

Calcolo delle Probabilità a.a. 2016-2017
soluzioni prova scritta del 19-1-2017

M. Isopi

Esercizio 1.

- a) $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B) - \mathbf{P}(A \cap B)$. Quindi $\mathbf{P}(A) + \mathbf{P}(B) = \frac{17}{15}$
- b) Scegliendo $A \supset B$, abbiamo $\mathbf{P}(A) = \frac{4}{5}$ e con $A \subset B$, abbiamo $\mathbf{P}(A) = \frac{1}{3}$
- c) $\mathbf{P}(A) = \mathbf{P}(A \cap B^C) + \mathbf{P}(A \cap B)$.
Quindi i valori massimo e minimo di $\mathbf{P}(A \cap B^C)$ sono $\frac{7}{15}$ e 0.
- d) $\frac{34}{45}$
- e) $\mathbf{P}(A^C \cup B^C) = \mathbf{P}((A \cap B)^C) = \frac{2}{3}$
- f) $\mathbf{P}(A^C \cap B^C) = \mathbf{P}((A \cup B)^C) = \frac{1}{5}$

Esercizio 2.

Ovviamente $\mathbf{E}(X) = \frac{5}{2}$; $\mathbf{E}(X^2) = \frac{15}{2}$

- a) $\mathbf{P}(B) = \sum_{k=1}^4 \mathbf{P}(B|X=k) \mathbf{P}(X=k) = \frac{1}{4} \sum_{k=1}^4 \frac{k}{6} = \frac{5}{12}$
- b) $\mathbf{P}(X=j|B) = \mathbf{P}(B|X=j) \frac{\mathbf{P}(X=j)}{\mathbf{P}(B)} = \frac{j \cdot \frac{1}{4}}{6 \cdot \frac{5}{12}} = \frac{j}{10}$
 $\mathbf{E}(X|B) = \frac{1}{4} \sum_{j=1}^4 j \cdot \mathbf{P}(X=j|B) = \frac{1}{10} \sum_{j=1}^4 j^2 = 3$
- c) $\mathbf{P}(Y=0|X) = \frac{(6-X)(5-X)}{30}$; $\mathbf{P}(Y=1|X) = 2 \frac{X(6-X)}{30}$;
 $\mathbf{P}(Y=2|X) = \frac{X(X-1)}{30}$
 $\mathbf{E}(Y|X) = \mathbf{P}(Y=1|X) + 2\mathbf{P}(Y=2|X) = \frac{X}{3}$
 $\mathbf{E}(Y) = \mathbf{E}(\mathbf{E}(Y|X)) = \mathbf{E}\left(\frac{X}{3}\right) = \frac{5}{6}$
 $\mathbf{E}(XY) = \mathbf{E}(\mathbf{E}(XY|X)) = \mathbf{E}(X\mathbf{E}(Y|X)) = \mathbf{E}\left(\frac{X^2}{3}\right) = \frac{5}{2}$
 $\text{Cov}(X, Y) = \mathbf{E}(XY) - \mathbf{E}(X)\mathbf{E}(Y) = \frac{5}{12}$