Modeling and Complexity Reduction in PDEs for Multiphysics

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The numerical solution of complex physical problems often requires a great deal of computational resources.

Sometimes, the numerical problem is so large that a reduction of its complexity becomes mandatory. This can be achieved by a manifold strategy with the attempt of simplifying the original mathematical model, devising novel numerical approximation methods, developing efficient parallel algorithms that exploit the dimensional reduction paradigm.

Three possible approaches along this direction are illustrated below:

1. Mathematical models of complex physical problems can be based on heterogeneous partial differential equations (PDEs), i.e. on boundary-value problems of different kind in different subregions of the computational domain.

2. On the other hand, in different circumstances, especially in control and optimization problems for parametrized PDEs, reduced order models such as the reduced basis (RB) method can be used to alleviate the computational complexity by creating an offline-online cascade of reduced problems.

3. Finally, when addressing the algebraic problem that arises from the numerical approximation, efficient parallel algorithms may be developed by restricting the given problem to the interface separating the physical regions in which different kind of physical processes occur (like, e.g. in fluid-structure interaction problems).

After introducing some illustrative examples, in this serie of lectures several approaches will be proposed and a few representative applications to blood flow modeling, sports design, and the environment will be addressed.