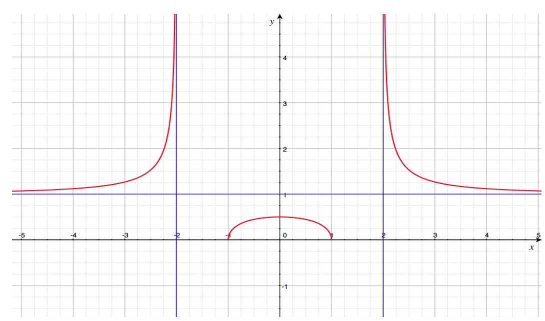
Soluzioni

Le soluzioni sono date come nel foglio 7.

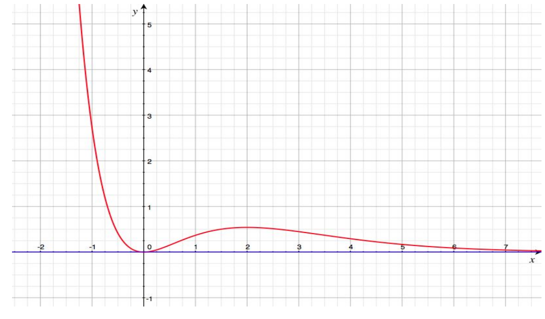
(i) $D_f = \mathbf{R}$
$y = 1$, $x = 1$; $(-\infty, 1)$
$+\infty, -\infty$
$f'(x) = \frac{-1}{3\sqrt[3]{(1-x)^2}}$; $\emptyset$
$f''(x) = \frac{-2}{9\sqrt[3]{(1-x)^5}}$; $(1, +\infty)$
$M = +\infty$; $m = -\infty$
$F : x = 1$
$r_0 : y = -\frac{1}{3}x + 1$



(ii) $D_f = (-\infty, -2) \cup (-1, 1) \cup (2, +\infty)$
$y = \frac{1}{2}$, $x = -1, 1$
$+\infty, -\infty$
a.o. $y = 0$, a.v. $x = 0$
$f'(x) = \frac{-3x}{(x^2-4)^2} \cdot \sqrt{\frac{x^2-4}{x^2-1}}$; $(-\infty, -2) \cup (-1, 0)$
$f''(x) = \frac{3(3x^4-2x^2-4)}{(x^2-1)^2(x^2-4)^2} \cdot \sqrt{\frac{x^2-1}{x^2-4}}$; $(-\infty, -2) \cup (2, +\infty)$
$M = +\infty$, $M_{loc} = f(0) = \frac{1}{2}$; $m = \lim_{x \to \pm 1} f(x) = 0$
$r_0 : y = \frac{1}{2}$



(iii) $D_f = \mathbf{R}$
$y = 0$, $x = 0$; $\mathbf{R} \setminus \{0\}$
$+\infty, 0$
a.o. $y = 0$
$f'(x) = x(2-x)e^{-x}$; $(0, 2)$
$f''(x) = (x^2 - 4x + 2)e^{-x}$;
$(-\infty, 2-\sqrt{2}) \cup (2+\sqrt{2}, +\infty)$
$M = +\infty$, $M_{loc} = f(2) = \frac{4}{e^2}$; $m = f(0) = 0$
$F : x = 2 \pm \sqrt{2}$
$r_0 : y = 0$



(iv) $D_f = (-1, +\infty)$

$y = 0,\ x = 0;\ \mathbf{R} \setminus \{0\}$

$+\infty, +\infty$

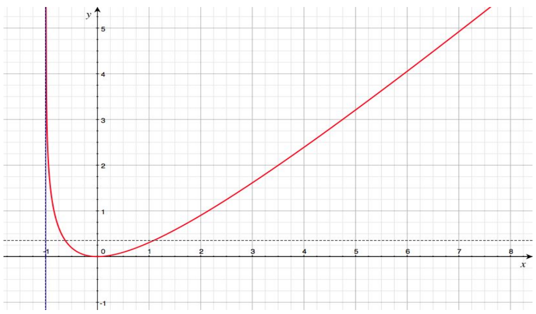
a.v. $x = -1$

$f'(x) = \frac{x}{1+x};\ (0, +\infty)$

$f''(x) = \frac{1}{(1+x)^2};\ (-1, +\infty)$

$M = +\infty;\ m = f(0) = 0$

$r_0 : y = 0$



(v) $D_f = \mathbf{R}$

$y = 0,\ x = 0;\ (0, +\infty)$

$-\infty, +\infty$

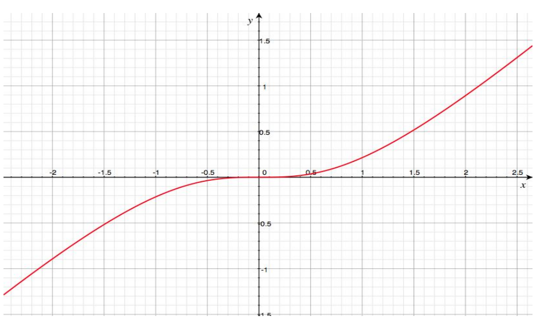
$f'(x) = \frac{x^2}{1+x^2};\ \mathbf{R}$

$f''(x) = \frac{2x}{(1+x^2)^2};\ (0, +\infty)$

$M = +\infty,\ m = -\infty$

$F : x = 0$

$r_0 : y = 0$



(vi) $D_f = \mathbf{R}$

$y = 0,\ x = k\pi;\ \mathbf{R} \setminus \{k\pi\}\ (k \in \mathbf{Z})$

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$f'(x) = 2\sin(x)\cos(x);\ (k\pi, k\pi + \frac{\pi}{2})$

$f''(x) = 2\cos(2x);\ (k\pi - \frac{\pi}{4}, k\pi + \frac{\pi}{4})$

$M = f(k\pi + \frac{\pi}{2}) = 1$

$m = f(k\pi) = 0$

$F : x = k\pi \pm \frac{\pi}{4}$

$r_0 : y = 0$