Variational Integrals and n-Harmonic Mappings

An Invitation to Geometric Analysis and Nonlinear PDEs

Tadeusz Iwaniec

(Syracuse University)

Abstract

Our primary theme running throughout these lectures is about the variational integrals of the form

$$\Xi[f] \;=\; \int_\Omega \mathbf{E}(x, \mathfrak{D}f)\, \mathrm{d}x$$

defined on functions, vector fields, tensors and so forth. Here $\mathfrak{D}$ is a linear differential operator and the integrand $\mathbf{E}(x, \cdot)$ is subjected to various convexity conditions. The intent is first to provide a comprehensive account of modern approaches to nonconvex variational problems. This leads us to Null Lagrangians as building blocks of so-called polyconvex functionals as well as to the concept of rank-one connections and quasiconvexity by C.B. Morrey. Second, is the application of harmonic analysis to nonlinear differential operators, such as the Jacobian determinants and more general null Lagrangians. In particular, we shall discuss complex interpolation of nonlinear commutators of singular integrals and $\mathcal{H}^1$- theory of nonlinear elliptic PDEs. The benefit is a greater understanding of the very weak solutions of PDEs. Among them are n-harmonic mappings and deformations of finite distortion. We shall make use of Sobolev spaces and differential forms. The lectures are largely focus on questions for new scholars with the understanding that the further development of the methods will rest on them. Every effort will be made to reduce to a minimum the technical aspects of the subject, so that the lectures will be suitable to all graduate students.