INTERACTIONS BETWEEN MATHEMATICS AND PHYSICS

LIE THEORY AND REPRESENTATION THEORY

ALIAS: VITTORIO GATTI

Title and abstracts of talks

- **Drazen Adamović** (University of Zagreb)
  On the semi-simplicity of the category KL for affine vertex algebras at collapsing levels

  Abstract. In this talk we will report on recent results on the representation theory of simple affine vertex algebra $V_k(\mathfrak{g})$ at collapsing levels. When k is an admissible rational number and $\mathfrak{g}$ is a Lie algebra, then each $V_k(\mathfrak{g})$–module in the category $\mathcal{O}$ is completely reducible by the result of Arakawa. But it turned out that the Kazhdan-Lusztig category KL_k of $V_k(\mathfrak{g})$–modules can be also semi-simple for non-admissible levels k. We will present an approach which uses the representation theory of minimal affine W-algebras and the quantum Hamiltonian reduction functor. We proved this in joint papers with V. Kac, P. Möseneder Frajria, P. Papi and O. Perse that KL_k is semi-simple when the affine W-algebra $W_k^{min}(\mathfrak{g})$ is rational or when k is collapsing level for $W_k^{min}(\mathfrak{g})$. This result enables us to prove complete reducibility of modules in KL_k for some non-admissible levels. In the case when $\mathfrak{g}$ is a Lie superalgebra, the analysis of the category KL_k is subtler than in the Lie algebra case, since in the super case KL_k can contain indecomposable modules. But even in this case we have a complete reducibility result for collapsing levels. Finally we present some conjectures based on recent classifications of conformal and collapsing levels for some non-minimal affine W-algebras.

- **Tomoyuki Arakawa** (RIMS, Kyoto university) *Hilbert schemes of points on the plane and quasi-liise vertex algebras with N=4 symmetry.*

  Abstract. Motivated by the 4D/2D duality, Bonetti, Menegheli and Rastelli conjectured the existence of extensions of N=4 superconformal algebras labeled by Coxeter groups, whose associated varieties are isomorphic to the canonical symplectic varieties associated with the natural representations of the groups. We prove this conjecture when the Coxeter group is a symmetric group, by chiralizing the Hilbert schemes of points on the plane. This is a joint work with Toshoro Kuwabara and Sven Möller.

- **Lucia Bagnoli** (University of Zagreb)
  On the finite irreducible modules over the conformal superalgebras K_4' and CK_6

  In this talk we recall the definition and main properties of conformal superalgebras. Then we present the classification of all the finite irreducible modules over the conformal superalgebra K_4' obtained by their correspondence with finite conformal modules over the associated annihilation superalgebra $\mathcal{A}(K_4')$. This is achieved by a

complete classification of singular vectors in generalized Verma modules for $\mathcal{A}(K_4')$. We also show that morphisms between generalized Verma modules can be arranged in infinitely many bilateral complexes. This classification is a joint work with F. Caselli. Then we present a result on the homology of these complexes that provides an explicit realization of all irreducible quotients. Next we present some results on the conformal superalgebra CK_6: the first one provides a bound on the degree of singular vectors of finite Verma modules over the exceptional Lie superalgebra $E(1,6)$ that is isomorphic to the annihilation superalgebra associated with the conformal superalgebra CK_6; the second is the computation of the homology of the first and third quadrants of the complexes of finite Verma modules, that were classified by Boyallian, Kac and Liberati, over the annihilation superalgebra $\mathcal{A}(CK_6) = E(1,6)$. The computation of the homology provides an explicit realization of all irreducible quotients. Finally, we discuss some open problems.

- **Bojko Bakalov** (North Carolina State University)
  Logarithmic vertex algebras and non-local Poisson vertex algebras

Abstract. We introduce the notion of a logarithmic vertex algebra, which is a vertex algebra with logarithmic singularities in the operator product expansion of quantum fields; thus providing a rigorous formulation of the algebraic properties of quantum fields in logarithmic conformal field theory. Many results about vertex algebras can be extended to logarithmic vertex algebras. In particular, we show that the associated graded vector space of a filtered logarithmic vertex algebra has an induced structure of a non-local Poisson vertex algebra, a notion introduced previously by De Sole and Kac in relation to integrable systems. This talk is based on joint work with Juan Villarreal.

