John M. Ball

Mathematical Problems of Liquid Crystals

Abstract

The lectures will survey mathematical models for static configurations of liquid crystals. Such configurations are often modelled using the Oseen-Frank theory, in which the average orientation of the rod-like molecules is represented by a unit director vector field. On the other hand, the Landau - de Gennes model represents the orientation by a tensor that respects head-to-tail invariance. The connections between these theories will be explored.

Stefan Müller

Rigidity in nonlinear elasticity and the derivation of lower dimensional theories

Abstract

A fundamental problem in elasticity is to derive theories for lower dimensional objects such as plates, shells or rods from the fully nonlinear three dimensional theory. The usual approach is to make certain assumptions on the three dimensional solutions and then to deduce a lower dimensional theory by formal or rigorous asymptotical analysis. These has lead to large variety of theories, which are sometimes not mutually compatible.

Since the early 90's a new, mathematically rigorous, approach has emerged, which is based on the variational principle and the associated notion of Γ convergence. Le Dret and Raoult have used Γ convergence to derive a theory for elastic membranes (these have only stretching stiffness, but no bending stiffness and cannot resist compression). In these lectures I will report on ongoing work with G. Friesecke (Munich/ Warwick) and R.D. James (Minnesota) to derive a full hierarchy of limiting theories, which are distinguished by the scaling of the elastic energy as a function of thickness. In particular I will discuss the derivation of Kirchhoff's plate theory (which captures bending) and the much debated von Kármán theory.

A key mathematical ingredient is a quantitative rigidity estimate which generalizes results of F. John for deformations with small nonlinear strain. A classical result says that any Lipschitz map from a (bounded) connected set in $\mathbf{R}^n$ to $\mathbf{R}^n$ (we are interested in $n \geq 2$) whose derivative is an element of $SO(n)$ a.e. has in fact constant derivative. The quantative rigidity estimate says that this can be extended to a linear estimate in L^2. More precisely for every $u \in W^{1,2}$ and every Lipschitz domain Ω we have

$$\min_{Q \in SO(n)} \int_\Omega |\nabla u - Q|^2 \leq C(\Omega) \int_\Omega \operatorname{dist}^2(\nabla, SO(n)). \tag{1}$$

Vladimir Sverak

Aspects of Navier-Stokes and certain model equations

Abstract

The lectures will focus on incompressible Navier-Stokes equations. The solutions of the equations exhibit a rich variety of behaviors, some of which are not easy to understand. In the lectures I will explain some of the known results about the equations. I will also introduce simpler model equations which illustrate some of the mathematical difficulties arising in the study of Navier-Stokes.