Advanced Topics in Analysis

Laurea Magistrale in Matematica & Matematica per le Applicazioni

Lecture diary 2024/2025

November 21, 2024

Lecture 1-2 (September 26, 2024) Introduction to the course. Preliminaries to the notion of viscosity sub and supersolution. The vanishing viscosity method to approximate first order HJ equations. Definition of viscosity sub and supersolution for fully nonlinear second order elliptic PDEs.

Lecture 3-4 (October 1, 2024) Some equivalent choices of test functions. The key stability result of the notion of viscosity sub and super solution.

Lecture 5-6 (October 3, 2024) Viscosity solution theory for first order HJ equations: sub and superdifferentials and relation wtih C^1 sub and supertangents; definition of viscosity sub and supersolution via super and subdifferentials; properties of sub and superdifferentials. 1dimensional examples and discontinuity points of the derivative of a viscosity solution to $H(x, u') = 0$ in Ω.

Lecture 7-8 (October 8, 2024) 1dimensional examples and discontinuity points of the derivative of a viscosity solution to $H(x, u') = 0$ in Ω. Comparison principles for a first order HJ equation set on a bounded open set. First comparison principle: statement and some remarks on the assumptions. Proof of the first comparison principle.

Lecture 9-10 (October 10, 2024) Second comparison principle: statement and proof. Third comparison principle: statement.

Lecture 11-12 (October 15, 2024) Third comparison principle: statement and two proofs.

Lecture 13-14 (October 17, 2024) Comparison principles for a first order HJ equation set on the euclidean space $\mathbb{R}^N$.

Lecture 15-16 (October 22, 2024) Existence results for a class of HJ equation set on $\mathbb{R}^N$. Existence of solutions to Dirichlet problems for HJ equations set on a bounded open set.

Lecture 17-18 (October 29, 2024) Clarke generalized gradient in finite dimension. Convex Hamiltonians: equivalence of the notion of viscosity subsolution and almost everywhere subsolution.

Lecture 19-20 (October 31, 2024) Convex Hamiltonians: equivalence among four different notion of subsolution for locally Lipschitz functions; remarks on the stability of the notion of viscosity subsolution for the inf operator. Evolutive H-J equations (with autonomous Hamiltonians): notion of discontinuous viscosity sub

and supersolution. Statement of the comparison principle. Statement and proof of the existence of solutions.

Lecture 21-22 (November 5, 2024) Evolutive H-J equations (with autonomous Hamiltonians): end of the proof of the existence of solutions. Convex Analysis: definition of convex function; convexity vs. continuity/Lipschitz continuity.

Lecture 23-24 (November 7, 2024) Subdifferentials of a convex function. Local minima of a convex function are global minima. The finite dimensional case: properties of the subdifferential.

Lecture 25-26 (November 12, 2024) Subdifferential of a convex function vs. Clarke's generalized gradient. Fenchel transform and its properties. Bi-polar conjugate.

Lecture 27-28 (November 14, 2024) Fenchel's inequality. Relation between the strict convexity of H and the C^1 regularity of H^*. The Lax-Oleinik formula and the value function: motivation of the formula. The value function is a viscosity subsolution of the associated evolutive H-J equation.

Lecture 29-30 (November 19, 2024) Lipschitz regularity of the value function.

Lecture 31-32 (November 21, 2024) Lipschitz regularity of the value function. Existence of Lagrangian minimizers.