- **Roman Bezrukavnikov** (MIT) *Commuting pairs and unipotent characters*

Abstract. I will report on a joint project with Dan Ciubotaru, David Kazhdan and Yakov Varshavsky, in which we describe unipotent invariant distributions on a p-adic group in terms of the Langlands dual group. Time permitting, I will also discuss an alternative approach to the proof of a somewhat weaker result based on considering the asymptotic Hecke algebra and its geometric realization (joint with Ivan Karpov and Vasily Krylov).

- **Luca Casarin** (Università di Roma La Sapienza)

  A Feigin-Frenkel theorem with n singularities

Abstract. It is now a well know theorem by Feigin-Frenkel that the center of the completed enveloping algebra of the affine algebra $\hat{\mathfrak{g}}_k$ at the critical level is canonically isomorphic to the algebra of functions on the space of Opers over the pointed formal disc $Op_{G^L}(D^*)$. Starting from the work of Fortuna, Lombardo, Maffei and Melani I will introduce an analogue of the affine algebra with n singularities. We then proceed to discuss an analogue of the Feigin-Frenkel theorem in this new setting, which establishes an isomorphism with the center of the completed enveloping algebra in the case with n singularities with the algebra of functions on the space of Opers over the n-pointed formal disc $Op_{G^L}(D_n^*)$. I will focus on the main ingredients of the proof and the various compatibilities that these isomorphisms satisfy with respect to the original Feigin-Frenkel isomorphism.

- **Fabrizio Caselli (Università** di Bologna) *Lie conformal superalgebras and duality of representations of linearly compact Lie superalgebras*

  Abstract. I will introduce Lie conformal superalgebras and show how one can deduce a duality of modules over the corresponding annihilation Lie superalgebras. I will discuss in some detail the cases of exceptional Lie superalgebras E(5,10) and E(4,4).

- **Thibauld Damour** (IHES)
  Hidden Hyperbolic Kac-Moody Structures in Supergravity

  Abstract. A brief review will be presented of the various indications of the presence of hyperbolic Kac-Moody structures in supergravity. The recent results (obtained with Philippe Spindel) concerning the presence of $K(G_2^{++})$ in the Fermionic sector of $D = 5$ supergravity will be summarized.

- **Ben Davison** (University of Edinburgh)
  Kac polynomials and generalised Kac-Moody algebras

  Abstract. Victor Kac proved that for an arbitrary finite directed graph Q, and dimension vector d, the number of isomorphism classes of absolutely irreducible Q-representations with dimension vector dover a field of order q is a polynomial in q with integer coefficients: the Kac polynomial. He moreover conjectured that the constant term is the dimension of the dth weight space of an associated Kac-Moody Lie algebra, and all of the coefficients are positive. These conjectures were later proved by Hausel, and Hausel, Letellier and Rodriguez-Villegas, respectively. In this talk I will explain a recent result, which strengthens both of the above results: the entire Kac polynomial is the characteristic function of the dth weight space of a cohomologically graded, generalised Kac-Moody algebra. This result has numerous implications, among which is a decomposition of the entire cohomology of Nakajima quiver varieties (i.e. in all cohomological degrees) into irreducible lowest weight modules for the above generalised Kac-Moody algebra, with spaces of lowest weight vectors identified with intersection cohomology of singular quiver varieties. This is joint work with Hennecart and Schlegel Mejia.

- **Pavel Etingof** (MIT)
  Lie theory in tensor categories with applications to modular representation theory

  Abstract. Let G be a group and k an algebraically closed field of characteristic $p > 0$. If V is a finite dimensional representation of G over k, then by the classical Krull-Schmidt theorem, the tensor power $V^{\otimes n}$ can be uniquely decomposed into a direct sum of indecomposable representations. But we know very little about this decomposition, even for very small groups, such as $G = (\mathbb{Z}/2)^3$ for $p = 2$ or $G = (\mathbb{Z}/3)^2$ for $p = 3$. For example, what can we say about the *number* $d_n(V)$ of such summands of dimension coprime to p? It is easy to show that there exists a finite limit $d(V) := \lim_{n\to\infty} d_n(V)^{1/n}$, but what kind of number is it? For example, is it algebraic or transcendental? Until recently, there was no techniques to solve such questions (and in particular the same question about the *sum of dimensions* of these summands is still wide open). Remarkably, a new subject which may be called "Lie theory in tensor categories" gives methods to show that $d(V)$ is indeed an algebraic number, which moreover has the form

