

Calcolo delle Probabilità a.a. 2016-2017
prova scritta del 16-6-2017

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Soluzione esercizio 1.

a) $\mathbf{P}(2^{\text{a}}\text{cuori} \mid 1^{\text{a}}\text{cuori}) = \frac{12}{51} = \frac{4}{17}$

b) $\mathbf{P}(2^{\text{a}}\text{nera} \mid 1^{\text{a}}\text{picche}) = \frac{25}{51}$

c)

$$\begin{aligned}\mathbf{P}(\text{asso e quattro}) &= \mathbf{P}(\text{asso poi quattro}) + \mathbf{P}(\text{quattro poi asso}) = \\ 2\mathbf{P}(\text{asso poi quattro}) &= 2 \cdot \frac{1}{13} \cdot \frac{4}{51} = \frac{8}{663}\end{aligned}$$

d) $\mathbf{P}(\text{due picche}) = \frac{\binom{13}{2}}{\binom{52}{2}} = \frac{1}{17}$

Soluzione esercizio 2.

a)

$$\begin{aligned}\mathbf{P}(X \geq 3) &= 1 - \mathbf{P}(X = 0) - \mathbf{P}(X = 1) - \mathbf{P}(X = 2) = \\ &= 1 - e^{-3} - 3e^{-3} - \frac{9}{2}e^{-3} = 1 - \frac{17}{2}e^{-3} \simeq 0.577\end{aligned}$$

b)

$$\begin{aligned}\mathbf{P}(X \geq 3 | X \geq 2) &= \frac{\mathbf{P}(X \geq 3 \cap X \geq 2)}{\mathbf{P}(X \geq 2)} = \frac{\mathbf{P}(X \geq 2)}{\mathbf{P}(X \geq 1)} = \\ &= \frac{1 - e^{-3} - 3e^{-3} - \frac{9}{2}e^{-3}}{1 - e^{-3} - 3e^{-3}} = \frac{e^3 - \frac{17}{2}}{e^3 - 4} \simeq 0.72\end{aligned}$$

c)

$$\begin{aligned}\mathbf{E}(X | X \geq 2) &= \sum_{k=1}^{k=\infty} k \cdot \mathbf{P}(X = k | X \geq 2) = \sum_{k=1}^{k=\infty} k \cdot \frac{\mathbf{P}(X = k)}{\mathbf{P}(X \geq 2)} = \\ &= \frac{\sum_{k=1}^{k=\infty} k \cdot \mathbf{P}(X = k)}{\mathbf{P}(X \geq 2)} = \frac{3}{1 - 4e^{-3}}\end{aligned}$$

Esercizio 3.

Sia E l'evento "il dado è equilibrato". $\mathbf{P}(E) = \frac{2}{3}$. Abbiamo

$$\mathbf{P}(X = 1) = \mathbf{P}(X = 1|E)\mathbf{P}(E) + \mathbf{P}(X = 1|E^C)\mathbf{P}(E^C) = \frac{1}{6} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3} = \frac{5}{18}$$

e per $j = 2, \dots, 6$

$$\mathbf{P}(X = j) = \mathbf{P}(X = j|E)\mathbf{P}(E) + \mathbf{P}(X = j|E^C)\mathbf{P}(E^C) = \frac{1}{6} \cdot \frac{2}{3} + \frac{1}{10} \cdot \frac{1}{3} = \frac{13}{90}$$

a)

$$\mathbf{P}(X \geq 5) = \mathbf{P}(X = 5) + \mathbf{P}(X = 6) = \frac{13}{45}$$

b)

$$\mathbf{E}(X) = \sum_{j=1}^6 j\mathbf{P}(X = j) = \frac{5}{18} + (2 + 3 + 4 + 5 + 6)\frac{13}{90} = \frac{19}{6}$$

$$\mathbf{E}(X^2) = \sum_{j=1}^6 j^2\mathbf{P}(X = j) = \frac{5}{18} + (4 + 9 + 16 + 25 + 36)\frac{13}{90} = \frac{239}{18}$$

$$\mathbf{Var}(X) = \frac{13}{4}$$