Variational Problems from Micromagnetics

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Abstract

Micromagnetics provides a rich source of mathematical problems and physical phenomena. By focusing on this application area, my lectures will address – in a single physical framework – a broad variety of topics including (a) relaxation, (b) wall structures and wall energies, and (c) surface energy as a selection mechanism. Material will largely be drawn from the recent review article [1].

I'll begin by introducing the micromagnetic energy and some basic phenomenology, focusing on soft thin film elements. I'll discuss the origin of domain walls and the very different behavior of small vs large thin films. I'll also discuss how hysteresis reflects the existence of many local minima, and the role of relaxation in explaining the domain structure of a large ferromagnet.

Then I'll turn to problems involving the internal structure of walls. This topic has seen a lot of progress recently. A key question is to understand why some walls are one-dimensional (as in the Modica-Mortola problem), but others are not (for example the cross-tie wall, seen in relatively thick films). I'll discuss, in particular, the striking work by Alouges, Riviere, and Serfaty which explains the internal structure of the cross-tie wall.

The third lecture will turn to my work with DeSimone, Müller, and Otto on relatively large thin films. In this setting the exchange energy is a singular perturbation, and the asymptotic (Gamma-limit) variational problem has the relaxed constraint $|m| \leq 1$ rather than the usual constraint constraint $|m| = 1$. The solution m is nonunique, but its divergence $\operatorname{div} m$ is unique. It is natural to suppose that the wall energy acts as a singular perturbation to select a special choice of m; this is closely related to recent work on the Aviles-Giga problem [2].

The last lecture will turn to the minimization of action rather than energy, drawing from my recent work with F. Otto, M. Reznikoff, Y. Tonegawa and E. Vanden-Eijnden [3]. Here the focus is on switching due to thermal

fluctuation. According to large deviation theory, the switching pathways minimize a suitable action. The analysis of its sharp-interface limit is related to the classical Modica-Modica example, but requires rather different methods and many questions remain open.

References

[1] A. DeSimone, R. Kohn, S. Müller, and F. Otto, *Recent analytical developments in micromagnetics*, MPI-MIS Preprint 80/2004, available at http://www.math.nyu.edu/faculty/kohn and also on the MPI-MIS web page. This will be published soon as a chapter in the book The Science of Hysteresis, edited by G. Bertotti and I. Mayerogoyz, Elsevier.

[2] For example W. Jin and R. Kohn, *Singular perturbation and the energy of folds,* J. Nonlin. Science 10 (2000) 355-390 and subsequent work by Otto, DeLellis, and others.

[3] R. Kohn, F. Otto, M. Reznikoff, and E. Vanden-Eijnden, *Action minimization and sharp interface limits for the stochastic Allen-Cahn equation,* to appear in Comm. Pure Appl. Math.; also R. Kohn, M. Reznikoff and Y. Tonegawa, *The sharp interface limit of the action fucntional for Allen-Cahn in one space dimension,* to appear in Calc. Var. PDE. Preprints of both are posted at http://www.math.nyu.edu/faculty/kohn/.