$$d(V) = \sum_{1\le j\le p/2} n_j(V)[j]_q,$$

where $n_j(V) \in \mathbb{N}$, $q := \exp(\pi i/p)$, and $[j]_q := \frac{q^j - q^{-j}}{q-q^{-1}}$. Moreover,

$$d(V \oplus W) = d(V) + d(W),\ d(V \otimes W) = d(V)d(W),$$

i.e., d is a character of the Green ring of G over k. Furthermore,

$$d_n(V) \ge C_V d(V)^n$$

for some $0 < C_V \le 1$ and we can give lower bounds for C_V. In the talk I will explain what Lie theory in tensor categories is and how it can be applied to such problems. This is joint work with K. Coulembier and V. Ostrik.

- **Maria Gorelik** (The Weizmann Institute of Science)

  On the simplicity of minimal W-algebras.

  Abstract. This is a report on an on-going project, joint with V. Kac, concerning simplicity of minimal W-algebras. Minimal W-algebras are the simplest conformal vertex algebras. For non-integral central levels the simplicity criterion was established 15 years ago in papers by Gorelik-Kac and Hoyt-Reif. The case of integral central levels for superalgebras of non-zero defect is still open, and I will report on recent progress in this area.

- **Reimundo Heluani** (IMPA)

  PBW bases of Ising modules

  Abstract. We describe PBW bases of the unique three irreducible modules of the Virasoro Lie algebra with central charge c = 1/2. We use these bases to find new bi-variable character formulas for these modules and describe new Rogers-Ramanujan-type identities from them. This is a report on the thesis of Diego Salazar Gutierrez (IMPA).

- **Vasily Krylov** (MIT)

  Subregular nilpotent orbits and explicit character formulas for modules over affine Lie algebras.

  Abstract. The talk is based on the joint work with Roman Bezrukavnikov and Victor Kac (arXiv:2209.08865). Let $\mathfrak{g}$ be a simple Lie algebra and let $\hat{\mathfrak{g}}$ be the corresponding affine Lie algebra. It is known that characters of irreducible (highest weight) representations of $\hat{\mathfrak{g}}$ can be computed in terms of values at q=1 of affine (inverse) Kazhdan-Lusztig polynomials. These values can be computed recursively, but there are no explicit formulas for them in general. The goal of this talk is to describe certain cases when we can compute the values above explicitly resulting in explicit formulas for characters of certain irreducible $\hat{\mathfrak{g}}$-modules (partly generalizing results of Kac and Wakimoto).

- **Roberto Longo** (Università di Roma Tor Vergata) *Signal communication and modular theory*

  Abstract. I propose a conceptual frame to interpret the prolate differential operator, which appears in Communication Theory, as an entropy operator; indeed, I write its expectation values as a sum of terms, each subject to an entropy reading by an embedding suggested by Quantum Field Theory. This adds meaning to the classical work by Slepian et al. on the problem of simultaneously concentrating a function and its Fourier transform, in particular to the "lucky accident" that the truncated Fourier transform commutes with the prolate operator. The key is the notion of entropy of a vector of a complex Hilbert space with respect to a real linear subspace, recently introduced by the author by means of the Tomita-Takesaki modular theory of von Neumann algebras, and studied in collaboration with Ciolli, Morsella, and Ruzzi. I consider a generalization of the prolate operator to the higher dimensional case and show that it admits a natural extension commuting with the truncated Fourier transform; this partly generalizes the one-dimensional result by Connes to the effect that there exists a natural selfadjoint extension to the full line commuting with the truncated Fourier transform.

- **Anne Moreau** (Université Paris Saclay)
  Functorial constructions of double Poisson vertex algebras

  Abstract. The notion of double Poisson vertex algebras was introduced by De Sole, Kac and Valeri. To any double Poisson algebra we produce a double Poisson vertex algebra using the jet algebra construction. We show that this construction is compatible with the representation functor which associates to any double Poisson (vertex) algebra and any positive integer a Poisson (vertex) algebra. We also consider related constructions, such as Poisson reductions and Hamiltonian reductions. This allows us to provide various interesting examples of double Poisson vertex algebras, in particular from double quivers. This is joint work in progress with Tristan Bozec and Maxime Fairon.

- **Pierluigi Möseneder Frajria** (Politecnico di Milano)
  Unitary minimal W-algebras

  Abstract: we classify the unitary minimal W-algebras and, up to a few extremal cases, their unitary highest weight modules.

- **Claudio Procesi** (Università di Roma La Sapienza)

  Swap polynomials and tensor polynomial identities

  Abstract. Swap polynomials is a concept coming from Quantum Information Theory, I will show how to connect this to the central polynomials of Formanek-Regev and the Formanek-Weingarten function.

- **Daniele Valeri** (Università di Roma La Sapienza)
  Integrability of classical affine W-algebras

  Abstract. Classical affine W-algebras $W(\mathfrak{g}, \mathcal{O})$ are algebraic structures associated to a simple Lie algebra g and a nilpotent orbit $\mathcal{O}$. In this talk we will first review the definition of classical affine W-algebras within the framework of Poisson vertex algebras. Then, for each W(g,O) (aside for seven nilpotent orbits for exceptional Lie algebras) we will show how to construct a suitable infinite dimensional abelian Lie algebra. This Lie algebra allows to define an integrable hierarchy: infinitely many evolutionary partial differential equations whose flows pairwise commute. When $\mathcal{O}$ is the principal nilpotent orbit one gets the Drinfeld-Sokolov hierarchy, which gives the famous Korteweg-de Vries hierarchy for $\mathfrak{g} = sl_2$. The talk is based on joint works with Alberto De Sole, Mamuka Jibladze and Victor G. Kac (alias Vittorio Gatti).

- **Jethro van Ekeren** (Universidade Federal Fluminense)
  Moonshine, the Leech lattice, and Victor's very strange formula

  Abstract. The rank of a unimodular even integral lattice must be divisible by 8, and the classification of such lattices of rank 24 is especially intriguing. There are exactly 24 such lattices, indexed by 24 distinct root systems, and all "controlled" in a certain sense by a special lattice known as the Leech lattice. In this talk I will describe a similar story for the natural generalisation of unimodular lattices to holomorphic vertex algebras. We find a conceptual construction and classification of such algebras in rank 24, controlled in a similar sense by the Leech lattice. A key role is played by the very strange formula of Victor Kac, tying together ideas from Lie algebras and modular forms. This talk is based on joint work with various subsets of: C.-H. Lam, S. Moeller, N. Scheithauer and H. Shimakura.

- **Efim Zelmanov** (Southern University of Science and Technology, Shenzhen, China).
  Superconformal algebras on affine varieties.

  Abstract. Let R be an affine commutative superalgebra. Let $G(n)$ be the Grassmann algebra on n Grassmann variables. Let $A(R:n) = R \otimes G(n)$. If the algebra R is equipped with a contact bracket then this bracket and the Poisson bracket on $G(n)$ extend to a contact bracket [,]$]on$A(R:n)$.ThisgivesrisetotheLiesuperalgebra$K(R:n) $=$ (A(R:n), [,])$.Ifdisanevenderivationof Rthen$CK(R,d) is the corresponding Cheng-Kac superalgebra (see [2, 3]). We show that except for some small cases the universal central extensions of superalgebras $W(R:n) = Der A(R:n), [K(R:n), K(R:n)], CK(R,d)$ are finitely presented. This and the results of V.G.Kac and J. Van de Leur [1] imply that all known superconformal algebras are finitely presented.

  1. V. G. Kac and J. W. van de Leur, On classification of superconformal algebras, Strings '88 (College Park, MD, 1988), World Sci. Publ., Teaneck, NJ, 1989, pp. 77-106.

  2. S.J. Cheng and V.G. Kac, A new N = 6 superconformal algebra, Comm. Math. Phys 186 (1997), n. 1, 219-231.

  3. C. Martinez nd E. Zelmanov, Simple finite-dimensional Jordan superalgebras in prime characteristic , Journal of Algebra 236 (2001), no.2, 575-629.

- **Gui Zhengping** (ICTP)
  Quadratic Duality for Chiral Algebras
  Abstract. The notion of Koszul duality is ubiquitous in mathematics. In the original context of quadratic algebra, it can be understood as the quadratic duality studied by Stewart Priddy. In this talk, we introduce a notion of quadratic duality for chiral algebras. Subsequently, we explain the relationship between this duality notion and the Maurer-Cartan equations for chiral algebras, which turns out to be parallel to the associative algebra case. We will also present some explicit examples. This talk is based on joint work with Si Li and Keyou Zeng.